

41 (a) At intersection A incoming traffic is equal to $x + 5$. The outgoing traffic is given by $y + 7$. Therefore, $x + 5 = y + 7$, which is the first equation. The incoming traffic at B is $z + 6$ and the outgoing traffic is $x + 3$, so $z + 6 = x + 3$. Finally at intersection C, the incoming flow is $y + 3$ and the outgoing flow is $z + 4$, so $y + 3 = z + 4$.

(b) These three equations can be written as

$$x - y = 2$$

$$x - z = 3$$

$$y - z = 1$$

This system of linear equations can be represented by the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 3 \\ 0 & 1 & -1 & 1 \end{array} \right]$$

Begin by subtracting the first row from the second, followed by subtracting the second row from the third. Gaussian elimination results in the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The last row of zeros indicates that the linear system is dependent and has an infinite number of solutions. Back solving produces $y - z = 1 \Rightarrow y = z + 1$. Substituting into the first equation gives $x - (z + 1) = 2 \Rightarrow x = z + 3$. Thus, the solution can be written as

$$\{(z + 3, z + 1, z) \mid z \text{ is any nonnegative real number}\}.$$

(c) There are an infinite number of solutions to the system. However, solutions such as $z = 1000$, $x = 1003$, and $y = 1001$ are unlikely, unless a large number of people are simply driving around the block. In reality there is an average traffic flow rate for z that could be measured. From this, values for both x and y could be determined.