

19 This triangle is of the form SSA. It is ambiguous.

$$\frac{\sin \beta}{b} = \frac{\sin \alpha}{a} \Rightarrow \sin \beta = \frac{b \sin \alpha}{a} = \frac{9 \sin 20^\circ}{7} \Rightarrow \beta_R = \sin^{-1}\left(\frac{9 \sin 20^\circ}{7}\right) \approx 26.1^\circ$$

Thus,  $\beta \approx 26.1^\circ$  or  $\beta \approx 180^\circ - 26.1^\circ = 153.9^\circ$ .

**Solution 1:** Let  $\beta \approx 26.1^\circ$ . Then  $\gamma \approx 180^\circ - 20^\circ - 26.1^\circ \approx 133.9^\circ$ .

$$\frac{c}{\sin \gamma} = \frac{a}{\sin \alpha} \Rightarrow c = \frac{a \sin \gamma}{\sin \alpha} = \frac{7 \sin 133.9^\circ}{\sin 20^\circ} \approx 14.7$$

$\beta \approx 26.1^\circ$ ,  $\gamma \approx 134^\circ$ ,  $c \approx 14.7$

**Solution 2:** Let  $\beta \approx 153.9^\circ$ . Then  $\gamma \approx 180^\circ - 20^\circ - 153.9^\circ \approx 6.1^\circ$ .

$$\frac{c}{\sin \gamma} = \frac{a}{\sin \alpha} \Rightarrow c = \frac{a \sin \gamma}{\sin \alpha} = \frac{7 \sin 6.1^\circ}{\sin 20^\circ} \approx 2.17$$

$\beta \approx 154^\circ$ ,  $\gamma \approx 6.1^\circ$ ,  $c \approx 2.17$