

41 (a) Since  $0^\circ < \frac{\theta}{2} < 45^\circ$ , we know that  $\frac{\theta}{2}$  is in quadrant I.

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{4}{5}}{2}} = \sqrt{\frac{\frac{5-4}{5}}{2}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{\frac{5+4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

$$\tan \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \sqrt{\frac{1 - \frac{4}{5}}{1 + \frac{4}{5}}} = \sqrt{\frac{\frac{5-4}{5}}{\frac{5+4}{5}}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

(b) Noting that  $\theta = \cos^{-1} \frac{4}{5}$ , numerical support is shown in *Figures 41a*, *41b* and *41c*.

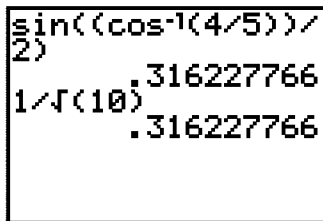


Figure 41a

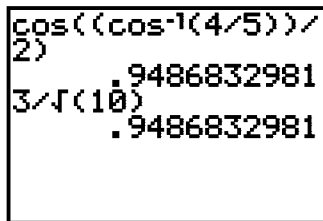


Figure 41b

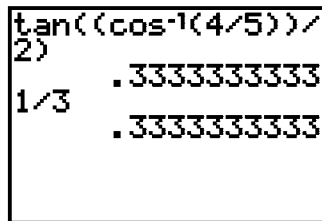


Figure 41c