

Chapter 9 Further Topics in Probability

§9.1 Permutations and Combinations

Factorial.

Factorial calculations, unless part of another set of commands or a program, should begin from the home screen. Input the value of n , access the factorial, and press **ENTER**.

For example, to calculate $10!$ on the TI-82 or TI-83/TI-83 Plus, from the home screen, type “10”, then press **MATH**, the left arrow key to access the PRB menu, then **4** **ENTER**. (See Figure 1.)

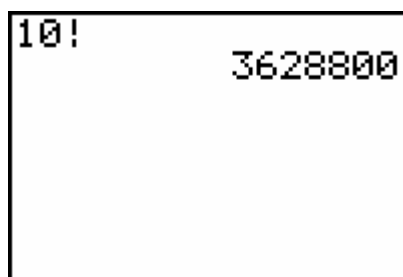


Figure 1.

On the TI-85 or TI-86, the factorial is located in the MATH PROB menu. To calculate $10!$, from the home screen, type “10,” then press **2nd** **×** to access the MATH menu, **F2** to access the PROB submenu, and **F1** for “!”. Press **ENTER** to execute the command. (See Figure 2.)



Figure 2.

Permutations

The permutation command on each of the calculators is located in the same menu as the factorial command.

For instance, to calculate ${}_8P_3$ with a TI-83/TI-83 Plus, as in Example 5 of the text, begin from the home screen and type “8”. Press **MATH**, the left arrow key to access the PRB menu, then **2** to access the nPr command. Type “3” and press **ENTER** to complete the calculation. (See Figure 3.)

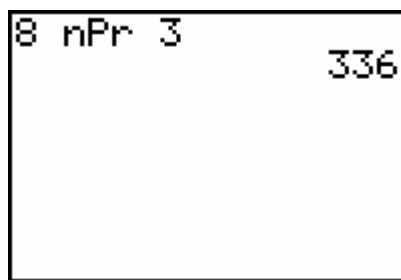


Figure 3.

Using the TI-85 or TI-86, calculate ${}_8P_3$ by typing “8” on the home screen. Press **2nd** **×** **F2** to access the MATH PROB menu, and **F2** for the nPr command. Type “3” and press **ENTER** to finish. (See Figure 4.)



Figure 4.

Combinations

The combination command is also located in the same menu as the two commands above.

For instance, to calculate ${}_8C_3$ with a TI-82 or TI-83/TI-83 Plus, as in Example 8 of the text, begin from the home screen and type “8”. Press **MATH**, the left arrow key to access the PRB menu, then **3** to access the nCr command. Type “3” and press **ENTER** to complete the calculation. (See Figure 5.)

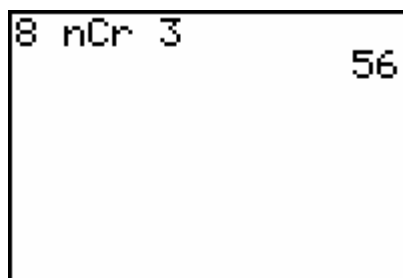


Figure 5.

Using the TI-85 or TI-86, calculate 8P_3 by typing “8” on the home screen. Press $\boxed{2nd}$ $\boxed{\times}$ $\boxed{F2}$ to access the MATH PROB menu, and $\boxed{F3}$ for the nCr command. Type “3” and press \boxed{ENTER} to finish. (See Figure 6.)



Figure 6.

§9.3 Binomial Probability.

The binompdf and binomcdf Commands on the TI-83.

The TI-83/TI-83 Plus has several probability functions built-in, including the binomial distribution. The appropriate commands are located in the DISTR menu of the calculator. The binompdf command is option 0. The command requires you to input the number of trials, n , followed by the probability, p , of one “success,” and the number of desired “successes,” x . When executed, this command returns the probability of x successes in n trials. The binomcdf command requires the same input, but instead returns the probability of x or fewer successes in n trials.

To use these commands to complete Example 3, begin from the home screen. In part (a), we are asked to find the probability that three people out of a group of fifteen will be “TV sleepers”, where the probability that a person is a “TV sleeper” is 0.25. Press $\boxed{2nd}$ \boxed{VARS} to access the DISTR menu, then $\boxed{0}$ to access the binompdf command. Type “15” followed by a comma, “.25” followed by another comma, then “3” for the number of desired “successes.” Press \boxed{ENTER} to execute the command. Part (b) of Example 3 asks for the probability of *at most*

3 “TV sleepers” in a group of 15. The `binomcdf` command will provide this information. It is option A (press `ALPHA` `MATH`) in the `DISTR` menu. (See Figure 7.)

The calculator screen displays the following commands and results:

```
binomPdf(15,.25,
3)
.2251990652
binomcdf(15,.25,
3)
.4612868817
```

Figure 7.

Although the TI-85, and TI-86 do not have a built-in binomial distribution function, it is not difficult to evaluate problems like Example 3(a) with these models, since the binomial probability function, $\binom{n}{x} p^x (1-p)^{n-x}$, can be typed directly onto the home screen. (See Figure 8.) These calculators can also be programmed to complete binomial distribution problems.

The calculator screen shows the manual calculation of the binomial probability function:

```
(15 nCr 3)(.25)^3*(.7
5)^12
.225199065171
```

Below the screen is a menu bar with the following options:

NUM	Frac	ANGLE	HYP	MISC
!	nPr	nCr	rand	randInt

Figure 8.

§9.4 Probability Distributions and Expected Value.

Drawing a Histogram.

Probability distributions can be drawn as histograms using the TI-83/TI-83 Plus, TI-85 and TI-86. You must first clear any functions in the function memory and set an appropriate window. To draw the histogram in Example 1 using the TI-83/TI-83 Plus or TI-86, use a window of $-0.5 \leq x \leq 2.5$ with a scale of 1 and $0 \leq y \leq 1$ with a scale of 0.25. For the TI-85, use a window of $-0.5 \leq x \leq 2.5$ with a scale of 1 and $0 \leq y \leq 4$ with a scale of 1.

To enter the probability distribution on the TI-83/TI-83 Plus, press `STAT` `1` and enter the outcomes in `L1` and the probabilities in `L2`. To graph the histogram press `2nd` `Y=` `1` to enter the `STAT PLOT` menu and select `PLOT1`. Highlight `On`, select the histogram option (the

third icon on the first row of Type) and enter L1 and L2 as the XList and Freq respectively. (See Figure 9.) Draw the histogram by pressing **GRAPH**. (See Figure 10.)

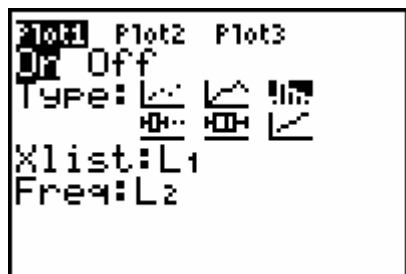


Figure 9.

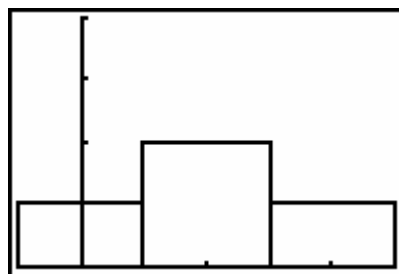


Figure 10.

To enter the probability distribution on the TI-86, press **2nd** **+** **F2** and enter the outcomes in xStat and the probabilities in yStat. To graph the histogram press **2nd** **+** **F3** to enter the STAT PLOTS menu and press **F1** to select Plot1. Highlight On, select the histogram option (press **F4**) and enter xStat and yStat as the Xlist and Freq respectively. (See Figure 11.) Draw the histogram by pressing **GRAPH** **F5** and press **CLEAR** to remove the menu. (See Figure 12.)

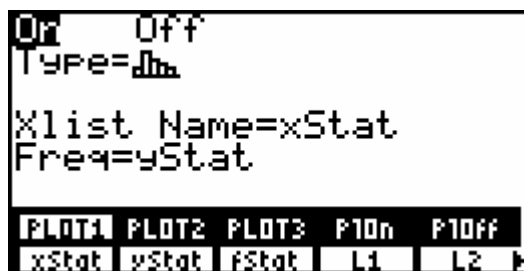


Figure 11.

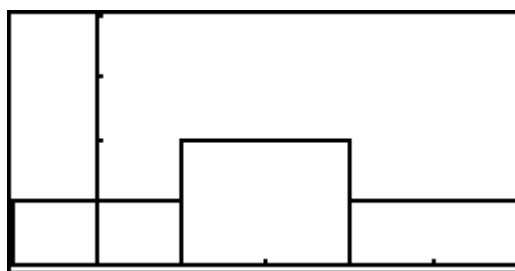


Figure 12.

To enter the probability distribution on the TI-85, press **STAT** **F2** and select xStat and yStat as the names of the xlist and ylist respectively. Enter the outcomes as the values of x and values 1, 2, 1 (the numerators of the probabilities $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{1}{4}$) for the probabilities as the values of y. To graph the histogram press **EXIT** **F3** **F1**, press **CLEAR** to remove the menu.

Chapter 10 Introduction to Statistics

§10.1 Frequency Distributions; Measures of Central Tendency

Frequency Histograms

The data in Example 1 of the text can be stored into and displayed by the graphing calculator. In this case, the data should be entered as one list and the frequencies entered as a second list. The TI-83/TI-83 Plus, TI-85, and TI-86 will use the value input for X_{sc1} as the width of the rectangles in a frequency histogram, so it is important to choose range variables that are appropriate for a given data set. In Example 1, X_{sc1} is set equal to 5; thus, each rectangle in the histogram drawn by the calculator will be 5 units in length.

After storing the data list into the TI-83/TI-83 Plus, we must define a statistics plot. Press $\boxed{2nd}$ $\boxed{Y=}$ and select a statistics plot name. Turn the plot “On;” beside *Type*, highlight the icon that resembles a histogram (the third icon on the first row for *Type*). Make sure that the list name under which the data is stored is chosen as the *Xlist*, and that the list name under which the corresponding frequencies are stored is chosen as *Freq*. If there is only one list for the data set, enter 1 as the *Freq*. Turn “off” any functions saved in the $\boxed{Y=}$ menu and set appropriate window values in the \boxed{WINDOW} menu. Press \boxed{GRAPH} to see the histogram. (See Figure 1.) The graph can be traced to determine the height and range of each rectangle. (See Figure 2.)

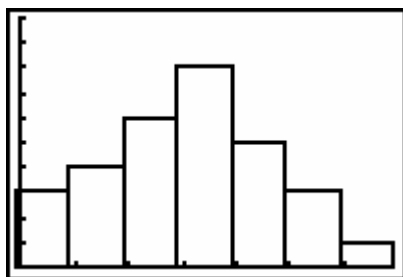


Figure 1.

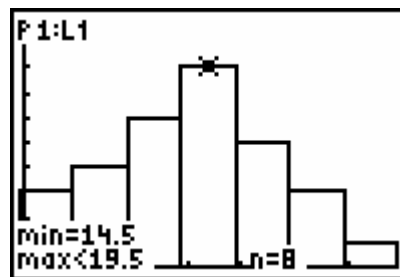


Figure 2.

If you have stored the data as a list in the TI-85, turn “off” any functions stored in the \boxed{GRAPH} $y(x)$ menu and set appropriate window values. Press \boxed{STAT} $\boxed{F3}$ to access the *STAT DRAW* menu. If any old drawings appear on the graphics screen, press $\boxed{F5}$ to clear them. Press $\boxed{F1}$ to see the histogram. You cannot trace a histogram on the TI-85.

If you are using the TI-86, enter the data values into *xStat* and the corresponding frequencies into *fStat*. To set up the statistics plot press $\boxed{2nd}$ $\boxed{+}$ $\boxed{F3}$ and select a plot. Turn the plot “On”, press $\boxed{F4}$ to select *HIST* as the plot *Type*, and select *xStat* as the *xList* and *fStat* as the frequency list. If there is only one list for the data set, enter 1 as the *Freq*. Turn

“off” any unnecessary functions, define appropriate window values, and press **GRAPH** **F5** to see the histogram. (See Figure 3.) You may trace the histogram to view the height and range of each rectangle. (See Figure 4.)

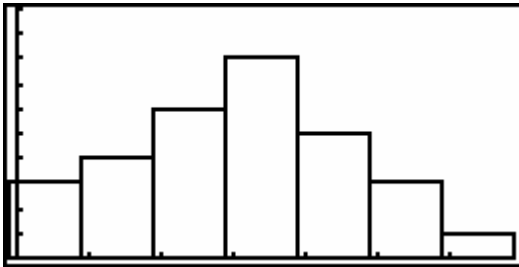


Figure 3.

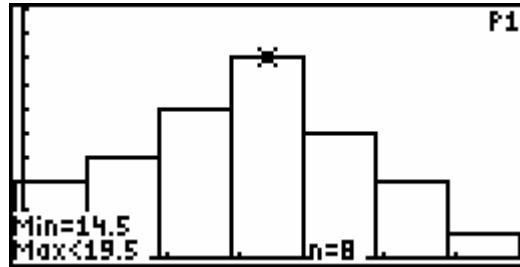


Figure 4.

For *grouped* data, a histogram can be created on your calculator by storing the *midpoints* of the groups as one list (to be used as the *Xlist*), and the corresponding frequencies as another (to be used as *Freq*).

Frequency Polygons

If data is entered as two lists, one containing the midpoint of each interval, and the other containing the corresponding frequencies, then a frequency polygon can be drawn on most calculators. For Example 2, to plot the frequency polygon along with the histogram on the TI-83/TI-83 Plus, first enter the midpoints and the frequencies in two new lists. Press **2nd** **Y=** and select a new plot, turn the plot “On;” beside *Type*, highlight the icon that resembles a frequency polygon (the second icon on the first row for *Type*). Make sure that the appropriate lists are indicated for *Xlist* (the midpoints) and *Ylist* (the frequencies). Select a *Mark* to represent the points, input the appropriate range values, and press **GRAPH**. (See Figure 5.)

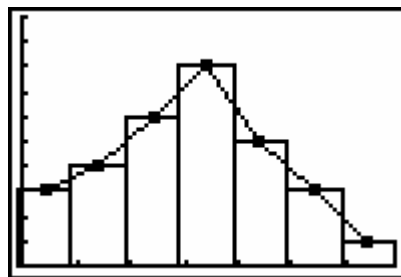


Figure 5.

On the TI-86 enter the midpoints and the frequencies in two new lists (you may need to create a new list name, say *fStat2*). Press **2nd** **+** **F3** and select a new plot, turn the plot “On”, press **F2** to select *xyLine* as the plot *Type* and the appropriate lists for *Xlist* (the

midpoints) and `Ylist` (the frequencies). Select a Mark to represent the points, input the appropriate range values, and press `GRAPH` `F5`. (See Figure 6.)



Figure 6.

On the TI-85, enter the midpoints and the frequencies in two new lists (you will have to create two new list names, say `xStat2` and `fStat`), set up the appropriate range variables, press `STAT` `F3` to access the `DRAW` submenu, then `F3` to draw the `xyline`.

Calculating Statistics

Once a data set has been entered, it is an easy task to find its mean and median on the TI-83/TI-83 Plus. From the homescreen, access the `LIST` menu by pressing `2nd` `STAT`, use the left/right arrow keys to move to the `MATH` submenu, and select option 3, `mean`. Type in the name of the list which contains the data (followed by a comma and the name of the list which contains the frequency, if necessary) and press `ENTER`. The fourth option in the list, `median`, will find the median of a saved list. See Figure 7 below for the mean and median of the data set from Example 10 of the text.

```

mean(L1)          3.2
median(L1)        3.5
  
```

Figure 7.

If other statistics are required, or if data is entered as two lists (one containing the frequencies), then the `1-Var Stats` command should be used on the TI-83/TI-83 Plus. From the homescreen, press `STAT` and the right arrow key to move to the `CALC` submenu; press `1`. Enter the name of the list containing the data. If a frequency list has been stored as well, then

type a comma followed by the name of the frequency list. Press **ENTER**. (Figures 8, 9 and 10 represent Example 10 from your textbook.)

```
1-Var Stats L1
```

Figure 8.

```
1-Var Stats
x̄=3.2
Σx=32
Σx²=116
Sx=1.229272594
σx=1.166190379
↓n=10
```

Figure 9.

```
1-Var Stats
↑n=10
minX=1
Q1=2
Med=3.5
Q3=4
maxX=5
```

Figure 10.

If you are using the TI-85, the mean of a stored data set can be calculated by pressing **STAT** **F1** to access the STAT CALC menu; select the name of the list containing the data and press **ENTER**; then the name of the list containing the frequencies and press **ENTER** again. Press **F1** to access the 1-VAR command, which calculates several one-variable statistics. The first statistic listed, \bar{x} , is the mean of the data set. The TI-85 does not have a command for calculating the median of a data set.

On the TI-86, press **2nd** **+** **F1** to access the STAT CALC menu, then **F1** for the OneVar command. Enter the name of the list containing the data. If a frequency list has been stored as well, then type a comma followed by the name of the frequency list. Press **ENTER**. The first statistic listed, \bar{x} , is the mean of the data set; use the down arrow key to scroll down to Med, which is the median of the data set. (Figures 11 and 12 represent Example 10 from your textbook.)

```
1-Var Stats
x̄=3.2
Σx=32
Σx²=116
Sx=1.22927259
σx=1.16619038
↓n=10
```

Figure 11.

```
1-Var Stats
↑n=10
minX=1
Qrt1=2
Med=3.5
Qrt3=4
maxX=5
```

Figure 12.

§10.2 Measures of Variation

Once a list of data is stored in the calculator, the standard deviation and variance can be calculated quickly. It is very important to remember that the calculator does not know whether the data entered represents a *sample* or a *population*. For that reason, most calculators will

calculate both the population and sample standard deviations. These are distinguished on the TI calculators by σ_x and S_x , respectively.

On the TI-83/TI-83 Plus, if data is saved in a single list, then option 7, `stdDev`, and option 8, `variance`, in the MATH submenu of the LIST menu can be used to calculate sample standard deviation and sample variance of that list.

The method described previously for calculating several statistics at once on the TI-83/TI-83 Plus, TI-85 and TI-86 will also calculate the population and sample standard deviations of a data set.

Notice that the variance is not included in the list of calculated statistics. Recall that the variance is the square of the standard deviation, so it can be obtained from the information given. The values calculated and shown in Figures 9 and 10 for the TI-83/TI-83 Plus or Figures 11 and 12 for the TI-86 are saved until you either perform a new statistical calculation or reset the calculator. To access these values on the TI-83/TI-83 Plus, press `VAR` `5`. The third and fourth options listed there are the sample and population standard deviations, respectively. To calculate the sample variance from the sample standard deviation, press `3` `x2` `ENTER`. (See Figure 13.)

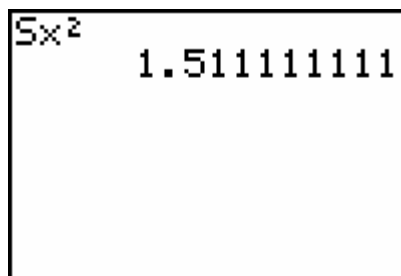


Figure 13.

To find the variance on the TI-85, press `STAT` `F5` `F3` to select the sample standard deviation, then press `x2` `ENTER`.

To find the variance on the TI-86, press `2nd` `CUSTOM` `MORE` `MORE` `F4` to access the STAT variables, use the down arrow to move the cursor down to `Sx` and press enter to select the sample standard deviation. Press `x2` `ENTER` to find the sample variance. (See Figure 14.)

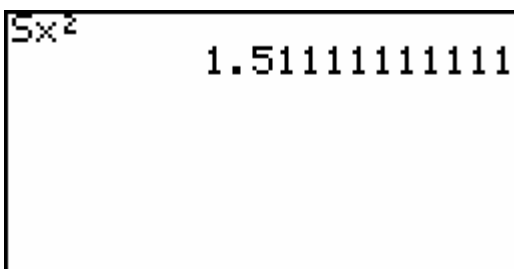


Figure 14.

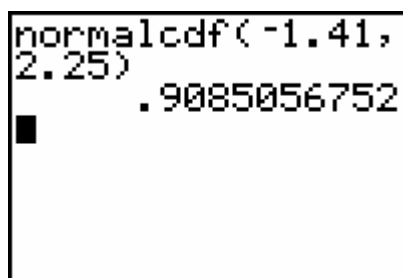
§10.3 Normal Distributions

The TI-83/TI-83 Plus come equipped with commands to calculate normal probabilities. While the TI-85 and TI-86 do not have such built-in functions, they can be programmed to perform the calculations (Programs are available at the textbooks website www.aw.com/mwa8);

the equation for the normal probability function, $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$, can be graphed by

entering it directly into the $\boxed{Y=}$ menus of the calculators, with appropriate values substituted for μ and σ .

To calculate the probability $P(a \leq z \leq b)$ on the TI-83/TI-83 Plus, use the `normalcdf` command, which is option 2 in the DISTR menu. This command must be followed by the left-hand endpoint, a , a comma, then the right-hand endpoint, b . To calculate $P(-1.41 \leq z \leq 2.25)$, as in Example 2(a) of the text, we would press $\boxed{2nd} \boxed{VAR}$ to access the DISTR menu, then press $\boxed{2}$ to select `normalcdf` (, followed by the bounds -1.41 and 2.25 separated by a comma, then press \boxed{ENTER} . (See Figure 15.)



```
normalcdf(-1.41,
2.25)
.9085056752
```

Figure 15.

To see a graph of the region, as well as to calculate the probability on the TI-83/TI-83 Plus, we can use the `ShadeNorm` command in the DISTR DRAW menu. First, we should turn off all unnecessary functions and plots in the $\boxed{Y=}$ and STAT PLOT menus. Next, we must set an appropriate window for the graph; for the *standard* normal distribution, we might set $X_{min} = -5$, $X_{max} = 5$, $X_{scl} = 1$, $Y_{min} = -.1$, $Y_{max} = .45$, and $Y_{scl} = .1$. You may also want to press $\boxed{2nd} \boxed{PRGM}$, followed by \boxed{ENTER} twice to clear any old graphs or drawings. Now, press $\boxed{2nd} \boxed{VAR}$ to access the DISTR menu and press the right arrow key to move to the DRAW submenu. Press $\boxed{1}$ to access the `ShadeNorm` command. As with the `normalcdf` command, we must follow this with the left-hand endpoint, a comma, and the right-hand endpoint. Press \boxed{ENTER} for the graph and the probability. (See Figure 16.)

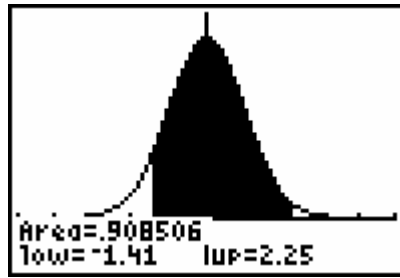


Figure 16.

For problems of the form $P(a \leq z)$ or $P(z \leq b)$, the TI-83/TI-83 Plus still requires both a left- and a right-hand endpoint in the `normalcdf` and `ShadeNorm` commands. In Example 1(a), we are asked to find $P(z \leq 1.25)$; since no left-hand endpoint is given, we should choose a very large negative number (a number to the far left of the graph) to use in the calculator command. Your text suggests using -1×10^{99} in this case. Example 2(c) asks us to find $P(2.09 \leq z)$; since no right-end point is given, we should choose a very large positive number to use in the calculator command. In such a case, your text suggests using 1×10^{99} . (See Figures 17 and 18.)

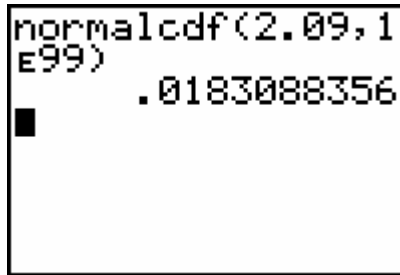


Figure 17.

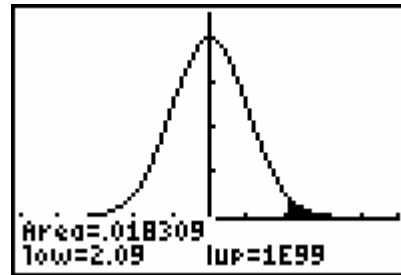


Figure 18.

All three commands discussed here can be used for any normal distribution by adding the mean and standard deviation of the distribution to the commands. For instance, Example 4(a) can be solved by using the command shown in Figure 19.

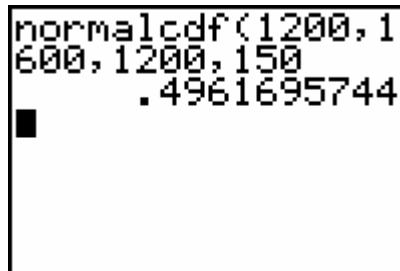


Figure 19.

In order to use the ShadeNorm command for other normal distributions, we must be careful to select an appropriate window. For X_{\min} and X_{\max} , choose values 4 or 5 standard deviations below and above the mean. To determine Y_{\max} , we can use the command `normalpdf` to find the y -value of the normal distribution curve at its highest point (for a normal distribution, this location is *always* the mean). For the Example 4(b), press `2nd` `VARS` `1` to access this function, and type “1200, 1200, 150”. These numbers represent the location of the highest point on the graph, the value of the mean, and the value of the standard deviation for the distribution; press `ENTER`. (See Figure 20.) Use a value slightly above the resulting number as Y_{\max} . Y_{\min} can always be chosen as 0, if you do not mind having text cover a portion of the graph. (See Figure 21.)

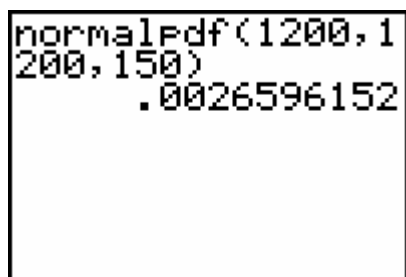


Figure 20.

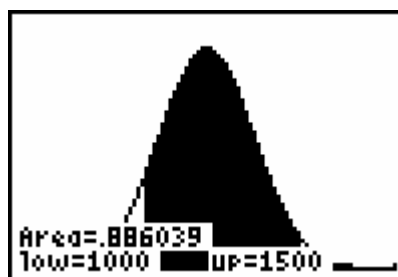


Figure 21.

If you are using the TI-85 or TI-86, probabilities involving the normal distribution can be approximated using the `fnInt` command. This command approximates the area under a curve which lies between two given x -values. First, press `GRAPH` `F1` and enter the appropriate formula for the normal distribution given by $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$ in the function memory.

To find the area, return to the homescreen on the TI-85 and TI-86 and press `2nd` `÷` `F5`. Follow this command with the name of the function where the normal distribution is stored, then a comma, the variable x , another comma, the left-hand x -value, a comma, and finally the right-hand x -value. Example 2(c), $P(2.09 \leq z)$, of the textbook can be approximated as shown in Figure 22. Note that the value 10 was used as the right-hand endpoint, and not $1E99$. This is because using too large a number will result in an overflow error on the calculator when it attempts to square the number. Always use a value at least 5σ away from the mean when solving problems similar to Example 2(c) with these calculators.

```

fnInt(91,x,2.09,10)
      .01830889985
  
```

Figure 22.

Example 4(a) can be solved on the TI-85 or TI-86 by first entering the formula for the normal distribution given by $f(x) = \frac{1}{150\sqrt{2\pi}} e^{-(x-1200)^2/(2 \times 150^2)}$ in the function memory, then use the command shown in Figure 23.

```

fnInt(91,x,1200,1600)
      .496169619429
  
```

Figure 23.

Drawing Boxplots

The TI-83/TI-83 Plus and TI-86 will draw a boxplot for a single list of data or a pair of lists, the data and the corresponding frequencies. Similar to drawing a histogram, first enter the data, turn off all unnecessary functions and plots in the $\overline{Y=}$ menu, and set an appropriate window.

On the TI-83/TI-83 Plus press $\overline{2nd}$ $\overline{Y=}$ and select a statistics plot name. Turn the plot “On;” beside Type, highlight the second icon that resembles a boxplot (the second icon on the second row for Type). Make sure that the list name under which the data is stored is chosen as the Xlist, and that the list name under which the corresponding frequencies are stored is chosen as Freq. If there is only one list for the data set, enter 1 as the Freq, and press \overline{GRAPH} . Figure 24 shows the boxplot for Example 6.

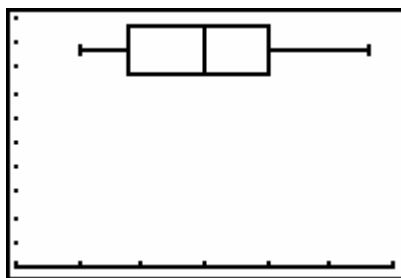


Figure 24.

To draw a boxplot on the TI-86, press $\boxed{2nd} \boxed{STAT} \boxed{F3}$ and select a statistics plot name. Turn the plot “On;” beside Type, press $\boxed{F5}$ to select boxplot, make sure that the list name under which the data is stored is chosen as the Xlist, and that the list name under which the corresponding frequencies are stored is chosen as Freq. If there is only one list for the data set, enter 1 as the Freq, and press $\boxed{GRAPH} \boxed{F5}$. Figure 25 shows the boxplot for Example 6.

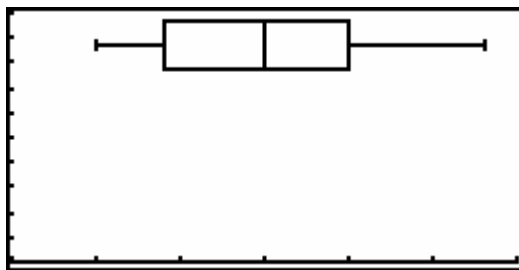


Figure 25.

The TI-85 does not draw boxplots.

§10.4 Binomial Distributions

See the discussion on §9.3 Binomial Probabilities for detailed instructions regarding this section of your text.

Chapter 11 Differential Calculus

§11.1 Limits

Graphically Estimating Limits

To graphically estimate the limit of a function, as in Example 3 of your text, first store the function as usual, then select a viewing window that is narrow and centered around the x -value where the limit is being taken. For Example 3, a good choice might be to set the minimum x -value to be 2.8 and the maximum x -value to be 3.2. You may have to adjust the range of y -values in order to get a good look at the graph. Trace the graph to get a good idea of what the limit is. The graphical solution to Example on the TI-83/TI-83 Plus is shown in Figures 1 and 2. The graphical solution using the TI-86 is shown in Figures 3 and 4.

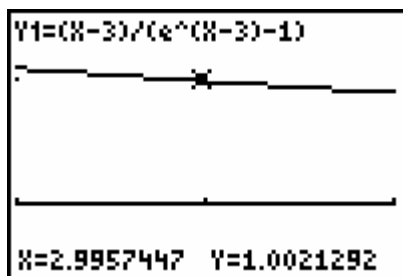


Figure 1.

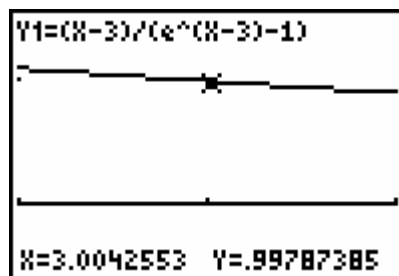


Figure 2.

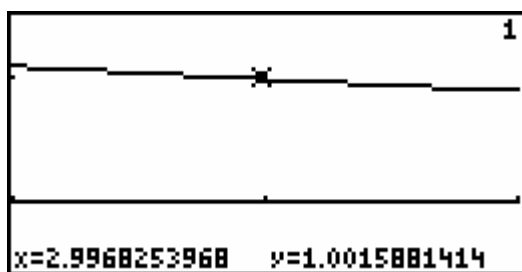


Figure 3.

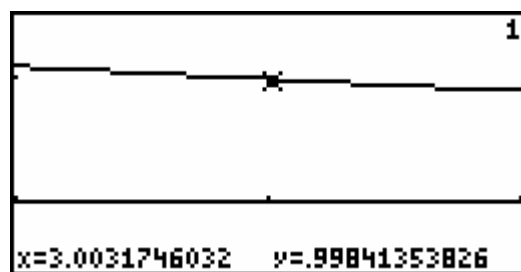


Figure 4.

You can use Table feature to numerically estimate a limit using the TI-83/TI-83 Plus or TI-86. Go to the TblSet menu, highlight Ask for Indpnt, then go to the Table and enter the desired values. The numerical solution to Example 3 using the TI-83/TI-83 Plus is shown in Figure 5 and the solution using the TI-86 is shown in Figure 6.

X	Y1	
2.99	1.005	
2.999	1.0005	
2.9999	1.0001	
ERROR		
3.0001	.99995	
3.001	.9995	
3.01	.99501	
X=3		

Figure 5.

X	Y1	
2.99	1.005008	
2.999	1.0005	
2.9999	1.00005	
ERROR		
3.0001	.99995	
3.001	.9995001	
X=3		
TBLST	SELCT	
x	y	

Figure 6.

§11.3 Tangent Lines and Derivatives

Graphing Tangent Lines

The tangent line to a function at a given point may be graphed on the same screen as the graph of the function itself, as indicated in the note after Example 1 of your text. Store the function under a function name and turn off other functions and/or plots. Set the window values so that the x -value of the point where the tangent line is to be drawn is *exactly* halfway between X_{min} and X_{max} . To duplicate Figure 11.14 of the text, try $X_{min} = -4$ and $X_{max} = 2$. Press **GRAPH**.

The TI-83/TI-83 Plus, TI-85 and TI-86 will draw a tangent line to a given function at a given point. If you are using the TI-83/TI-83 Plus, press **2nd** **PRGM** to access the DRAW menu and choose option 5, Tangent (. The graphics screen is displayed again, with the blinking cursor at the center; because of the values chosen for X_{min} and X_{max} , the cursor will already be located at the desired x -value; press **ENTER**. (See Figure 7.) Otherwise, for the TI-83/TI-83 Plus, type the x -value of the point after accessing the Tangent command for the tangent line to the function at any point between X_{min} and X_{max} . Press **ENTER** for the graph of the tangent line.

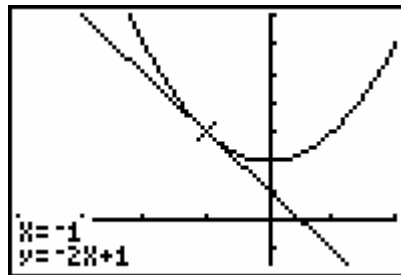


Figure 7.

If you are using the TI-85, while viewing the graph of the function, press **MORE** **F2** to access the DRAW menu, then **MORE** **MORE** **F1** for the TanLn command. The calculator returns to the home screen, where you should type the function name, followed by a comma, and the desired x -value of the location of the tangent line; press **ENTER**.

On the TI-86, while viewing the graph of the function, press **MORE** **F2** to access the DRAW menu, then **MORE** **MORE** **MORE** **F2** for the TanLn command. The calculator returns to the home screen, where you should type the function name, followed by a comma, and the desired x -value of the location of the tangent line; press **ENTER**. (See Figure 8.)

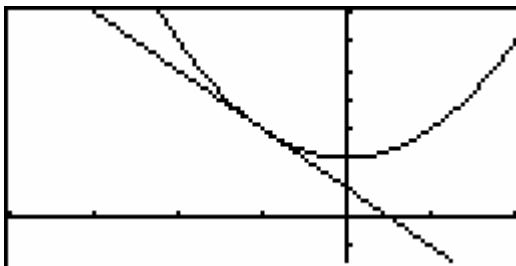


Figure 8.

Numerical Differentiation on the Calculator.

Graphing calculators are equipped with commands that calculate the numerical derivative of a function, when it exists, for a given value of the variable. If you are using the TI-83/TI-83 Plus, from the home screen press **MATH** and choose option 8, nDeriv. Type in the function, followed by a comma, the variable, followed by another comma, then the value at which the derivative is to be calculated, and press **ENTER**. Figure 9 shows the result of Example 9 of this section of the textbook.

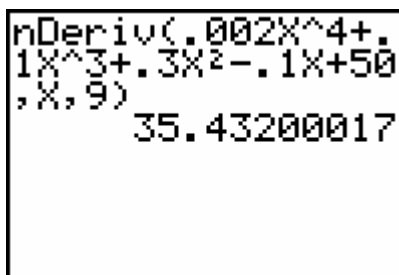


Figure 9.

If you are using the TI-85 or TI-86, press **2nd** **÷** to access the CALC menu, then **F2** for the nDer command. Type in the function, followed by a comma, the variable, followed by another comma, then the value at which the derivative is to be calculated, and press **ENTER**. Figure 10 shows the result of Example 9 of this section of the textbook for the TI-86.

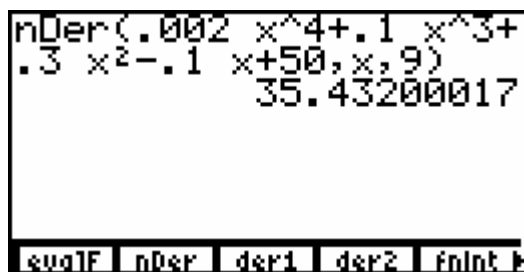


Figure 10.

The TI-85 and TI-86 also have a command which gives more exact values of derivatives for non-polynomial functions. This command, `der1`, is located in the same menu as the `nDer` command.

While viewing the graph of a function, you can also find an approximation for the derivative of the function at a given point. First, store the function, and set the range values as described for graphing tangent lines. On the TI-83/TI-83 Plus, press `2nd` `TRACE` to access the `CALC` menu and choose option 6, dy/dx . On the TI-85 or TI-86, press `MORE` `F1` for the `MATH` submenu, then `F4` on the TI-85 or `F2` on the TI-86 for the dy/dx command. If the range was set so that the desired point is in the center of the graphics screen, then the blinking cursor will already be located at that point. Press `ENTER` and the derivative will be displayed. If you are using the TI-83/TI-83 Plus or TI-86, you may also type the x -value of the point after accessing the dy/dx command, and press `ENTER` to calculate the derivative of the function at any point between X_{min} and X_{max} . The value of the derivative in Example 4(b) using the TI-83/TI-83 Plus is shown in Figure 11 and the value using the TI-86 is shown in Figure 12.

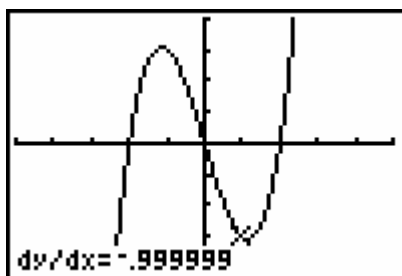


Figure 11.

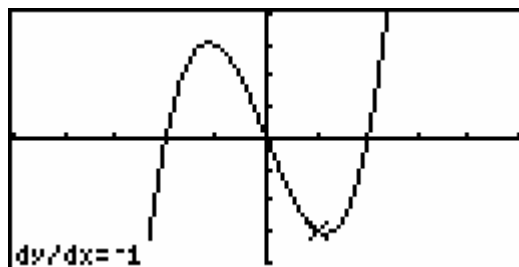


Figure 12.

Chapter 12 Applications of the Derivative

§12.1 Derivatives and Graphs

As discussed previously, the maximum and/or minimum of a function, within a certain range of x -values, can be approximated graphically with your calculator. Review the instructions regarding §3.5 Applications of Quadratic Equations, if necessary, and note the importance of choosing appropriate upper and lower bounds for the intervals containing the desired extrema. Also, answers can be checked by using the `value` or `Eval` command, as described in the instructions for §3.1 Functions.

§12.3 Optimization Applications

As stated above, you can use your graphing calculator to check your answers to optimization problems by using the built-in commands for finding extrema. Review the instructions regarding §3.5 Applications of Quadratic Equations, if necessary, and note the importance of choosing appropriate upper and lower bounds for the intervals containing the desired extrema.

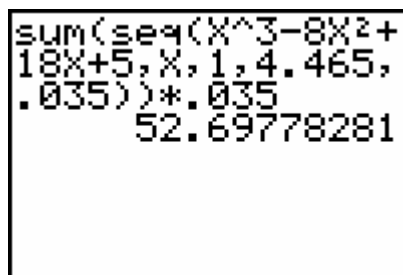
Chapter 13 Integral Calculus

§13.3 Area and the Definite Integral

Approximating Definite Integrals

You can use your calculator to approximate the definite integral using sums by combining the `sum(` and `seq(` commands. Both commands are found under the `LIST` menu on the TI-83/TI-83 Plus, TI-85 and TI-86. Example 4(a) requires you to approximate the definite integral using sums.

On the TI-83/TI-83 Plus, `sum(` is found under the `MATH` menu of `LIST`, while the `seq(` command is found under the `OPS` menu of `LIST`. After selecting `sum(` then `seq(`, enter the function, the variable, the initial value of the variable, the final value of the variable, and the increment, each separated by a comma. Press `)` twice, then `×`, followed by the increment `0.035`, and finally press `ENTER`. (See Figure 1.)



```
sum(seq(X^3-8X^2+
18X+5,X,1,4.465,
.035))*0.035
52.69778281
```

Figure 1.

On the TI-85 and TI-86, both `sum` and `seq(` are found under the `OPS` menu of `LIST`. When you enter the `OPS` menu, press `MORE` to display the menu with both commands. After selecting `sum` then `seq(`, enter the function, the variable, the initial value of the variable, the final value of the variable, and the increment, each separated by a comma. Press `)` once, then `×`, followed by the increment `0.035`, and finally press `ENTER`. (See Figure 2.)



Figure 2.

Option 9 in the **MATH** menu of the TI-83/TI-83 Plus, **fnInt**, can be used to approximate the value of a definite integral. This command can be found on the TI-85 or TI-86 by pressing **2nd** **÷** to access the **CALC** menu, then **F5**. The command must be followed by the function to be integrated (or the function name under which it is stored), a comma, the variable, followed by another comma, the lower limit of the integral, a comma, then the upper limit of the integral. Pressing **ENTER** executes the command. For instance, on the TI-83, the integral in Example 4(b) can be approximated by using the command shown in Figure 3 below. The same approximation on the TI-86 is shown in Figure 4.

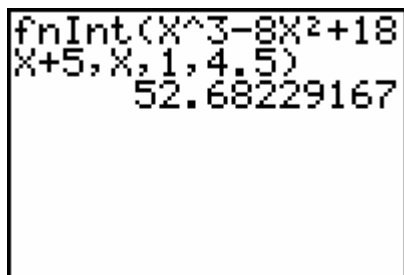


Figure 3.

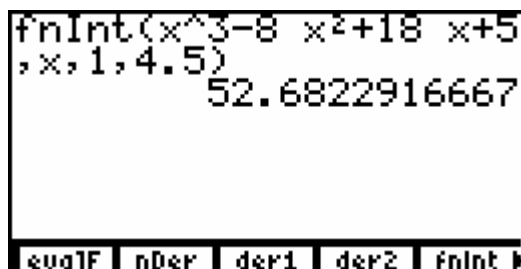


Figure 4.

Graphically Displaying Definite Integrals

Your calculator can approximate a definite integral of a graphed function, where the upper and lower limits of integration are between X_{min} and X_{max} . On the TI-83/TI-83 Plus and TI-86, this command also shades the region corresponding to the definite integral.

To use this command on the TI-83/TI-83 Plus or TI-86, first store the function and set appropriate window values. While viewing the graph of the function, access the $\int f(x) dx$ command; it is option 7 in the **CALC** menu on the TI-83/TI-83 Plus and may be accessed on the TI-86 by pressing **GRAPH** **MORE** **F1** **F3**. Type in the lower limit of integration, press **ENTER**, then type in the upper limit and press **ENTER** again. The definite integral is approximated and the corresponding region shaded. Figure 5 shows the results for Example 4(b) of the text, $\int_1^{4.5} x^3 - 8x^2 + 18x + 5 dx$ using the TI-83/TI-83 Plus. Figure 6 shows the same results using the TI-86.

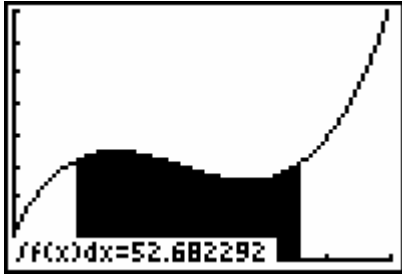


Figure 5.



Figure 6.

If you are using the TI-85, it is most convenient to set X_{min} and X_{max} to be the lower and upper limits of integration, respectively. The $\int f(x)$ command is accessed by pressing **GRAPH** **MORE** **F1**, then **F5**. When the graph is displayed again, use the left arrow key to move the cursor as close to the lower limit of integration as possible and press **ENTER**. Use the right arrow key to move the cursor as close as possible to the upper limit and press **ENTER** once more. The approximate value of the integral is displayed, but the corresponding region is not shaded.