

Chapter 4 Exponential and Logarithmic Functions

§4.1 Exponential Functions

Evaluating Powers of e

Powers of e can easily be approximated from the homescreen of your TI-83/TI-83 Plus, TI-85 or TI-86. Press $\boxed{2\text{nd}} \boxed{\text{LN}}$, followed by the desired exponent, and press $\boxed{\text{ENTER}}$. On the TI-83/TI-83 Plus, you can also raise e to a power by pressing $\boxed{2\text{nd}} \boxed{\div}$, to access e , then press $\boxed{\wedge}$ (to raise it to a power), followed by the appropriate exponent, and press $\boxed{\text{ENTER}}$.

§4.2 Applications of Exponential Functions

Entering Lists, Plotting Points and Exponential Regression

A set of points can be plotted, as in Example 3, by first entering the coordinates of the points into two lists—one list for the x -values and one for the y -values. On the TI-83/TI-83 Plus, there are six list names to choose from—L1 through L6. The TI-83/TI-83 Plus allows more lists to be added with user-defined names, as do the TI-85 and TI-86. You can also find the coefficient and base for the exponential regression curve using the TI-83/TI-83 Plus, TI-85, or TI-86. The steps for finding the exponential regression curve are similar to those used to find the linear, quadratic, cubic and quartic regression curves.

The steps for entering and graphing lists of points are almost identical on the TI-83/TI-83 Plus. Start by pressing $\boxed{\text{STAT}} \boxed{\text{ENTER}}$ to obtain the list editor. Decide which list name will contain the x -values and use the left/right arrow keys to move the cursor to that list. If there is old data in the list that is no longer needed, use the up arrow to move the cursor to the list name and press $\boxed{\text{CLEAR}} \boxed{\text{ENTER}}$ to delete the data. Enter the x -values, one at a time, in order, pressing $\boxed{\text{ENTER}}$ after each. Choose a second list for the y -values and repeat the same process, being careful that corresponding x - and y -values are in the same position within each list. (See Figure 1.)

L1	L2	L3	Z
50	20		
60	12.6		
65	10.6		
70	9.2		
75	7.6		
78	7.2		

L2(12) =			

Figure 1.

To plot the data values, we must go through a few extra steps to make sure that the graph does not contain information we do not want. Begin by pressing $\boxed{Y=}$ and turning “off” all functions stored there. To do this, move the cursor to the “=” beside any stored function and press \boxed{ENTER} to unhighlight the “=.” Now, press \boxed{WINDOW} and enter the appropriate range values or press \boxed{ZOOM} $\boxed{9}$ to set the values automatically. To define the plot, press $\boxed{2nd}$ $\boxed{Y=}$ to obtain the STAT PLOT menu. Press $\boxed{4}$ \boxed{ENTER} to turn off any other plots, then return to the STAT PLOT menu. Press $\boxed{1}$, $\boxed{2}$, or $\boxed{3}$ to select a plot name. Turn the plot “on” by pressing \boxed{ENTER} . Move the cursor to the TYPE line; select the first plot type, the *scatter plot*, by again pressing \boxed{ENTER} . Next, the lines titled Xlist and Ylist need to contain the appropriate list names. On the TI-83/TI-83 Plus, simply type in the name of the list containing the appropriate values. Finally, select one of three Marks that will represent the points on the graph. The plot is now defined; to see it, press \boxed{GRAPH} . (See Figure 2.)

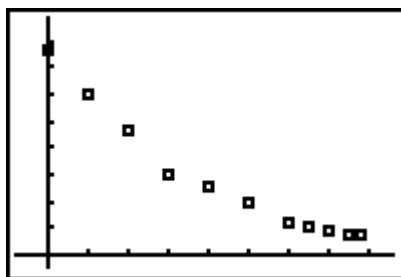


Figure 2.

To find the coefficient and base for the regression curve on the TI-83/TI-83 Plus press \boxed{STAT} followed by a right arrow to go to the CALC submenu. To select ExpReg on the TI-83/TI-83 Plus press $\boxed{0}$. This command should be followed by the names of the lists containing the x -values and y -values, separated by a comma, and then press \boxed{ENTER} . (See Figure 3.) If you wish to save the regression equation in the $\boxed{Y=}$ of the TI-83/TI-83 Plus, enter the function name before pressing \boxed{ENTER} .

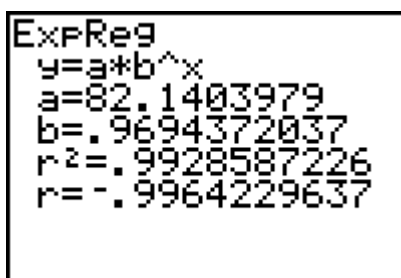


Figure 3.

Saving and graphing lists on the TI-85 is somewhat different. Begin by pressing **[STAT]** **[F2]** to obtain the list editor. Choose a name for the x list by typing in a descriptive name and pressing **[ENTER]**; repeat for the y list. Once the list names have been entered, the calculator moves to the editing screen. Press **[F5]** to clear any old data values. Entering the points, one at a time, as follows: x -value, **[ENTER]**, y -value **[ENTER]**. Before graphing the points, “turn off” any stored functions by pressing **[GRAPH]** **[F1]** **[MORE]** **[F2]**. Now press **[GRAPH]** **[F2]** and enter an appropriate set of window values. Finally, press **[STAT]** **[F3]** **[F2]** to draw the points. The top “menu” line can be removed by pressing **[CLEAR]**.

To find the regression equation on the TI-85, return to the CALC submenu after pressing **[STAT]**. Enter the names of the lists containing the x - and y -values followed by **[F4]** to select **ExpR**. The coefficient is given by a and the base is b .

If you are using the TI-86, press **[2nd]** **[+]** to access the STAT menu, and **[F2]** to EDIT. (Lists can be cleared in the same way as described for the TI-83/TI-83 Plus.) Enter the x -values in the list $xStat$, one at a time, pressing **[ENTER]** after each. Use the right arrow key to move to $yStat$ and enter the corresponding y -values, in order. Finally, be sure to enter a 1 in the $fStat$ column for each corresponding ordered pairs. To define a statistics plot, press **[2nd]** **[+]** to return to the STAT menu then **[F3]** to access the PLOT submenu. To define the plot as $Plot1$, press **[F1]** and press **[ENTER]** to turn the plot “on.” Use the down arrow key to move to the Type line; press **[F1]** to select SCAT for the scatter plot. Press **[ENTER]** to move to the Xlist Name line; select $xStat$ for the Xlist name and press **[ENTER]** again. Select $yStat$ for the Ylist name and press **[ENTER]**. Choose one of three Marks to represent the points in the graphics screen. Turn off any stored functions by pressing **[GRAPH]** **[F1]** **[MORE]** **[F2]**. Now press **[GRAPH]** **[F2]** and enter the appropriate range values. Finally, press **[F5]** to see the graph. Or press **[GRAPH]** **[F3]** **[MORE]** **[F5]** and the calculator will choose an appropriate window. (See Figure 4.)

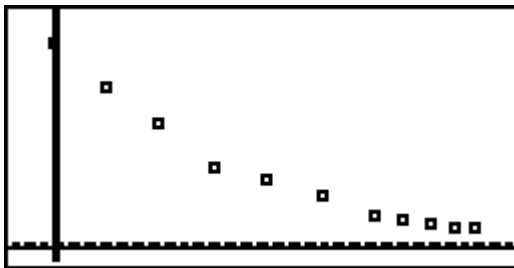


Figure 4.

To find the regression equation on the TI-86, return to the CALC submenu of the STAT menu. To find the exponential regression equation select **ExpReg** by pressing **[F5]** **[ENTER]**. (See Figure 5.)

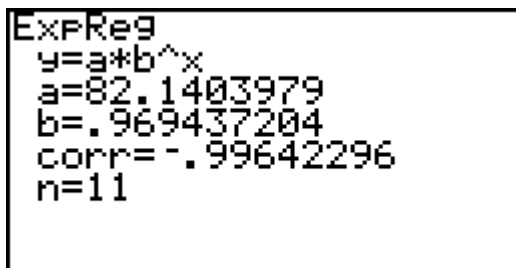


Figure 5.

§4.3 Logarithmic Functions

Evaluating Expressions Involving Logarithms

Your calculator has a built in natural logarithm command. You must, however, be careful when evaluating expressions involving logarithms. For instance, in Example 9 of this section, you must evaluate $\frac{\ln 3}{\ln 7}$. To do this and similar problems on your calculator, it is good practice to *always* enclose the arguments in parentheses to avoid calculator error. (See Figure 6.)

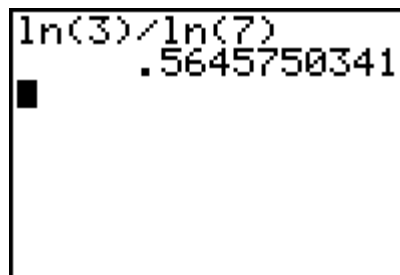


Figure 6.

§4.4 Logarithmic and Exponential Equations

Graphically Solving Logarithmic and Exponential Equations

You can solve an equation graphically either by using the intersection feature or the root finder feature. For instance, in Example 6 of this section, you must solve the equation $3e^{x^2} = 200$. If you wish to use the intersection finder, graph the two equations $y_1 = 3e^{x^2}$ and $y_2 = 200$ on an appropriate that includes the point(s) of intersection. Then use the Intersection or Isect command to find the intersection point(s). If you wish to use the root finder, graph the equation $y = 3e^{x^2} - 200$ on an appropriate window that includes the x -intercept(s), then use the Root or Zero command to find the intercept(s).

Chapter 5 Mathematics of Finance

§5.2 Compound Interest

Compounded Interest

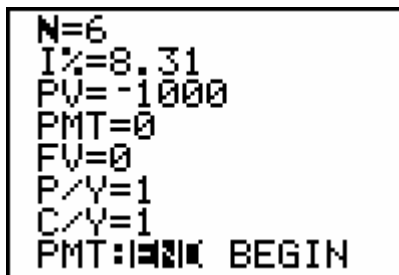
The TI-83/TI-83 Plus are equipped with a TVM Solver. This can be found on the TI-83 by pressing $\boxed{2\text{nd}} \boxed{x^{-1}} \boxed{\text{ENTER}}$ and on the TI-83 Plus by pressing $\boxed{\text{APPS}} \boxed{1} \boxed{\text{ENTER}}$. TVM stands for *time-value-of-money*. This solver will be handy throughout Chapter 5 of your text. There are also several financial commands available, many of which are discussed below. (Note: On the TI-83/TI-83 Plus, it is very important to remember that these financial commands cannot be used until values have been entered for the variables in the TVM Solver.)

If you are using the TI-83/TI-83 Plus, the variables are as follows:

- N** Number of payment periods.
- I%** The percentage rate, given as a percent.
- PV** The present value of the account. If money is being paid *into* the account, PV is entered as a negative number; otherwise, PV is entered as a positive number.
- PMT** The amount of each payment; if money is being paid *out*, PMT is entered as a negative number; if money is being *earned* or *received*, then PMT is entered as a positive number.
- FV** Future value of the account.
- P/Y** Number of payments per year.
- C/Y** Number of compounding per year.

On the TI-83/TI-83 Plus, you will also see the line PMT:END BEGIN. If payments are paid at the end of the compounding period, choose END. If payments are made at the beginning of the compounding period, as is the case in many annuity problems, choose BEGIN.

Example 1 of this section of the text can be solved with the TI-83/TI-83 Plus by entering the values shown in Figure 1:



```
N=6
I%=8.31
PV=-1000
PMT=0
FV=0
P/Y=1
C/Y=1
PMT:END BEGIN
```

Figure 1.

In part (a), we are being asked to find the future value, A of the account. To do this with the TI-83/TI-83 Plus solver, use the arrow keys to move the cursor beside FV and press **ALPHA** **ENTER** to execute the SOLVE command. (See Figure 2.)

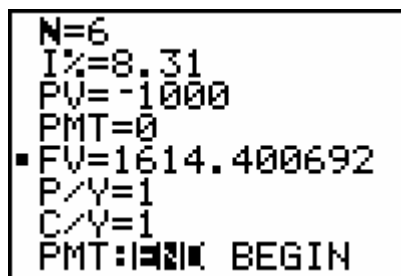


Figure 2.

Note: Often, the calculator's answers will differ slightly from those of the book because of the amount of precision used during calculation.

Entering Values in a Table

You can enter values individually in the TABLE on the TI-83/TI-83 Plus, and TI-86. Go to the TblSet menu and highlight Ask for Indpnt. To calculate the various effective rates in Figure 5.3 of the text, enter $\left(1 + \frac{.10}{x}\right)^x - 1$ into the function memory (be sure to clear or turn off any other functions in the function memory), go to the Table and enter the values. (See Figure 3 and Figure 4.)

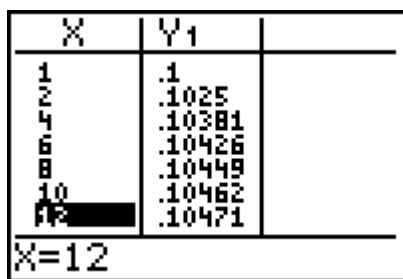


Figure 3.

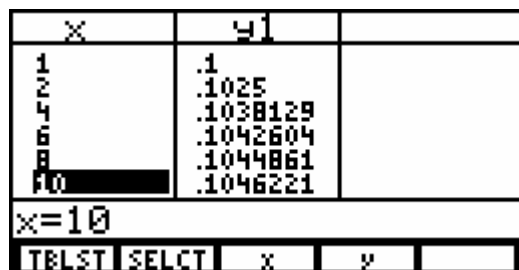


Figure 4.

Effective Interest Rate

Other commands in the FINANCE menu of the TI-83/TI-83 Plus can be helpful in this section. The Eff command can be used to find the effective rate in Example 4. From the home screen of the TI-83, press **2nd** x^{-1} and choose option C (by pressing **ALPHA** **PRGM**). On the TI-83 Plus, press **APPS** **1** and choose option C. Type in the rate of compounded interest, as a

percent, followed by a comma, then the number of compoundings per year. Press **ENTER** to complete the command. (See Figure 5.)

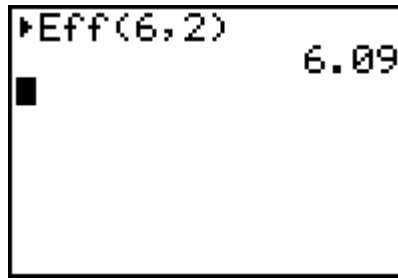


Figure 5.

Present Value of an Account

Example 8 can also be solved with the TVM Solver. Set $N = 18$, $I\% = 6$, $PV = 0$ (since it is unknown), $PMT = 0$ (no additional payments will be made into the account), $FV = 16000$, $P/Y = 2$, $C/Y = 2$ (since interest is compounded semiannually), and choose **END**. If you are using the TI-83/TI-83 Plus, move the cursor beside PV and press **ALPHA** **ENTER** to find the amount to be deposited. (See Figure 6.)

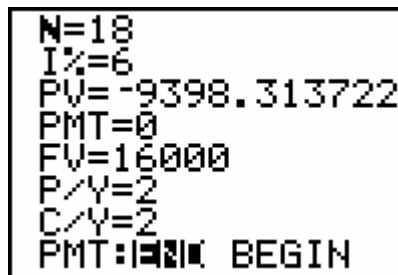


Figure 6.

Solving for N.

To solve Example 9, set $I\% = 8$, $PV = -1$, $PMT = 0$, $FV = 2$, $P/Y = 1$, $C/Y = 1$, and choose **END**. If you are using the TI-83, move the cursor beside N and press **ALPHA** **ENTER**. (See Figure 7.)



Figure 7.

There are no built-in commands on the TI-82, TI-85, or TI-86 similar to those described in this section, however the functions can be programmed into the calculators. (Programs are available at the textbooks website www.aw.com/mwa8.) Alternatively, values can be found by entering the appropriate formula as an equation in the SOLVER menu of the TI-85 and TI-86. For instance, Example 9 can be completed on the TI-85 or TI-86. Press **2nd** **GRAPH** to obtain the SOLVER menu, type in the equation to be solved, $2 = 1.08^N$, and press **ENTER**. Type in a reasonable guess for the variable N and press **F5** to solve. (See Figure 8.)

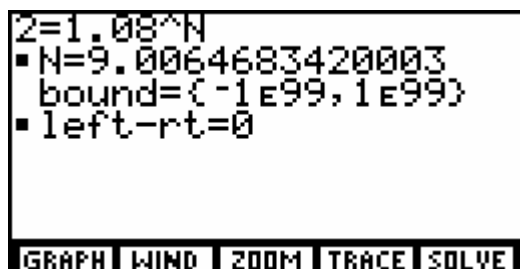


Figure 8.

§5.3 Annuities

Geometric Sequences

You can generate a list of terms of a sequence on the TI-83/TI-83 Plus, TI-85 and TI-86 using the `seq` command. This command is found under the OPS submenu of the LIST menu. You must input the n th term of the sequence, the variable, initial value of the variable, the final value of the variable, and the increment of the variable, each separated by a comma. The increment is optional on the TI-83/ TI-83 Plus and TI-86, if it is omitted, the default is 1.

To calculate the first four terms of the sequence shown in Figure 5.4 of the text, press **2nd** **STAT** to access the LIST menu on the TI-83/TI-83 Plus. Select the OPS submenu and press **5** to select `seq`. Type in the expression $6(4)^{(N-1)}$, the variable N, the initial value of 1, the final value of 4 and the increment of 1, each separated by a comma. Press **ENTER** to find the first four terms of the sequence. (See Figure 9.)

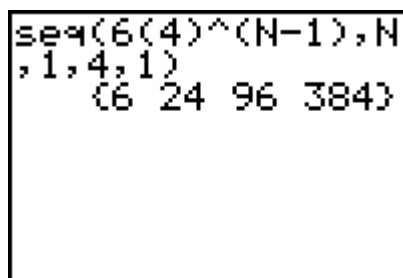


Figure 9.

On the TI-85 and TI-86, press $\boxed{2nd}$ $\boxed{-}$ to access the LIST menu. Press $\boxed{F5}$ to select the OPS submenu, followed by \boxed{MORE} $\boxed{F3}$ to select seq. Type in the expression $6(4)^{(N-1)}$, the variable N, the initial value of 1, the final value of 4, and the increment of 1, each separated by a comma. Press \boxed{ENTER} to find the first four terms of the sequence. (See Figure 10.)

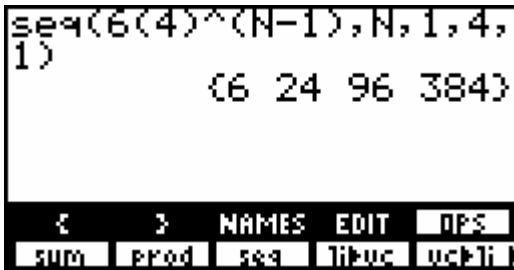


Figure 10.

You can find the sum of a list of terms TI-83/TI-83 Plus, TI-85 and TI-86 using the calculate sum command. To calculate the sum of the first n terms of a geometric sequence, use the sum command in combination with the seq command.

To find the sum in Example 3 on the TI-83/TI-83 Plus, return to the LIST menu and select the MATH submenu and press $\boxed{5}$ to select sum, followed by the command to find the terms of the sequence (using the seq command discussed in the previous paragraph). (See Figure 11.)

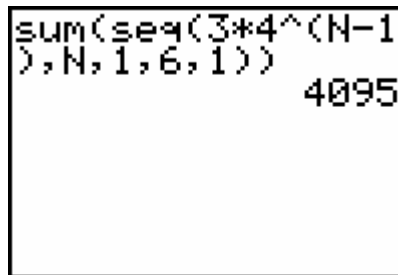


Figure 11.

On the TI-85 and TI-86, return to the OPS submenu of the LIST menu and press \boxed{MORE} $\boxed{F1}$ to select sum, followed by the command to find the terms of the sequence (using the seq command discussed in the previous paragraph). (See Figure 12.)



Figure 12.

Ordinary Annuities

By entering the values shown in Figure 13, and solving for FV, the future value of the annuity in Example 4 of this section can be determined.

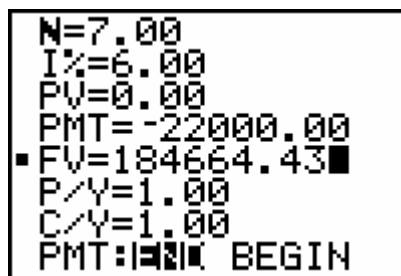


Figure 13.

To solve Example 5(a) with the TI-83/TI-83, set $N = 12 \times 20 = 240$, $I\% = 7.2$, $PV = 0$ (since it is irrelevant here), $PMT = -200$, $FV = 0$ (since it is unknown), $P/Y = 12$, $C/Y = 12$ (since interest is compounded monthly), and choose END. Solve for FV. For part (b) of the same example, replace the value of FV by 130000, and solve for $I\%$.

The SOLVER menu of the TI-85 or TI-86 can also be used to solve parts (a) and (b) of Example 5, as previously described. The TI-85, and TI-86 can be programmed to calculate the future value of an ordinary annuity. (Programs are available at the textbooks website www.aw.com/mwa8.) You can also solve part (b) graphically, using the technique of find intersection points graphically discussed in section §3.3 Applications of Linear Equations.

Annuities Due

To solve this type of problem, select BEGIN in the TVM Solver menu on the TI-83. For instance, Figure 14 represents Example 7 of the text:

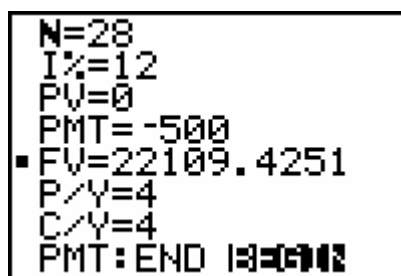


Figure 14.

Again, neither the TI-85, nor the TI-86 has built-in functions to perform the financial calculations of this section. However, they can be programmed to calculate the future value of an annuity. The SOLVER menu of the TI-85 or TI-86 can also be used as described previously.

§5.4 Present Value of an Annuity; Amortization

Present Value of an Annuity

The TVM Solver can again be used to perform the calculations necessary in this section of the text. To solve Example 1, enter the values shown in Figure 15. Note that FV is set equal to 0 since it is irrelevant to the problem. Solve for PV.

```

N=9.00
I%=7.50
■ PV=3189.44■
PMT=-500.00
FV=0.00
P/Y=1.00
C/Y=1.00
PMT: [ ] [ ] BEGIN

```

Figure 15.

Amortization

The TVM Solver and other financial functions of the TI-83/TI-83 Plus can be used to calculate the size of the periodic payments required to amortize a loan, as well as other information. These functions can be programmed into the TI-85 or TI-86.

To complete Example 3(a) set $N = 12 \times 30 = 360$, $I\% = 9.6$, $PV = 78000$ (the size of the mortgage), $PMT = 0$ (since this is unknown), $FV = 0$, $P/Y = 12$, $C/Y = 12$ (since interest is compounded monthly), and choose END. Solve for PMT to see that the payment size needs to be \$661.56 per month in order to pay off the mortgage in 30 years. (See Figure 16.)

```

N=360
I%=9.6
PV=78000
■ PMT=-661.564777
FV=0
P/Y=12
C/Y=12
PMT: [ ] [ ] BEGIN

```

Figure 16.

To solve part (b) of this example with the TI-83/TI-83 Plus, exit to the home screen by pressing $\boxed{2nd} \boxed{MODE}$. Press $\boxed{2nd} \boxed{x^{-1}}$ on the TI-83 or $\boxed{APPS} \boxed{1}$ on the TI-83 Plus, to obtain the FINANCE menu and select option A (by pressing $\boxed{ALPHA} \boxed{MATH}$), Σint . This function computes the sum of the interest paid between any two payments. Since, in part (b), we want the total interest paid throughout the 30-year mortgage, we need to apply the function to the 1st through 360th payments. So, press $\boxed{1}$, then a comma, then $\boxed{3} \boxed{6} \boxed{0}$ and press \boxed{ENTER} . (See Figure 17.)



Figure 17.

Remember that the negative sign indicates that the interest is being *paid*, not *earned*. Part (c) can be solved with the same function, this time finding the amount of interest between the 1st and 1st payments; that is, evaluate $\Sigma\text{int}(1,1)$ on the TI-83/TI-83 Plus.

To find the remaining balance on the loan at any given payment, required in Example 4, we can use the `bal` function on the TI-83/TI-83 Plus. First, if you are using the TI-83/TI-83 Plus, first verify that the payment is \$88.85 using the TVM Solver. From the home screen, press `2nd` `x-1` on the TI-83 or `APPS` `1` on the TI-83 Plus to obtain the FINANCE menu, and select option 9. Enter the payment number, in this case, 3, and press `ENTER`. (See Figure 18.)

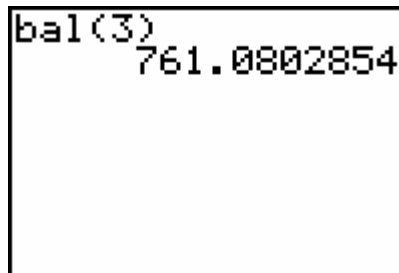


Figure 18.

Chapter 6 Systems of Linear Equations and Matrices

§6.1 Systems of Linear Equations

Entering Matrices

To enter the matrix of Example 6 on the TI-83, press **MATRX** and press the left arrow key to move to the EDIT menu. On the TI-83 Plus, press **2nd** **x⁻¹** followed by the left arrow key to move to the EDIT menu. There are ten different matrix names to choose from on the TI-83/TI-83 Plus. To begin editing matrix [A], press **1**. The dimension of the matrix must be entered first; type in the number of *rows*, press **ENTER**, then the number of *columns*, and press **ENTER** again. In Example 6, there are 3 rows and 4 columns. Type in the matrix entries, from left to right, top to bottom, pressing **ENTER** after each one. (See Figure 1.) Press **2nd** **MODE** to return to the homescreen.

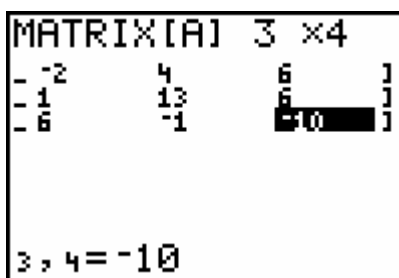


Figure 1.

To enter the same matrix on the TI-85 or TI-86, press **2nd** **7** to access the MATRX menu, then **F2** to begin the editing process. Type in a name for the matrix; for example, "A," and press **ENTER**. Follow the same steps as outlined in the previous paragraph for typing in the dimension and the individual matrix entries. (See Figure 2.) Press **EXIT** to return to the homescreen.

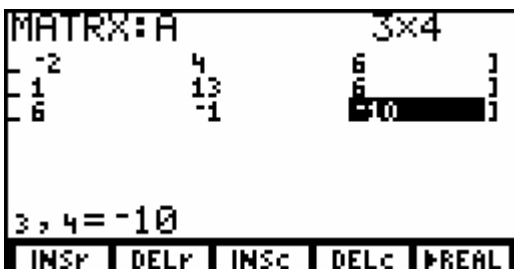


Figure 2.

The `ref` command

As mentioned in Example 6, many calculators have a single command which will perform all of the steps to transform a matrix into row echelon form using the `ref` command. First, of course, the matrix must be entered into the calculator as described above.

Once the matrix has been entered into the TI-83/TI-83 Plus, press `MATRIX` on the TI-83 or `2nd` `x-1` on the TI-83 Plus, followed by the right arrow key to move to the MATH menu, and select option A (`ALPHA` `MATH` on the TI-83/TI-83 Plus); enter the matrix name from the NAMES menu of `MATRIX` and press `ENTER` to execute the command. (See Figure 3.) Use the right arrow key to scroll through the rest of the matrix.

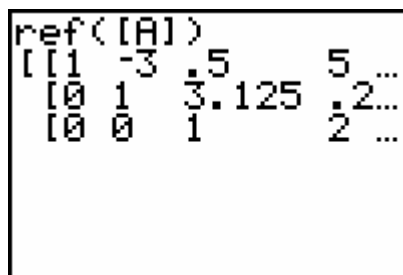


Figure 3.

If you are using the TI-85 or TI-86, press `2nd` `7`, then `F4` to access the OPS menu, and `F4` to access the `ref` command; type in the name of the matrix and press `ENTER` to complete the process. (See Figure 4.)

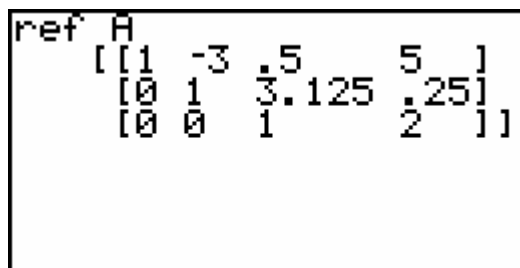


Figure 4.

§6.2 The Gauss-Jordan Method

The `rref` command

As mentioned in Example 2, many calculators have a single command which will perform all of the steps to transform a matrix into reduced row echelon form using the `rref` command. First, of course, the matrix must be entered into the calculator as described above.

Once the matrix has been entered into the TI-83/TI-83 Plus, press **MATRX** on the TI-83 or **2nd** **x⁻¹** on the TI-83 Plus, followed by the right arrow key to move to the MATH menu, and select option B (**ALPHA** **MATRX** on the TI-83 and **ALPHA** **APPS** on the TI-83 Plus); enter the matrix name from the NAMES menu of MATRX and press **ENTER** to execute the command. (See Figure 5.)

```

rref([A])
[[1 0 0 1 ]
 [0 1 0 2 ]
 [0 0 1 -1]
 [0 0 0 0 ]]

```

Figure 5.

If you are using the TI-85 or TI-86, press **2nd** **7**, then **F4** to access the OPS menu, and **F5** to access the `rref` command; type in the name of the matrix and press **ENTER** to complete the process. (See Figure 6.)

```

rref A
[[1 0 0 1 ]
 [0 1 0 2 ]
 [0 0 1 -1]
 [0 0 0 0 ]]

```

NAMES	EDIT	MATH	OPS	CPLX
dim	Fill	ident	rrf	rref

Figure 6.

Row Operations

Once a matrix has been entered, the row operations required for the Gauss-Jordan method can be performed on the calculator. The steps for performing the row operations are given below.

The row operations for completing the Gauss-Jordan process are located in the **MATRX** MATH menu of the TI-83 (**MATRX** followed by the right arrow key), in the MATRX MATH menu of the TI-83 Plus (**2nd** **x⁻¹** followed by the right arrow key), and the MATRX OPS (**2nd** **7** **F4** **MORE**) menu of the TI-85 and TI-86. The individual command names, their locations within the above menus, and their results are summarized below.

Interchange two rows

TI-83: rowSwap	Location:	Option C (ALPHA PRGM)
TI-85: rSwap	Location:	F2
TI-86: rSwap	Location:	F2

Multiply a row by a nonzero number

TI-83: *row	Location:	Option E (ALPHA SIN)
TI-85: multR	Location:	F4
TI-86: multR	Location:	F4

Add two rows

TI-83: row+	Location:	Option D (ALPHA x^{-1})
TI-85: rAdd	Location:	F3
TI-86: rAdd	Location:	F3

Add a nonzero multiple of one row to another row

TI-83: *row+	Location:	Option F (ALPHA COS)
TI-85: mRAdd	Location:	F5
TI-86: mRAdd	Location:	F5

Since the syntax for using commands grouped together above is the same for the TI-83/TI-83 Plus, TI-85 and TI-86, we will work through Example 6 using the TI-83/TI-83 Plus steps.

Once the matrix has been entered, and the calculator has been returned to the homescreen, the matrix name can be copied to an expression or command. On the TI-83, first press **MATRX**, followed by the number corresponding to the name of the matrix. On the TI-83 Plus, press **2nd** **x^{-1}** , followed by the number corresponding to the name of the matrix. (On the TI-85 or TI-86, matrix names can be typed in directly from the keypad of the calculator.) To get the required zeros and ones in the first and second columns, use the following sequence of commands, pressing **ENTER** after each (See Figures 7 through 10):

*row+(-2,[A],1,2)→[A] Multiplies row 1 of [A] by -2; adds the result to row 2.

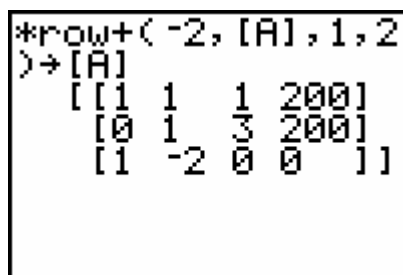


Figure 7.

*row+(-1,[A],1,3)→[A] Multiplies row 1 of [A] by -1; adds the result to row 3.

```
*row+(-1,[A],1,3)
)→[A]
[[1 1 1 200 ]
 [0 1 3 200 ]
 [0 -3 -1 -200]]
```

Figure 8.

*row+(-1,[A],2,1)→[A] Multiplies row 2 of [A] by -1; adds the result to row 1.

```
*row+(-1,[A],2,1)
)→[A]
[[1 0 -2 0 ]
 [0 1 3 200 ]
 [0 -3 -1 -200]]
```

Figure 9.

*row+(3,[A],2,3)→[A] Multiplies row 2 of [A] by 3; adds the result to row 3.

```
*row+(3,[A],2,3)
)→[A]
[[1 0 -2 0 ]
 [0 1 3 200 ]
 [0 0 8 400]]
```

Figure 10.

Note: Any time that decimals appear in a calculation, we can convert the matrix entries into fractions by accessing the `Frac` command. The `Frac` command is the first option in the `MATH` menu on the TI-83/TI-83 Plus; it is in the `MATH` MISC menu of the TI-85 and TI-86.

To complete the Gauss-Jordan method, we use the following commands, pressing `ENTER` after each (See Figures 11 through 13):

*row(1/8,[A],3)→[A] Multiplies row 3 of [A] by 1/8.

```
*row(1/8, [A], 3)→
[A]
[[1 0 -2 0 1
 [0 1 3 200]
 [0 0 1 50 1]]
```

Figure 11.

*row+(2,[A],3,1)→[A] Multiplies row 3 of [A] by 2; adds the result to row 1.

```
*row+(2, [A], 3, 1)
→[A]
[[1 0 0 100]
 [0 1 3 200]
 [0 0 1 50 1]]
```

Figure 12.

*row+(-3,[A],3,2)→[A] Multiplies row 3 of [A] by -3; adds the result to row 2.

```
*row+(-3, [A], 3, 2)
→[A]
[[1 0 0 100]
 [0 1 0 50]
 [0 0 1 50 1]]
```

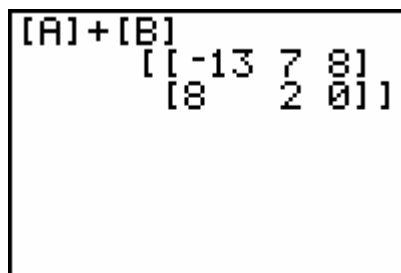
Figure 13.

§6.3 Basic Matrix Operations

Finding the Sum or Difference of Two Matrices and the Scalar Product of a Matrix

You can use the $+$, $-$, and \times keys to find the sum, difference, or scalar product of matrices. To find the sum shown in Figure 6.11 in your text, first enter the two matrices (use the names A and B respectively) and return to the homescreen.

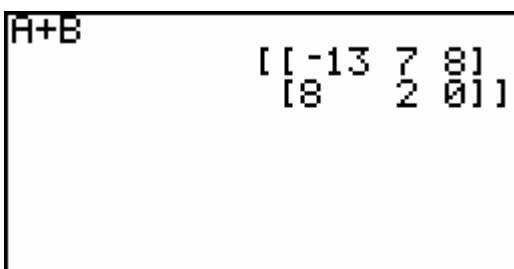
To continue on the TI-83/TI-83 Plus, return to the MATRX menu (press $\boxed{\text{MATRX}}$ on the TI-83 or $\boxed{2\text{nd}} \boxed{x^{-1}}$ on the TI-83 Plus), press $\boxed{1}$ to select matrix [A], followed by $\boxed{+}$, then return to the MATRX menu and press $\boxed{2}$ to select matrix [B] and $\boxed{\text{ENTER}}$. (See Figure 14.)



The calculator display shows the expression $[A] + [B]$ at the top left. Below it, the resulting matrix is displayed as $\begin{bmatrix} -13 & 7 & 8 \\ 8 & 2 & 0 \end{bmatrix}$.

Figure 14.

On the TI-85 and TI-86, press $\boxed{\text{ALPHA}}$ $\boxed{\text{LOG}}$, then $\boxed{+}$, followed by $\boxed{\text{ALPHA}}$ $\boxed{\text{SIN}}$ and $\boxed{\text{ENTER}}$. (See Figure 15.)



The calculator display shows the expression $A+B$ at the top left. Below it, the resulting matrix is displayed as $\begin{bmatrix} -13 & 7 & 8 \\ 8 & 2 & 0 \end{bmatrix}$.

Figure 15.

To find the difference, use the $\boxed{-}$ key, to find the scalar multiple of a matrix, use the $\boxed{\times}$ key, and to find the opposite of a matrix, use the $\boxed{(-)}$ key.

§6.4 Matrix Products and Inverses

Finding the Product of Two Matrices

You can find the product of two matrices (if it exists) using the $\boxed{\times}$ key. First, enter the matrices, then return to the homescreen and type in the operation, using $\boxed{\times}$ for multiplication. If the product does not exist, your calculator will display an error message.

In Example 3, the product BA cannot be found. If you attempt to find it on the TI-83/TI-83 Plus, you would receive the error message shown in Figure 16.



Figure 16.

On the TI-85 and TI-86 you would receive the error message shown in Figure 17.



Figure 17.

Finding the Inverse of a Matrix

You can find the inverse of a matrix using the x^{-1} key on the TI-83/TI-83 Plus or the x^{-1} operation on the TI-85 and TI-86. To find the inverse in Example 7, first enter the matrix, then return to the homescreen and type in the matrix name.

To continue on the TI-83/TI-83 Plus press x^{-1} and ENTER . (See Figure 18.)

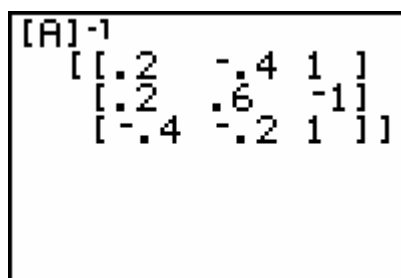


Figure18.

Similarly, on the TI-85 and TI-86, press 2nd EE and ENTER . (See Figure 19.)

$$A^{-1} = \begin{bmatrix} .2 & -.4 & 1 \\ .2 & .6 & -1 \\ -.4 & -.2 & 1 \end{bmatrix}$$

Figure 19.

A Note About Round-off Errors and Matrix Inverses

Occasionally, when performing calculations involving matrix inverses, one or more entries in the resulting matrix may look like “1 E -10,” or something similar. Keep in mind that this represents 1 times 10 to the -10th power, or 0.0000000001. A result like this in a problem that originally contained no numbers of this form is due to round-off error in the calculator. Usually, it is safe to treat any similar results as “0.”

§6.5 Applications of Matrices

Creating an Identity Matrix

As mentioned before Example 6 in the text, graphing calculators can be used to determine production for an input-output model in essentially one step. Once matrices A and D have been entered into the calculator, the appropriate identity matrix can be created, and the expression $(I - A)^{-1}D$ can be calculated.

On the TI-83/TI-83 Plus, an identity matrix can be created using the *identity* command, located in the *MATRX MATH* menu. Access this command by pressing $\boxed{5}$. The command must be followed by a number, n , which represents the number of rows of the desired identity matrix. (Remember to follow this number with $\boxed{)}$ on the TI-83/TI-83 Plus.) In Example 6, the appropriate identity matrix will have dimension 2×2 , so we would type “*identity* (2)” for this matrix. The necessary command to complete Example 6 is shown in Figure 20.

$$(identity(2) - [A])^{-1} * [D] = \begin{bmatrix} 818.905007 & \\ 1427.234441 & \end{bmatrix}$$

Figure 20.

If you are using the TI-85 or TI-86, an identity matrix can be created by pressing $\boxed{2nd} \boxed{7}$ to access the MATRX menu, then $\boxed{F4}$ for the OPS menu and $\boxed{F3}$ for the ident command. Type in the number of desired rows for the matrix to finish defining the identity matrix. If you are using an identity matrix in an expression, you should enclose the identity command in parentheses. For instance, the expression for Example 6 should appear on the home screen as shown in Figure 21.

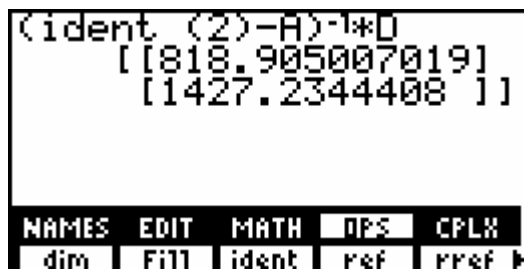


Figure 21.

Chapter 7 Linear Programming

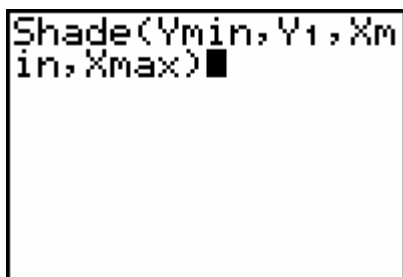
§7.1 Graphing Linear Inequalities in Two Variables

Graphing One Linear Inequality in the Plane

Example 3 in this section of your text requires the use of a graphing calculator. All of the calculators discussed in this manual can complete this problem.

If you are using one of the TI models, first set an appropriate range for the viewing window and store “.4x+2” as a function. Make sure all plots and other unnecessary functions are turned “off,” then view the graph to see if you need to make any changes in the viewing window.

Since we want to shade *below* the graph of the line, on the TI-83/TI-83 Plus, press **2nd** **PRGM** to access the DRAW menu and select option 7: Shade. Use the following command line: `Shade(Ymin, function name, Xmin, Xmax)`. (See Figures 1 and 2.) You can either access the names Xmin, Xmax and Ymin in the Window submenu of the **VAR** menu, or type in the actual values you chose. If, instead, you need to shade *above* the graph of a function, use the command line `Shade(function name, Ymax, Xmin, Xmax)`.



```
Shade(Ymin, Y1, Xmin, Xmax)
```

Figure 1.

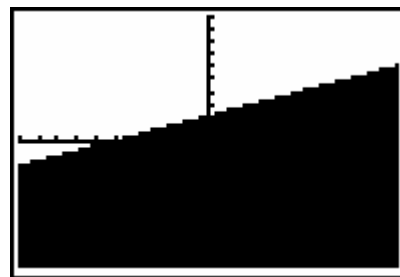


Figure 2.

On the TI-85 and TI-86, press **GRAPH** **MORE** **F2** to access the DRAW menu and press **F1** to select Shade (. Use the same command line: `Shade(yMin, function name, xMin, xMax)`. (See Figures 3 and 4.) You can either access the names xMin, xMax and yMin in the Wind submenu of CATLG-VARS on the TI-86 or the Range submenu of Vars on the TI-85, or type in the actual values you chose. If, instead, you need to shade *above* the graph of a function, use the command line `Shade(function name, yMax, xMin, xMax)`.

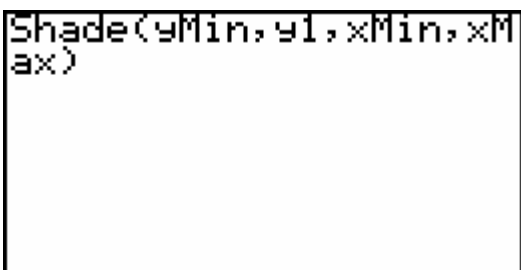


Figure 3.

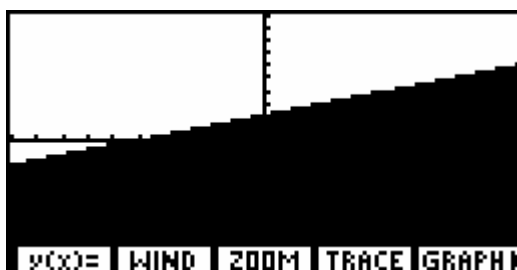


Figure 4.

Systems of Inequalities

To graph systems of inequalities on the TI models, first graph the equations suggested by the inequalities (replace the inequality symbol by an equality symbol), find the point of intersection, and use the following command line: *Shade(lower function name, upper function name, left endpoint, right endpoint)*.

§7.4 The Simplex Method: Maximization

Row Operations

See the detailed instructions for Chapter 6 for the location of commands and necessary syntax for performing row operations on the TI models. Programs are available at the textbooks website www.aw.com/mwa8, which includes Simplex method programs for all the calculators.

§7.6 The Simplex Method: Duality and Minimization

In this section, you are required to find the transpose of a matrix. To do so with your calculator, first enter the original matrix under a matrix name and return to the home screen.

If you are using one of the TI-83/TI-83 Plus, access the name of the matrix you just stored. Now access the MATRX menu and select the MATH submenu. Press $\boxed{2}$ to select the T option. This will return you to the home screen and place the T symbol immediately after the matrix name. Press $\boxed{\text{ENTER}}$ to find the transpose of the matrix. (See Figure 3, which represents Example 2(a).)

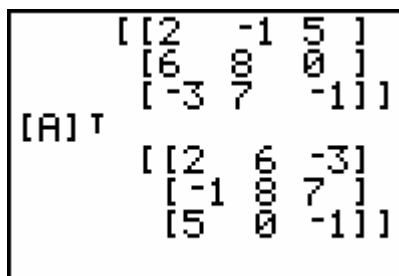


Figure 5.

On the TI-85 and TI-86, access the MATRX menu and select the MATH submenu. Press **F2** to select the T option. This will return you to the home screen and place the T symbol immediately after the matrix name. Press **ENTER** to find the transpose of the matrix. (See Figure 6, which represents Example 2(a).)

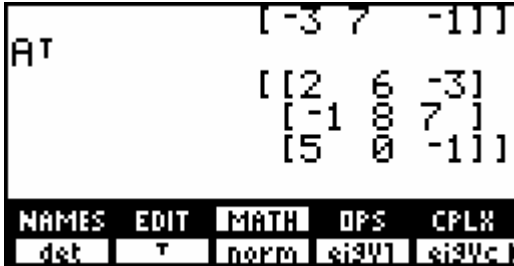


Figure 6.