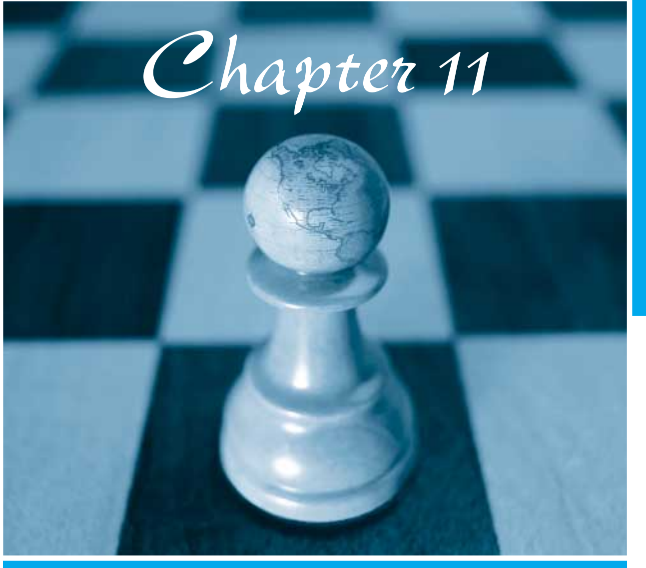


Chapter 11

The World of Oligopoly: Preliminaries to Successful Entry

The economy that we are studying in this book is still extremely primitive. At the present time, it has only a few productive enterprises, all of which are monopolies. This economy is certainly far from the type of highly competitive free enterprise system that we are familiar with in the United States. It is unlikely that such a system will grow out of the situation described in Chapter 10, where we saw a government-regulated natural monopoly develop to supply pure water to the community. To find the origins of competitive markets, we will have to investigate other industries in which the technology is such that several firms, if not many firms, can survive simultaneously at the equilibrium.

In this chapter, we will see a new industry that is not a sustainable or natural monopoly develop in our primitive society. This industry produces a recently discovered product called a gadget. The first entrepreneur in the industry quickly realizes that once she establishes her firm and begins to sell her product, other firms will attempt to imitate the product and enter the industry. Unless she can prevent the entry of such potential competitors, the industry will rapidly undergo a transformation from a monopoly to a *duopoly* to an *oligopoly*. As we investigate the events that occur in this industry, we will examine the theory of duopoly and oligopoly—the theory of markets with two or a few competing firms. This theory will be of major importance in Chapter 12 when we see our gadget maker plan a strategy to keep potential entrants out of her market.

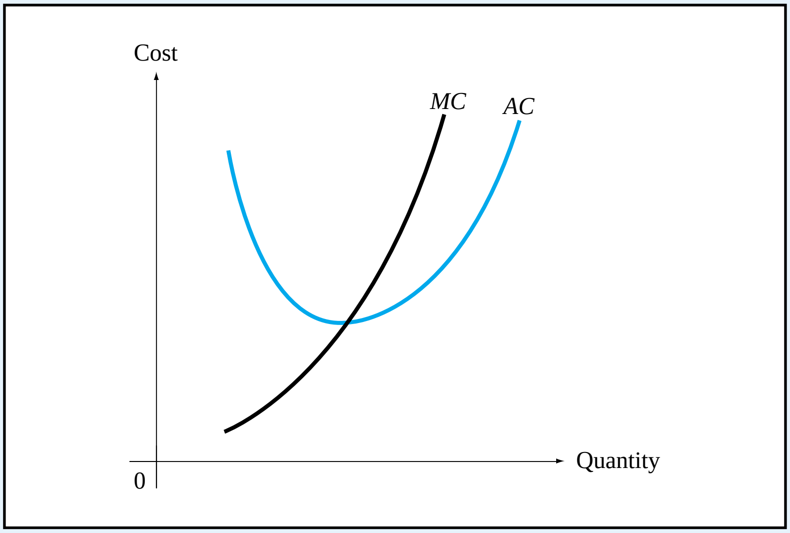
11.1 Production in a Nonnatural Monopoly Situation

Let us assume that an agent in our primitive society comes upon a technology to make a product that we will call a gadget. This technology is such that the marginal cost of producing gadgets rises as more are produced and rises at an increasing rate. Figure 11.1

Figure 11.1

Marginal and Average Cost Curves for a Firm That Produces Gadgets.

Marginal cost rises at an increasing rate as gadget output rises.



shows such a marginal cost curve along with the assumed average cost curve for the firm producing the gadgets.

Remember that we usually depict marginal cost curves as being U-shaped, like the average cost curve shown in Figure 11.1. We could easily have done so again, but, for the sake of simplicity, we will assume that in this case the marginal cost is always increasing. There is no contradiction between a U-shaped average cost curve and a constantly rising marginal cost curve because the firm presumably has fixed costs that make the average cost of production high at low levels of output. As these average fixed costs fall with the increase in output, so does the average cost until the rising marginal cost of production pulls the average cost up again.

We will assume that the inverse demand for gadgets is a simple linear function $p = A - bq$, where p is the market price of the good, A is a constant, b is the slope of the inverse demand function, and q is the total output placed on the market. As we saw in Chapter 9, this function tells us the maximum price attainable for any given quantity sold and is therefore called the *inverse demand function.*

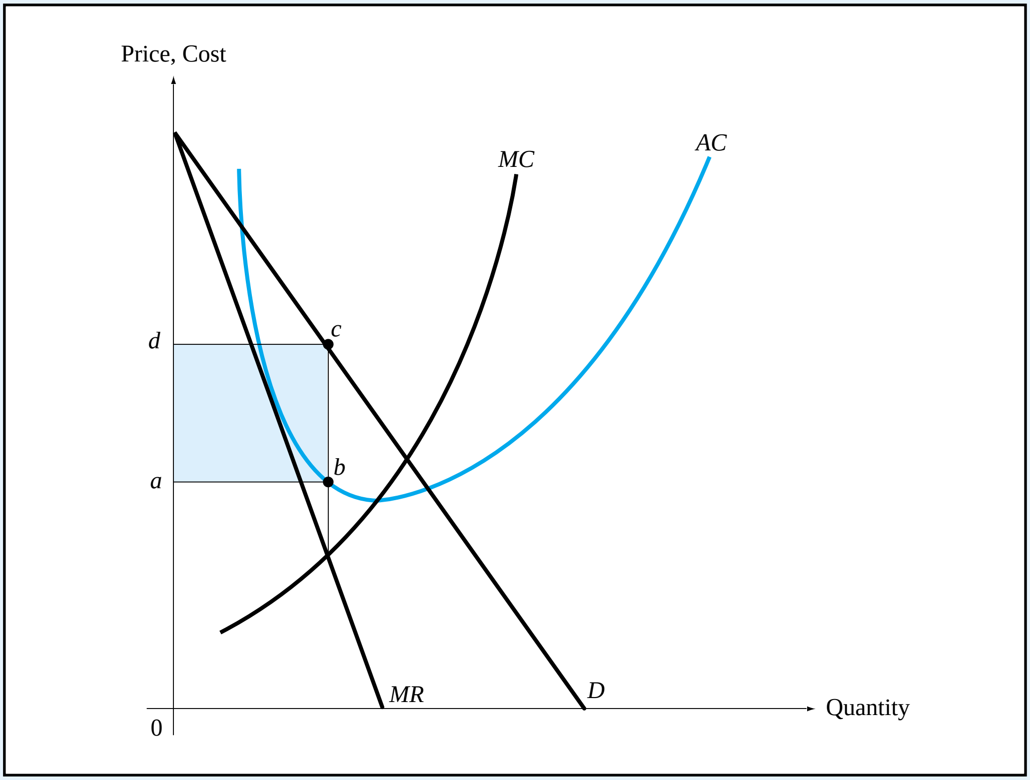
Our gadget maker sees immediately that if she can keep competitors out of her market, she will be able to make a substantial profit for herself. Figure 11.2 demonstrates this fact by showing her demand function superimposed on her cost function.

Figure 11.2 also indicates the monopoly price that the gadget maker can set in the absence of any competitors in the market. If she uses this price, it will yield profits equal to the area *dcba*. Unfortunately, however, her technology does not give her a natural monopoly. She will have to battle to keep competitors from entering her market because, presumably, two or even more firms will be able to profitably coexist in this market.

Figure 11.2

The Profit-Maximizing Output for the Gadget Monopoly.

If there are no other market entrants, the entrepreneur can earn monopoly profits that are equal to the area *dcba.*



11.2 The Cournot Theory of Duopoly and Oligopoly

As our gadget maker starts to plan a strategy for keeping other firms out of her market, she realizes that in order to know how to prevent entry, she needs an understanding of what the industry might be like with competition present. For example, what kind of market equilibrium might she face if the industry is a **duopoly**—that is, if there are two firms selling the product? To obtain this information, our gadget maker hires a consulting firm, which bases its opinions on **Cournot's theory of duopoly,** a famous theory that was devised in the nineteenth century to analyze the behavior of quantities, prices, and profits in a two-firm market.[1]

The consultants issue a report that summarizes the essential features of the **Cournot model.** (Although this model is for a duopoly, it can be, and often is, extended to an **oligopoly**—a market that is dominated by a few sellers of a product.)

To understand the Cournot model better, let us assume that there are two firms in the gadgets market and that these firms face a linear inverse demand function of $p = A - b(q_1 + q_2)$. Clearly, the price that will prevail in the market will be a function of the outputs of *both* duopolists, q_1 and q_2. We can express the cost function for firm 1 as $C(q_1)$ and the cost function for firm 2 as $C(q_2)$.

Note that while the demand function and the price depend on the output levels of both duopolists, each duopolist's cost function is determined by its own output level. We will assume that both cost functions have marginal and average costs as depicted in Figure 11.1. Marginal costs rise as output grows and rise at an increasing rate, while average costs are U-shaped.

Given a two-firm industry with the linear demand function and cost functions shown in Figure 11.1, we want to use the Cournot model to find the answers to the following questions: What will be the *equilibrium* output levels of the two duopolists? What price will prevail in the market? What profit will each firm make? By equilibrium, we mean a pair of output levels, one for each firm, which are such that after they are chosen, neither firm has any incentive to change its output level. This type of equilibrium is called a **Cournot equilibrium.** It is simply the Nash equilibrium that we defined in Chapter 7 applied to a model in which duopolistic or oligopolistic firms compete with each other by choosing output levels. To understand how a Cournot equilibrium is reached, we must examine the concept of reaction functions.

Reaction Functions

In every market, there is a strategic interaction among firms. Each firm in the market will respond to the actions of the other firms in some manner. These responses are summarized by what are called **reaction functions** or **best-response functions.** The reaction function specifies a firm's optimal choice for some variable such as output, given the choices of its competitors.

Using the Cournot model, let us assume that both firms in a duopolistic market want to make as much profit as they possibly can. However, each firm has a problem because its profit depends on the output level its competitor chooses as well as the output level it chooses, and it does not know what the choice of its competitor will be when it makes its own choice. We can summarize this situation by saying that both duopolists (firms 1 and 2) want to maximize their profits, as indicated by the following profit functions: $\pi_1 = (A - b(q_1 + q_2)) \cdot q_1 - C(q_1)$ and $\pi_2 = (A - b(q_1 + q_2)) \cdot q_2 - C(q_2)$.

---

[1] Antoine Augustin Cournot (1801–1877) was a French mathematician, economist, and philosopher. He was one of the first scholars to use mathematical techniques to analyze economic problems. In his most noted work, which was published in 1838, Cournot examined problems of pricing in monopolistic, duopolistic, oligopolistic, and perfectly competitive markets. This work appeared in English as *Researches into the Mathematical Principles of the Theory of Wealth,* translated by Nathaniel Bacon (New York: Macmillan, 1897).

Consulting Report 11.1

Using the Cournot Model to Determine an Equilibrium for a Duopolistic Market

The consultants explain that the Cournot model is based on two key concepts about the firms in a duopolistic market: that each will behave in a profit-maximizing manner and that each will assume that the other firm will keep its output constant at the existing level when it changes its own output. We can think of the Cournot model as one in which firms alternate making decisions about the quantity they wish to produce. First, one firm chooses what it considers to be a profit-maximizing level of output. Then, given that firm's choice of a quantity and assuming it will not change, the other firm sets its own profit-maximizing quantity.

This process of adjustment continues through several stages of action and reaction until the two firms reach an equilibrium and have no further incentive to change their outputs. During the entire process of adjustment, each firm believes that the other firm's current level of output is fixed and uses this assumption in selecting its own level of output.

In the Cournot model, the quantity, price, and profits produced at the equilibrium for a duopolistic market will be between those that occur in a monopolistic market and those that occur in a perfectly competitive market.

Note that the price that either duopolist faces $(A - b(q_1 + q_2))$ and the profit it will earn depend on the output of both duopolists. Note also that each profit function is composed of two parts. The first part is the revenue component of profit, which is represented by the price of the good $(A - b(q_1 + q_2))$ times the output of the firm, q_1 or q_2. The second part of the profit function is simply the duopolist's cost, $C(q_1)$ or $C(q_2)$.

Let us now say that firm 2 decides to produce $\overline{q}_2$ units of output. Under the Cournot model, firm 1 assumes that no matter what output choice it makes, firm 2 will not change its own output choice in response. Economists today call this assumption the **Cournot conjecture.** More generally, we will let the **conjectural variation** denote the change that a firm expects in its competitor's choice of an output level in response to a change the firm made in its own output level. Using this definition, we can say that the conjectural variation in the Cournot model is zero. (Later we will consider oligopoly models with nonzero conjectural variations.)

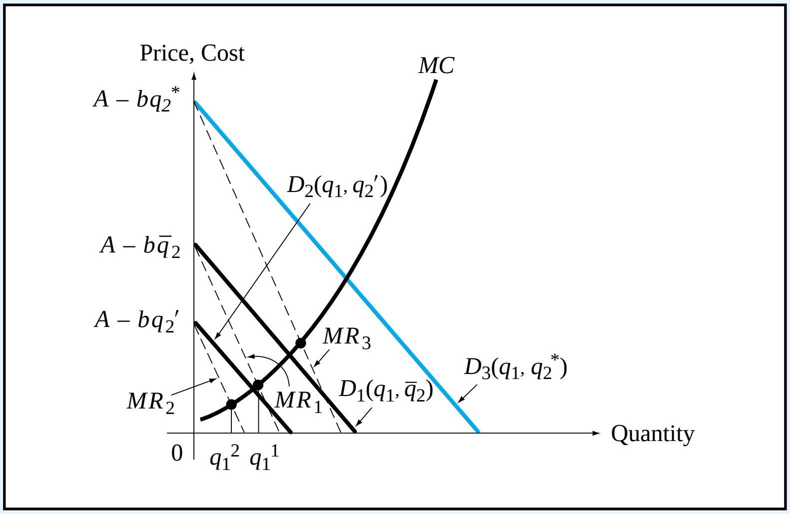
Given firm 2's decision, firm 1's profit is now solely determined by its own output choice, which means that we can express its profit function as $\pi_1 = (A - b(q_1 + \overline{q}_2)) \cdot q_1 - C(q_1)$. In a sense, firm 1 is now a monopolist because with firm 2's output fixed at $\overline{q}_2$, the price of the good is only a function of the output choice of firm 1. Given firm 2's decision, firm 1 should now choose an output level that equates its marginal revenue to its marginal cost. To see how the optimal output for firm 1 is derived, consider Figure 11.3.

Figure 11.3 shows the demand function and the associated marginal revenue function facing firm 1 at different levels of output set by firm 2. To understand how these functions

Figure 11.3

The Optimal Output for a Gadget Duopolist.

Given firm 2's production of $\bar{q}_2$, firm 1 maximizes its profit by choosing output level q_1^1, which equates its marginal cost to its marginal revenue given that $\bar{q}_2$ units are already in the market.



are determined, consider what happens when firm 2 chooses an output of $\bar{q}_2$. If the inverse demand curve for the product is $p = A - b(q_1 + q_2)$, then with an output of $\bar{q}_2$ for firm 2, the inverse demand curve facing firm 1 will be $p = (A - b\bar{q}_2) - bq_1$. In other words, if firm 2's output is fixed at $\bar{q}_2$ and firm 1 sets an output of zero, then the price will be $A - b\bar{q}_2$. The output of firm 2 reduces the price from A, which is what it would be if *both* firms set a zero output. As firm 1 raises its output, the price will fall even further and the slope of the demand curve will be b.

In Figure 11.3 the demand curve labeled D_1 is the one that will result when firm 2 sets its output at $\bar{q}_2$. If firm 2 chooses a higher level of output, say q_2', then the demand curve for firm 1 will shift toward the origin, as shown by D_2. The reason for this shift is that the price that will result now if firm 1 sets an output of zero is $A - bq_2'$, which is less than $A - b\bar{q}_2$, because $q_2' > \bar{q}_2$. Note, however, that the slope of the demand curve remains the same. If firm 2 chooses a level of output that is lower than $\bar{q}_2$, say q_2^*, then the demand curve for firm 1 will be further from the origin, as depicted by D_3. In this case, if firm 1 sets an output of zero, the market price will be $A - bq_2^*$, which is greater than $A - b\bar{q}_2$, because $q_2^* < \bar{q}_2$.

In Figure 11.3 we also see the marginal cost curve for firm 1. Depending on the output level chosen by firm 2, we can now define the output level that represents the

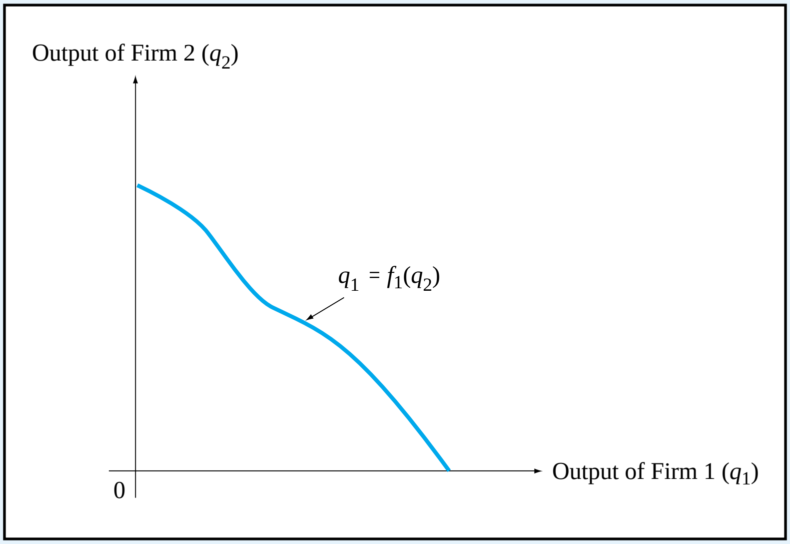


Figure 11.4

The Reaction (Best-Response) Function for Gadget Duopolist 1.

Given firm 2's choice of q_2, firm 1's optimal response is $q_1 = f_1(q_2)$.

best response—the profit-maximizing choice—for firm 1. Finding the best response is a simple matter. First we locate the demand curve for firm 1 that is associated with the quantity chosen by firm 2, and then we find the output level at which firm 1's marginal revenue equals its marginal cost. For example, say that $\bar{q}_2$ is the quantity set by firm 2. We therefore look at the demand curve labeled D_1 and the marginal revenue curve labeled MR_1 in Figure 11.3 and see that q_1^1 is the optimal level of output for firm 1. It is firm 1's best response because it is the output level that maximizes the firm's profit. Similarly, if firm 2 sets a quantity of q_2', the relevant demand and marginal revenue curves for firm 1 are D_2 and MR_2, and its best response is to choose an output level of q_1^2. In this way, we can define a best response for firm 1 to every hypothetical output level of firm 2. The reaction function for firm 1 is formally presented in Figure 11.4.

Note that in Figure 11.4 the output level of firm 1 is inversely related to the output level of firm 2. The more firm 2 produces, the less firm 1 will produce. It is important to realize that each point on this reaction function represents the *optimal (profit-maximizing) choice or best response* of firm 1 to a possible output level of firm 2.

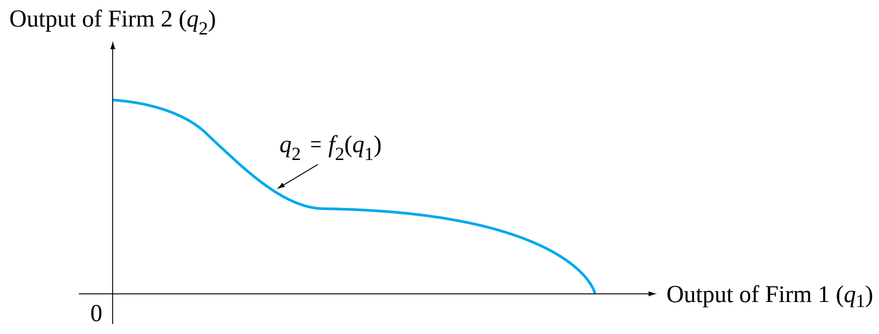
A similar analysis can be made for firm 2 in order to derive its reaction function and find its best response to each level of output that firm 1 might choose. Such a curve appears in Figure 11.5.

In Figure 11.5, we again see that the optimal or profit-maximizing output level for one duopolist (firm 2) is a decreasing function of the output level chosen by the other duopolist (firm 1), given the Cournot conjecture. For convenience, we can express these reaction functions for the duopolists in the gadgets market as $q_1 = f_1(q_2)$ and $q_2 = f_2(q_1)$.

Figure 11.5

The Reaction (Best-Response) Function for Gadget Duopolist 2.

Given firm 1's choice of q_1, firm 2's optimal response is $q_2 = f_2(q_1)$.



Solved Problem 11.1

Question (Application and Extension: The Best Responses)

Assume that there are two firms, each producing with a constant marginal cost function of $MC = 2$. Assume that the demand function for the product is defined as follows: $p = 100 - .5(q_1 + q_2)$. If firm 2 sets a quantity of $q_2 = 100$, what is the best response quantity for firm 1.

Answer

If firm 2 sets a quantity of $q_2 = 100$, then the residual demand curve facing firm 1 is $p = 100 - .5(q_1 + 100) = 100 - .5(100) - .5q_1 = 50 - .5q_1$. If the marginal cost of production is constant at 2, then the firm should set a quantity at which the marginal revenue associated with the residual demand curve is equal to the constant marginal cost. We know that the marginal revenue curve associated with this demand curve has the form $MR = 50 - q_1$, since it has a slope twice the steepness of the demand curve. Setting $MR = MC$ means setting $50 - q_1 = 2$. Solving, we see that $q_1 = 48$, and this is the best response.

An Alternative Derivation of Reaction Functions

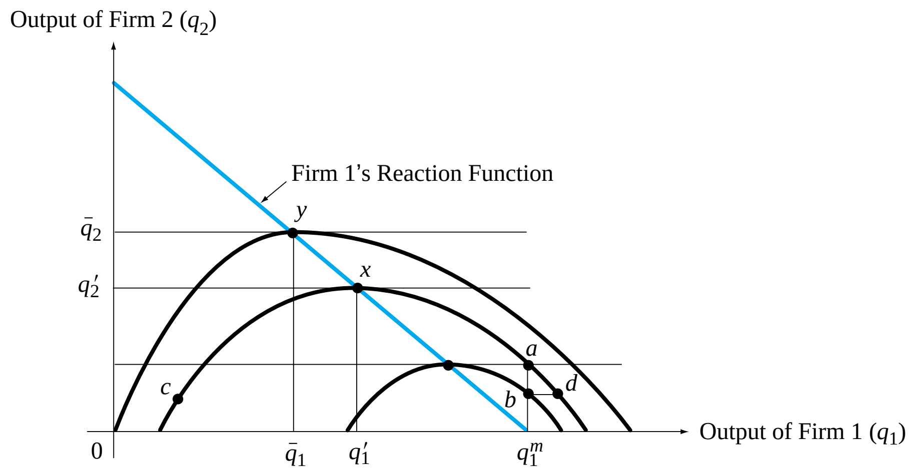
There is an alternative, and perhaps simpler, way to derive the reaction function for a firm. Consider Figure 11.6.

In Figure 11.6 each point, like point x, represents output levels for firm 1 and firm 2. For example, at point x, firm 1 produces q_1' and firm 2 produces q_2'. At point q_1^m, the output

Figure 11.6

Reaction Functions.

For each level of q_2, firm 1's reaction function gives the level of q_1 that places firm 1 on the lowest attainable isoprofit curve (that is, the level of q_1 determined by the tangency of the isoprofit curve and a horizontal line). Given $q_2 = q_2'$, firm 1 chooses $q_1 = q_1'$; given $q_2 = \bar{q}_2$, firm 1 chooses $q_1 = \bar{q}_1$.



of firm 1 is q_1^m, and the output of firm 2 is zero. In other words, q_1^m is firm 1's monopoly output. This output combination of (q_1^m, 0), in which firm 1 produces its monopoly output and firm 2 produces zero output, yields profits for firm 1 that are greater than the profits it can earn with any other output combination.

Now look at the output combination at point *a* of Figure 11.6. At that point, firm 1 continues to produce its monopoly output of q_1^m, but now firm 2 produces a positive output. Clearly, firm 1 will receive lower profits at point *a* than it will at point q_1^m, because the positive output of firm 2 decreases the price that firm 1 can obtain for its output. Let us now locate the **isoprofit curves** in this space—the sets of points that yield the same profits to firm 1. We will start by looking for those output combinations that yield the same profits as point *a*. Let us examine point *b*, where again firm 1 produces the monopoly output of q_1^m, but now firm 2 is producing less than it did at point *a*. Clearly, the reduction in firm 2's output raises firm 1's profits above what they were at point *a*. To bring the profits of firm 1 back to what they were at point *a*, we have two options. We can either move to point *c* or point *d*. At point *c*, firm 1 has lower output than it did at point *b*, which raises the price of gadgets but lowers the firm's profits because it is now selling a smaller quantity of the product. At point *d*, firm 1 has higher output than it did at point *b*, which allows it to sell a greater quantity, but the additional output lowers the price and also increases the firm's costs. The net result is that points *c*, *a*, and *d* all have the same profit levels. In general, the isoprofit curves for firm 1 have the shape shown in Figure 11.6. The curves closer to the horizontal axis (and closer to the monopoly output level) contain higher levels of profit.

To derive the reaction functions for firms 1 and 2, let us look first at firm 1. For any given output level chosen by firm 2, the reaction function should tell us the profit-maximizing output level for firm 1. Say that firm 2 sets an output level of q_2'. Given this choice by firm 2, firm 1 will want to choose the output level that places it on the lowest possible isoprofit curve because profits increase as firm 1 moves toward the horizontal axis. This output level will be characterized by the tangency of the isoprofit curve and the line drawn parallel to the horizontal axis at the height of q_2'. Such a tangency occurs at point x in Figure 11.6, where the output level is q_1'. When firm 2 chooses a higher level of output, such as $\bar{q}_2$, tangency occurs at point y and the optimal level of output for firm 1 falls to $\bar{q}_1$. By successively choosing different levels of output for firm 2 and finding the tangency points for firm 1, we can trace firm 1's reaction function. A similar analysis can produce firm 2's reaction function.

Deriving a Cournot Equilibrium

To find the Cournot equilibrium for this duopolistic market, we can simply take the two reaction functions for firms 1 and 2, place them on the same diagram, and see where they intersect. The point of intersection represents the equilibrium, as shown in Figure 11.7.

Note that the two reaction functions intersect at point e in Figure 11.7. At this point, firm 1 is producing an output of q_1^e and firm 2 is producing an output of q_2^e. If firm 1 produces q_1^e, then the best response for firm 2 is to produce q_2^e. Similarly, if firm 2 produces q_2^e, then the best response for firm 1 is to produce q_1^e. In short, q_1^e and q_2^e are the best responses of the two firms to each other. If both firms choose these output levels, neither firm will have an incentive to change its choice. Put differently, if these output levels are set, they will remain unchanged—the market will be in equilibrium.

Stable and Unstable Cournot Duopolies

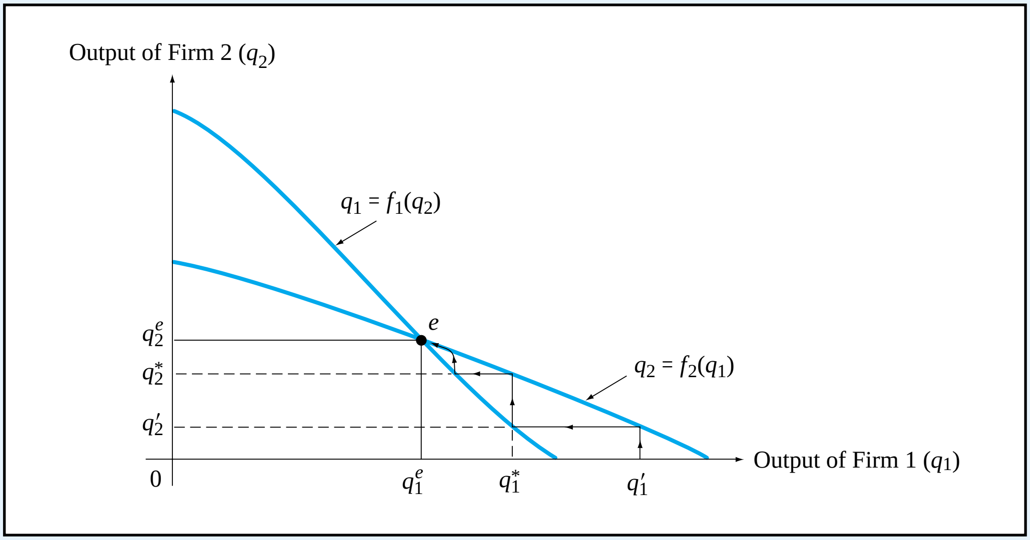
At this point in our analysis, the following question arises. According to the Cournot theory of duopoly, the gadgets market will be in equilibrium *if* firms 1 and 2 choose output levels q_1^e and q_2^e, but what guarantee do we have that the two firms will actually choose these output levels? There are two responses to this question. The first response is that the Cournot theory does not claim that the duopolists will choose these output levels. All it says is that *if* they do choose these output levels, the market will be in equilibrium. The second response goes further and says that we can actually expect that the output levels in the market will eventually reach q_1^e and q_2^e.

Let us examine the reasoning behind the second response. Say that we are not at the equilibrium in Figure 11.7. Also say that firm 1 chooses an output level that is higher than its equilibrium output level of q_1^e, such as q_1'. From firm 2's reaction function, we see that it will then choose an output level of q_2', which is lower than its equilibrium output level of q_2^e. However, when firm 2 chooses q_2', firm 1's reaction function indicates that it will decrease its output from q_1' to q_1^*. With the output of firm 1 at q_1^*, the reaction function of

Figure 11.7

The Reaction Function Equilibrium (Nash Equilibrium) for the Gadget Duopoly.

The intersection of the two firms' reaction functions at ($q_1{}^e$, $q_2{}^e$) is the point at which each firm is responding optimally to the other's choice.



firm 2 shows that it will now increase its output from q_2' to q_2^*, and so on. This process is *convergent.* If allowed to continue, it will lead the firms to converge on q_1^e and q_2^e, which are the equilibrium output levels for the market.

Does convergence on the equilibrium depend on how we draw the reaction functions? The answer to this question is yes. In Figure 11.7 the reaction functions are drawn in such a way that the one for firm 2 is flatter than the one for firm 1. If the opposite is true, then we will have the situation depicted in Figure 11.8.

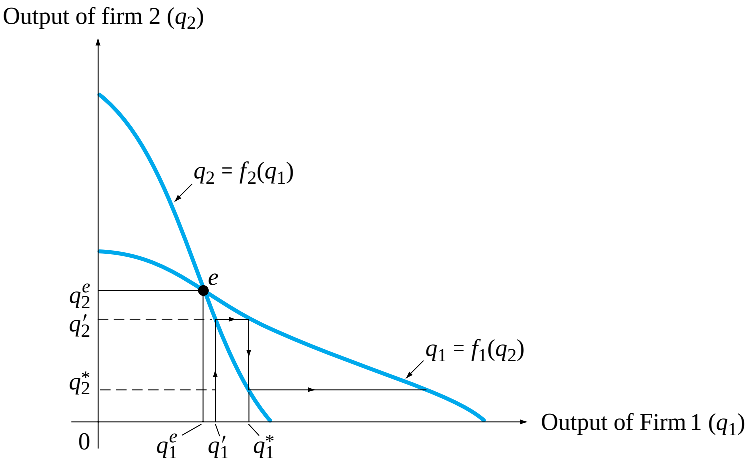
If the reaction functions are shaped as shown in Figure 11.8, it will be difficult for the market to converge to the equilibrium. In this case, it is still true that if the market ever reaches the equilibrium, it will remain there, but we cannot rely on the process of convergence to bring about the equilibrium.

To illustrate this point, let us say that the two firms are not at the equilibrium in Figure 11.8. We will assume that firm 1 chooses an output that is higher than its equilibrium output of q_1^e, such as q_1'. From firm 2's reaction function, we can see that it will then choose q_2', which is lower than its equilibrium output of q_2^e. When firm 2 chooses q_2', firm 1's reaction function shows that it will increase its output from q_1' to q_1^*, moving it further away from the equilibrium. With the output of firm 1 now at q_1^*, the reaction function of firm 2 indicates that it will decrease its output from q_2' to q_2^*, moving it further away from the equilibrium as well, and so on. This process is *divergent.* At each stage, it moves the two firms further away from the equilibrium.

Figure 11.8

An Unstable Cournot Equilibrium.

The reaction function of the firm whose output is measured on the vertical axis is steeper than the reaction function of the firm whose output is measured on the horizontal axis. In this case, the reactions of the two firms to a disequilibrium situation will take them further and further away from the equilibrium point.



Solved Problem 11.2

Question (Application and Extension: Nash Equilibrium Quantities)

Assume that you have two firms competing in a market and that these firms have the following reaction functions:

$$q_1 = 50 - \frac{1}{2}q_2$$

$$q_2 = 50 - \frac{1}{2}q_1$$

a) For what quantity produced by firm 2 would firm 1 prefer to shut down and produce nothing?
b) What quantity would firm 1 produce if firm 2 never existed?
c) Verify that $q_1 = 33.3$ and $q_2 = 33.3$ is a Nash equilibrium.

Answers

a) Obviously, if firm 2 set a quantity of 100, then $q_1 = 50 - \frac{1}{2}(100) = 0$ and firm 1 would shut down.

b) If firm 2 never existed, then $q_2 = 0$ and the best output for q_1 would be 50.

c) We can verify that 33.3 for each firm is a Nash equilibrium by seeing if those quantities constitute a best response for each firm. Plugging 33.3 into each reaction function we see that $33.33 = 50 - \frac{1}{2}(33.33)$; thus, if firm 1 is choosing 33.33, firm 2 will also want to choose 33.33, and vice-versa. That is the definition of an equilibrium.

Using Game Theory to Reinterpret the Cournot Equilibrium

Our previous analysis of the Cournot equilibrium can be restated in terms of game theory. We can think of the strategic interaction between firms in a duopolistic market as a game, which we might call the **simultaneous-move quantity-setting duopoly game.** In this game, there are two players, firms 1 and 2; and each player has a strategy set from which it can choose a feasible strategy whenever it must make a move. These strategy sets are equivalent to all the positive output levels in the Cournot model. However, to simplify the game, it might make sense to restrict the strategy sets to those output levels between 0 and A/b. Because A/b drives the price of the good to zero, we can presume that rational firms will never choose output levels above A/b. The payoff functions for this game are presented in two equations that we used previously: $\pi_1 = (A - b(q_1 + q_2)) \cdot q_1 - C(q_1)$ for firm 1 and $\pi_2 = (A - b(q_1 + q_2)) \cdot q_2 - C(q_2)$ for firm 2. These are the components of the game's normal form.

The game is played as follows: First, both firms choose their output levels simultaneously, with neither firm knowing what level the other firm has chosen. Once these quantities are placed on the market, the demand curve tells the players what the price will be, and each firm calculates its payoffs (profits) accordingly. The equilibrium defined by the Cournot model is nothing more than the Nash equilibrium in this simultaneous-move quantity-setting duopoly game.

Solved Problem 11.3

Question (Application and Extension: Best Response Functions and Nash Equilibria)

Bob and Art live next door to each other and share a driveway between their houses. Both men are botanists and love flowers. Each has decided to plant pansies on his side of the driveway. On one Saturday morning, each announces to the other that he is going to his own favorite gardening shop to purchase pansies. Since each man will plant the flowers so close to each other, both will get enjoyment from his own and his neighbor's pansies.

Art's preferences for how many pansies to purchase depend on how many Bob has bought and can be described by the best-response function

$$A = 20 - \frac{1}{2}B$$

where B is the number of pansies bought by Bob. Bob's preferences can be described by his best-response function

$$B = 24 - \frac{1}{4}A$$

where A is the number of pansies Art buys.

a) If Bob finds out that Art is going to buy 12 pansies, how many should he buy?
b) What is the equilibrium amount of pansies the men should buy if neither knows how many the other is going to purchase?

Answer

a) If Art buys 12 pansies, Bob's best response is

$$B = 24 - \frac{1}{4}(12) = 21$$

Thus, Bob should buy 21 pansies.

b) The equilibrium is found by solving the two best-response functions simultaneously—that is, solving the following two equations:

$$A = 20 - \frac{1}{2}B$$

$$B = 24 - \frac{1}{4}A$$

for A and B.

This can easily be done by substituting Bob's function into Art's function and solving,

$$A = 20 - \frac{1}{2}\left(24 - \frac{1}{4}A\right)$$

$$= 8 + \frac{1}{8}A$$

$$\frac{7}{8}A = 8$$

$$A = \frac{64}{7}$$

and substituting back into Bob's best-response function

$$B = 24 - \frac{1}{4}\left(\frac{64}{7}\right)$$

$$= \frac{152}{7}$$

Thus, assuming that pansies (like all other goods) are perfectly divisible, in equilibrium Art buys $\frac{64}{7}$ pansies and Bob buys $\frac{152}{7}$. Since Art's best response is to buy $\frac{64}{7}$ pansies if Bob is going to buy $\frac{152}{7}$ pansies, and Bob's best response is to buy $\frac{152}{7}$ pansies if Art is going to buy $\frac{64}{7}$, each pansy decision is a best response to the other. This is the definition of an equilibrium.

11.3 Criticisms of the Cournot Theory: The Stackelberg Duopoly Model

An Asymmetric Model

Our gadget maker thinks that she now understands the Cournot theory and its game theory interpretation quite well. However, she questions the relevance of the Cournot theory to her situation because she is worried about preventing potential competitors from entering the market in which she is already entrenched. As a result of this entrenchment, she believes that any entrant will view her as a kind of leader and will view himself, a relative upstart, as a follower. In other words, she feels that the Cournot theory treats firms or players symmetrically, but, in reality, the situation she faces is *asymmetric*, with her firm established as the leader and any firm that enters the market now taking the role of a follower. (Imagine a new firm starting to manufacture automobiles in the United States and having to compete with General Motors and Ford. Clearly, a theory that treats all firms in this type of market symmetrically would be unrealistic.) To obtain information about how an asymmetric market functions, our gadget maker turns to another consulting firm, one that bases its opinions on the work of Heinrich von Stackelberg, another economist who studied the problem of duopoly and developed a well-known duopoly model.[2] The new consulting firm issues the following report in which it describes the main features of the **Stackelberg model.**

Let us now take a closer look at the Stackelberg model by applying it to the gadgets market. We will assume the same demand, cost, and profit functions as we did with the Cournot model. We will say that demand is linear, that marginal costs are strictly increasing, and that profits are represented by the following two equations: $\pi_1 = (A - b(q_1 + q_2)) \cdot q_1 - C(q_1)$ for firm 1 and $\pi_2 = (A - b(q_1 + q_2)) \cdot q_2 - C(q_2)$ for firm 2. We will also assume

[2] Heinrich von Stackelberg was a German economist who examined market organization and the strategic interaction of firms. He proposed the leader-follower concept for duopolistic markets in *Marktform und Gleichgewicht* (Vienna: Julius Springer, 1934). This work appeared in English as *The Theory of the Market Economy*, translated by A.T. Peacock (New York: Oxford University Press, 1952).

Consulting Report 11.2

Using the Stackelberg Model to Determine an Equilibrium for a Duopolistic Market

The consultants explain that the Stackelberg duopoly model is an extension of the Cournot model but allows for asymmetric behavior by the two firms in a duopolistic market. The Stackelberg model assumes that one firm will play an aggressive role in the market (be the leader) and the other firm will play a passive role (be the follower). The leader will choose its level of output first. It will set a profit-maximizing quantity, taking into consideration the quantity it expects the follower to set in reaction to its own choice. The leader assumes that the follower will also want to maximize its profits but that it will accept the leader's output choice as a given. This assumption permits the leader to predict the follower's output choice and take that choice into account when it makes its own output choice.

that each firm has a Cournot reaction function defining its best response to any given output level chosen by its competitor. We will denote these reaction functions as $q_1 = f_1(q_2)$ for firm 1 and $q_2 = f_2(q_1)$ for firm 2.

To make our example asymmetric, assume that firm 1 chooses its quantity first. Then, firm 2, *knowing what firm 1 has done,* makes its choice. After both firms have sequentially chosen their outputs, the price of the good on the market and the profits of both firms are determined. Because firm 1 moves first, it is the **Stackelberg leader** and can commit itself to a fixed output. Firm 2, the **Stackelberg follower,** then takes firm 1's output as a given and chooses a best response. In such a model, there is an advantage to moving first, as depicted by Figure 11.9.

In Figure 11.9 we see the reaction function of firm 2, the Stackelberg follower in this market. Firm 1, the Stackelberg leader, knows that for any output level it might set, firm 2 will set the output level that represents its best response to firm 1's choice. Therefore, firm 1 can predict firm 2's choice of an output level from its reaction function. If firm 1 is rational, it will choose the output level that maximizes its profits *after taking into consideration firm 2's best response to that output level.*

The Stackelberg Equilibrium

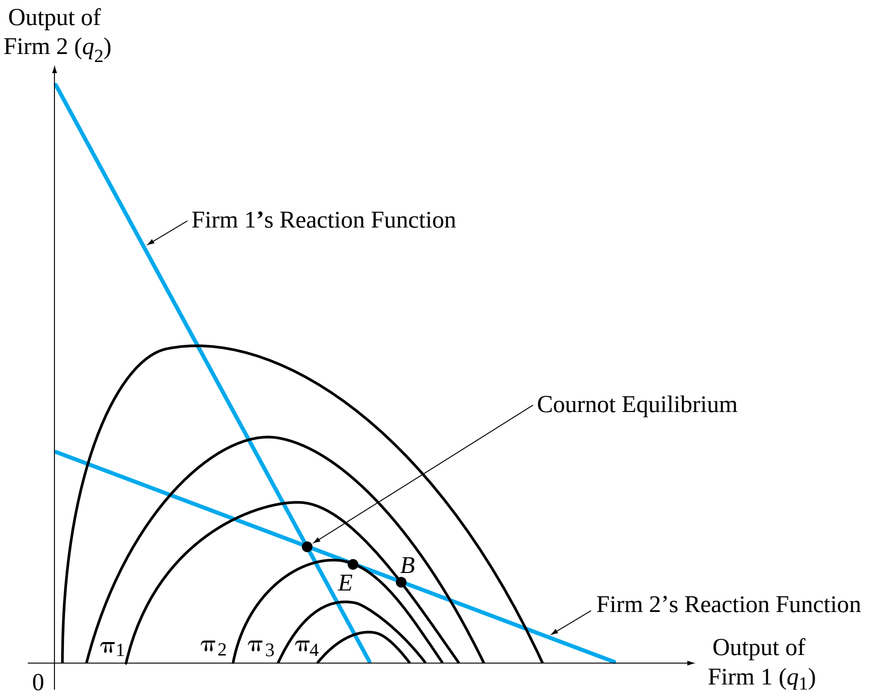
To understand what output level would be consistent with a Stackelberg equilibrium, let us look at Figure 11.9 again. Note that the isoprofit curves of firm 1 are superimposed on firm 2's reaction function. The task for firm 1 is to choose the output level that will place it on the lowest possible isoprofit curve consistent with firm 2's choice of an optimal (profit-maximizing) quantity. In short, by moving first, firm 1 can actually choose at which point on firm 2's reaction function it wants the market to be—the point where its own profits will be greatest. In Figure 11.9 this profit maximization occurs at point *E,* where firm 1's isoprofit curve is tangent to firm 2's reaction function. Such a point represents a **Stackelberg equilibrium.**

To demonstrate that point *E* in Figure 11.9 must be a Stackelberg equilibrium, consider any other point such as *B.* Note that point *B* is on isoprofit curve π_1, which involves

Figure 11.9

The Stackelberg Solution.

Firm 1 (the leader) chooses the point on the reaction function of firm 2 (the follower) that is on the lowest attainable isoprofit curve of firm 1: point *E*.



lower profits than isoprofit curve π_2, the one where point E is located. No output combination on isoprofit curves like π_3 and π_4 can be the equilibrium because it is not on firm 2's reaction function and therefore does not fit our assumption that firm 2 will act in a rational manner.

Algebraically, we can describe the Stackelberg model as follows: Let $q_2 = f(q_1)$ be firm 2's reaction function and let $\pi_1 = (A - b(q_1 + q_2)) \cdot q_1 - C(q_1)$ be firm 1's profit function. However, because firm 1 knows that firm 2 will respond to its output choice by choosing a best response, we can replace q_2 in firm 1's profit function by $f_2(q_1)$. Firm 1's profit function now reads $\pi_1 = (A - b(q_1 + f_2(q_1)) \cdot q_1 - C(q_1)$. Firm 1's problem is simply to maximize this profit function by choosing q_1, knowing that firm 2 will respond optimally to its choice.

As we see in Figure 11.9, at the Stackelberg equilibrium, firm 1 chooses a higher level of output than it previously did at the Cournot equilibrium and receives greater profits. This is the essence of the **first-mover advantage** that the leader has in the Stackelberg model.

The Stackelberg model matches our gadget maker's view of her market as an asymmetric one, in which her firm will be the leader and an entrant will be the follower. Remember that we characterized the Cournot model as a simultaneous-move quantity-setting duopoly game. Similarly, we can think of the Stackelberg model as a **sequential-move quantity-setting duopoly game** that results in a greater payoff for the leader and a smaller payoff for the follower than they would receive at the Cournot equilibrium of the same market.

11.4 The Welfare Properties of Duopolistic Markets

As our primitive society begins to develop markets where two or a few firms compete, a question naturally arises about the welfare aspects of the Cournot equilibrium. Do such markets produce a better welfare outcome than monopolistic markets? The answer to this question is as follows: The Cournot equilibrium outputs for firms in duopolistic markets yield better welfare results than those that occur in monopolistic markets (when welfare is measured in terms of consumer surplus plus producer surplus), but the welfare results in such markets are not optimal. They are in between the welfare levels produced in perfectly competitive markets and those that occur in monopolistic markets. To prove this statement, let us again turn our attention to the gadgets market—a duopolistic market. As we did previously, we will assume that the inverse demand for gadgets is linear and is represented by $p = A - b(q_1 + q_2)$. For the sake of simplicity, we will also assume that the marginal cost of production is zero. (The results that we derive would not be different if we were to assume that the marginal cost is U-shaped or strictly rising, as in Figure 11.1.) Because each firm has zero marginal cost, it will set its marginal revenue equal to zero when the other firm chooses a level of output. Reformulating the problem, with the assumption of zero marginal cost, we can express the profit function for the duopolists as follows: $\pi_1 = (A - b(q_1 + q_2)) \cdot q_1$ for firm 1 and $\pi_2 = (A - b(q_1 + q_2)) \cdot q_2$ for firm 2. Note that when the marginal cost is zero, maximizing profits is the same as maximizing revenue.

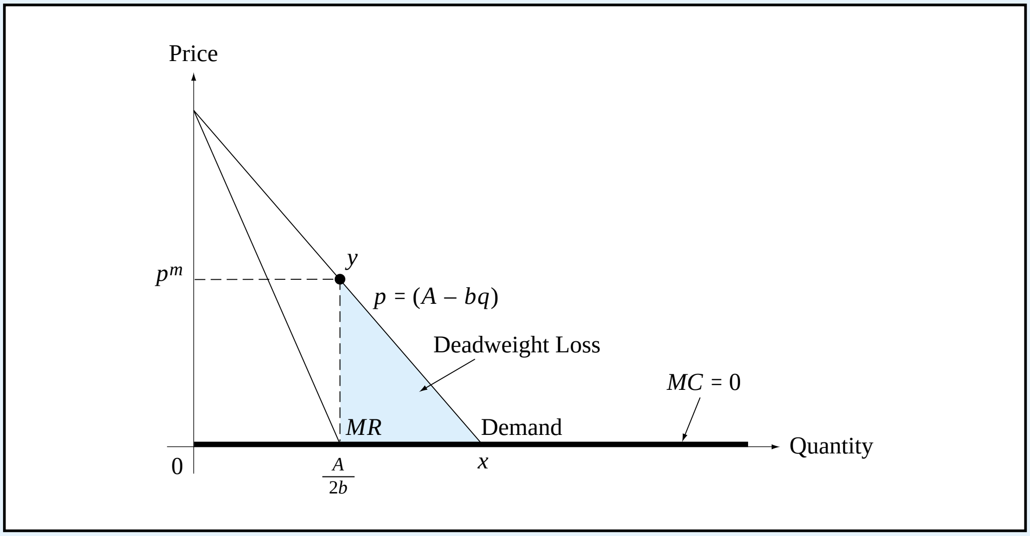
We can derive the reaction functions for the two firms from these profit functions by equating the marginal revenue for each firm to zero after the other firm has set its level of output. Using partial derivatives, we find that the marginal revenue is $MR_1 = A - 2bq_1 - bq_2$ for firm 1 and $MR_2 = A - 2bq_2 - bq_1$ for firm 2. Solving for q_1 and q_2 will give us the reaction functions of the two firms. These reaction functions will specify the profit-maximizing output that each firm should set for any given output of the other firm. We can express the reaction functions as follows: $q_1 = (A - bq_2)/2b$ for firm 1 and $q_2 = (A - bq_1)/2b$ for firm 2.

If a monopolist with the same cost structure were to provide all the gadgets for this market, then she would have to determine how much of output q to produce so as to maximize her profits. This problem can be stated as: Max $\pi = (A - bq)q$. Note that because there is only one producer, the total output (q) for the market is the same as the sum of the

Figure 11.10

The Monopoly Solution with Zero Marginal Costs.

The monopolist will choose output $A/2b$, at which the marginal revenue equals the marginal cost of zero. At the welfare-optimal output level, x, the price equals zero. The deadweight loss is area $(A/2b)xy$ under the demand curve and between the monopoly and welfare-optimal output levels.



firms' output ($q = q_1 + q_2$). To maximize this function, we take the derivative and make it equal to zero. We then find that the optimal monopoly output is $q = A/2b$.

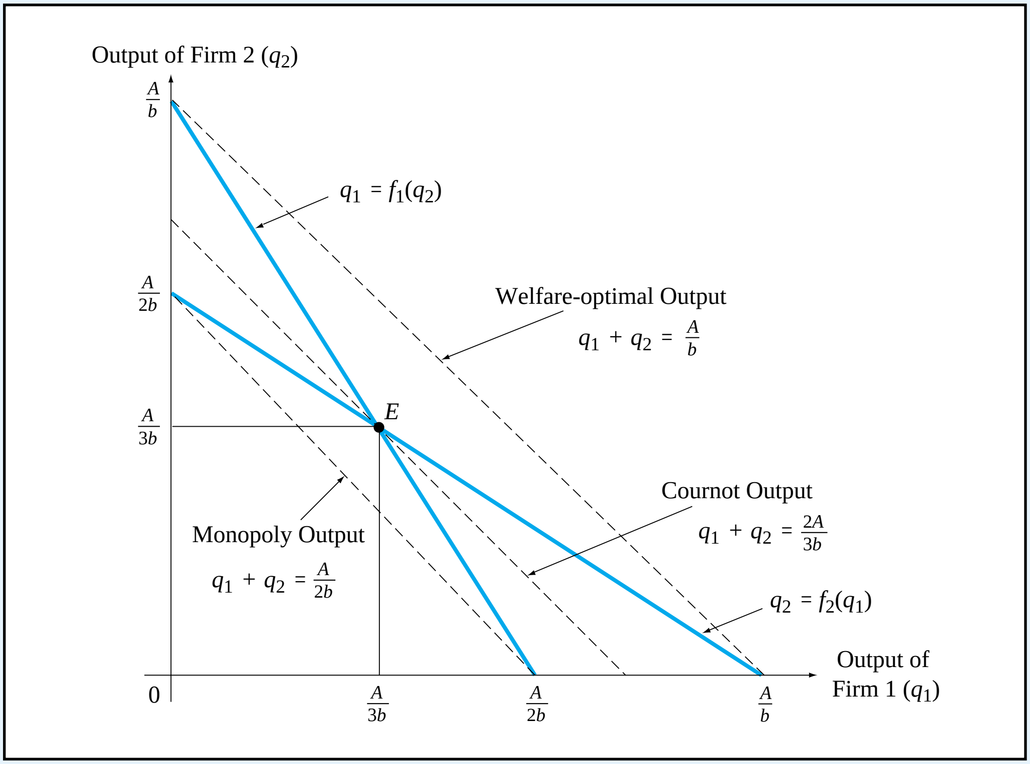
Figure 11.10 represents the monopolistic market by our familiar inverse demand and marginal revenue curves. However, note that in this example, the marginal cost curve is flat and moves along the horizontal axis.

In Figure 11.10 we see the market or aggregate demand for the product along with the marginal cost curve of the monopolist (which is the same as the marginal cost curve of each duopolist). We also see that the monopolist will choose an output of $A/2b$ and a price of p^m. This monopoly price-quantity combination will create a deadweight loss of consumer surplus equal to the area in the triangle $(A/2b)xy$. The welfare-optimal price will be zero, and the welfare-optimal quantity will be x units. Note that with an inverse demand curve of $p = A - bq$, a zero price results when $A = bq$ so that $q = A/b$, which is the output at point x. In other words, because the marginal cost is zero, any consumer willing to pay more than a price of zero should be allowed to buy gadgets, which means that the optimal price must be zero. The monopoly and welfare-optimal price-quantity combinations therefore represent

Figure 11.11

The Equilibrium Price and Quantity Compared.

The Cournot equilibrium, with a total quantity of $2A/3b$, is on an iso-output line strictly between the monopoly line (a total quantity of $A/2b$) and the welfare-optimal line (a total quantity of A/b).



the two extremes between which prices and quantities can fall. Let us now demonstrate that in a duopolistic market, the price and the quantity will be between these two extremes. Consider Figure 11.11.

Solved Problem 11.4

Question (Application and Extension: The Equilibrium Price and Quantity Under Duopoly)

a) The town of Deadeye has only one chicken farmer to supply eggs. Assume that the marginal cost of egg production is zero and that in Deadeye the inverse demand for eggs (measured in cents) is $p = 100 - 2q$. At what price and quantity will the chicken farmer sell eggs?

b) Now assume that a second chicken farmer moves to Deadeye who also has zero marginal cost. Taking demand into consideration, this means that chicken farmer 1 has a reaction function

$$q_1 = 25 - \frac{1}{2}q_2$$

where q_1 and q_2 are the quantities of eggs supplied by chicken farmers 1 and 2, respectively. Chicken farmer 2, likewise, has a reaction function

$$q_2 = 25 - \frac{1}{2}q_1$$

What will the quantities supplied and price for eggs be now?

Answer

a) The chicken farmer will set the quantity that equates marginal revenue to marginal cost. Since demand is linear, marginal revenue will be $MR(q) = 100 - 4q$. Thus, to find the optimal quantity

$$100 - 4q = 0$$
$$q = 25$$

Putting the quantity back into the demand curve, we find that the price of eggs is

$$100 - 2(25) = 50¢$$

b) To find the quantities, simultaneously solve the two reaction functions.

$$q_1 = 25 - \frac{1}{2}\left(25 - \frac{1}{2}q_1\right)$$
$$.75q_1 = 12.5$$
$$q_1 = 16\frac{2}{3}$$

Substituting back into farmer 2's reaction function, we find that q_2 is also equal to $16\frac{2}{3}$. This is a result of the fact that the two chicken farmers are symmetric; they face identical demands and marginal costs. Whenever this occurs, both firms in the Cournot duopoly game will produce identical amounts.

The price of eggs is, then,

$$p = 100 - 2\left(16\frac{2}{3} + 16\frac{2}{3}\right) = 33\frac{1}{3}¢$$

Notice that the price has gone down and the total quantity supplied of eggs has gone up as compared to the case when there was only one chicken farmer. In the case when there was only one chicken farmer, consumer surplus was \$6.25. When the second chicken farmer entered the market, consumer surplus increased to \$11.11. This shows that increased competition lowers the price for the consumer.

In Figure 11.11 we see our familiar reaction functions, this time for firms that have a zero marginal cost. Note that the horizontal axis represents the output of firm 1, while the vertical axis represents the output of firm 2. From the reaction functions of these two firms, which we previously saw in equation form as $q_1 = (A - bq_2)/2b$ and $q_2 = (A - bq_1)/2b$, we find that when firm 2 produces a zero output, firm 1's best response is to set the monopoly output because, in effect, firm 1 is a monopolist. The monopoly output for firm 1 occurs at point $A/2b$ along the horizontal axis. If firm 1 produces a zero output, firm 2's best response is to choose the monopoly output at point $A/2b$ along the vertical axis. Similarly, if either firm were ever to choose the welfare-optimal output of A/b, the other firm's best response would be to set an output of zero. For example, suppose that firm 1 is at point A/b, on the horizontal axis. At this point the price would be zero, so firm 2's best response is the produce zero (see point *x* in Figure 11.10).

Now consider the line drawn between the two monopoly outputs in Figure 11.11 (the line between points $A/2b$ and $A/2b$). Any output combination along this line is such that the *total* output on the market will be the monopoly output. The points along this line differ only according to the amount that each firm individually supplies to the market. A line of this type is called an **iso-output line.** Because the market price depends on the *sum* of the outputs produced, the price at any point on the iso-output line will be the same. For example, the price at any point on the line between the monopoly outputs $A/2b$ will be the monopoly price. Similarly, consider the line between the welfare-optimal outputs A/b. At any point along this iso-output line, the *total* output is equal to the welfare-optimal quantity and the price is equal to the welfare-optimal price. The only thing that differs at any point on the line is the portion of the welfare-optimal quantity that each firm supplies.

Finally, note that point *E* in Figure 11.11, the Cournot equilibrium quantity ($2A/3b$), is on an iso-output line strictly between the monopoly line and the welfare-optimal line[3] Because price decreases as quantity produced increases, it must be true that at the Cournot equilibrium, the price and the quantity are between the monopoly and welfare-optimal levels. This proves our premise that in a duopolistic market, the outcome for society (as measured by the consumer surplus plus the producer surplus) will be between the monopoly outcome and the welfare-optimal outcome. Note that this result will occur for any technologies that produce reaction functions with the same general shape as the ones in Figure 11.11, not just for technologies with zero marginal costs.

[3]Note that the Cournot output $(2/3)\ (A/b)$ can be written as $(n/(n+1)) \cdot A/b$, where n is the number of firms in the market. As n becomes large and moves toward infinity, $(n/(n+1)) \cdot A/b$ approaches A/b, the competitive result. We will derive this result more formally in Chapter 14.

Solved Problem 11.5

Question (Application and Extension: Monopoly, Duopoly, and Consumer Surplus)

Consider a market with a demand curve for a product produced by two firms, firms 1 and 2. The demand for the products can be summarized by the following demand function:

$$p = 50 - \frac{1}{2}(q_1 + q_2)$$

There is zero marginal cost.

Given this demand function, the reaction function for each firm is $q_1 = 50 - \frac{1}{2}q_2$ and $q_2 = 50 - \frac{1}{2}q_1$. As discussed in Solved Problem 11.2 earlier in the chapter, the Nash equilibrium for this market is one where each firm sets a quantity of 33.3. The monopoly solution is one where the total quantity on the market is 50. What is the improvement in consumer surplus by having the market organized as a duopoly?

Answer

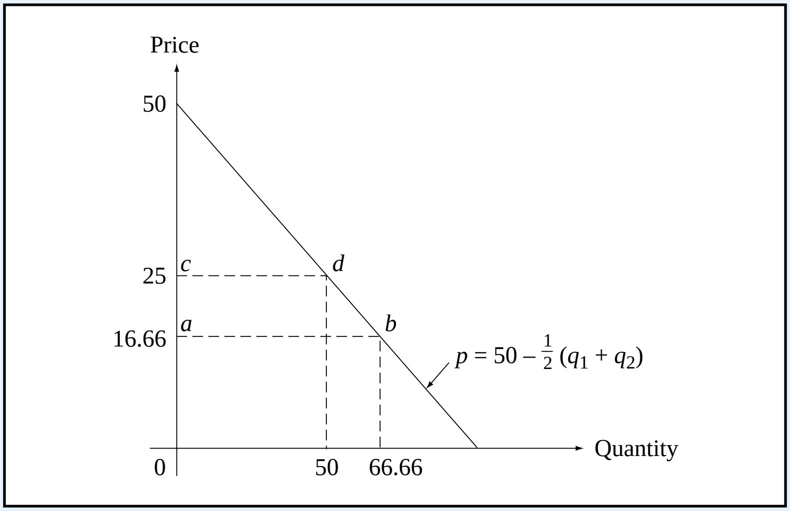
Let us answer this question with a diagram (see Figure 11.12). In this diagram we have the market demand curve

$$p = 50 - \frac{1}{2}(q_1 + q_2)$$

and we have the two quantities of 66.66 and 50. As we see, if the price for the good in the market is 50 or more, the quantity demanded is zero. No one is willing to pay more than 50 for the good. Using the demand curve, we see that if both firms produce 33.33, the total

Figure 11.12

Welfare Comparisons of Monopoly and Duopoly.



quantity is 66.66 and the market price is $p = 50 - \frac{1}{2}(66.66) = 16.66$. At the monopoly quantity, the market price is $p = 50 - \frac{1}{2}(50) = 25$. Now in the diagram we see that the consumer surplus is represented by the triangle above the line *ab* for the duopoly situation (that is, the line representing a quantity of 66.66 and a price of 16.66) and the triangle above the line *cd* in the monopoly situation (that is, the line representing a price of 25 and a quantity of 50). The areas of these triangles are as follows:

$$CS_{monopoly} = \frac{1}{2}(50 - 25)(50) = 625$$

$$CS_{duopoly} = \frac{1}{2}(50 - 16.66)(66.66) = 1{,}111.22$$

11.5 Criticisms of the Cournot Theory: The Bertrand Duopoly Model

One feature of the Cournot model that often strikes people as odd is the fact that it assumes that firms compete in a market by choosing the quantities of a good they will produce. Our usual perception is that firms compete through the prices they charge for their goods. For example, when we look at the advertisements in a newspaper, we see automobile dealers, consumer electronics stores, supermarkets, and many other types of firms competing on the basis of price. Clearly, the view that firms compete through the prices they choose is at variance with the assumption of the Cournot model that they compete through the quantities they decide to produce. Our gadget maker is one of the people who questions the validity of the Cournot model for just this reason. She therefore asks for advice about the nature of a duopolistic market from still another consulting firm. This firm bases its opinions on the work of Joseph Bertrand, who had the same reservations about the Cournot model and, as a result, developed a different kind of duopoly model.[4] In their report, the consultants summarize the basic features of the **Bertrand model.**

Let us use a simple example to look at the Bertrand model more closely. Say that two firms, *i* and *j*, sell an identical product. According to the Bertrand model, these firms can think of themselves as facing a demand function with the following characteristics: If one firm charges a price that is above the price set by its competitor, the demand for its product will be zero. If it charges a price below the price set by its competitor, it will capture all the demand in the market. (In the Bertrand model, it is assumed that all firms have enough production capacity to supply the entire market. Later, we will see what happens when this assumption is violated.) If both firms set the same price for the product, they will split the demand in the market. This type of demand function can be depicted in algebraic terms as follows:

[4]Joseph Bertrand was a nineteenth-century French mathematician and economist. His critique of the Cournot model and presentation of an alternative model appears in "Theorie Mathematique de la Richesse Sociale," *Journal des Savants* (September 1883): 499–508.

Consulting Report 11.3

Using the Bertrand Model to Determine an Equilibrium for a Duopolistic Market

The consultants tell our gadget maker that the Bertrand model is based on a number of assumptions. The most important of these assumptions is that if two firms are selling an identical product, consumers will buy from the firm that charges the lower price. Therefore, in the Bertrand model, firms set prices and allow the market to determine the quantities. Of course, this is the opposite of what happens in the Cournot model.

According to the Bertrand model, each firm in a duopolistic market sets a profit-maximizing price in the belief that the price chosen by its rival will not change. This belief encourages the two firms to engage in a process of competitive price-setting until the market arrives at an equilibrium. Thinking that the price set by its rival is fixed, first one firm and then the other firm changes its price in order to take customers and profits away from its rival. Eventually, the two firms reach an equilibrium at which neither has an incentive to change its price any further. This equilibrium occurs when the price of the product falls to its marginal cost.

In the Bertrand model, a duopolistic market produces the same equilibrium as a perfectly competitive market in terms of price, quantity, and profits. The two firms share the market and earn zero profits.

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j \\ (1/2)(D(p_i)) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

The first line of this demand function tells us that if firm i sets a lower price than firm j, all consumers will buy from firm i at price P_i, and firm i will be able to sell as much as the market wants at that price, $D(p_i)$. The second line of the demand function tells us that if firms i and j set the same price, they will split the market demand between them at that price. Finally, the third line of the demand function tells us that if firm i sets a higher price than firm j, it will sell nothing. If we assume that each firm can produce the product at a constant marginal cost of c, then the payoff to each firm will be $\pi_i = p_i(D_i(p_i, p_j)) - c(D_i(p_i, p_j))$.

The Bertrand Equilibrium

If we think of our example of the Bertrand model in terms of a game, the players are firms i and j, their strategy sets are all the pairs of positive prices they can charge, and their payoffs are the ones just described. We can characterize this game as a **simultaneous-move price-setting duopoly game.** In the first move of the Bertrand game, each firm sets a price. Seeing these prices, the consumers then decide which firm to buy from. They do so according to the demand function specified previously. The payoffs from the game are determined by the pricing decisions of the two firms and the buying decisions of the consumers. An equilibrium for this game is a pair of prices that, once they are set, are such that neither firm has any incentive to change its price given the price of its opponent. Just as the Cournot equilibrium is a Nash equilibrium for the quantity-setting game, the **Bertrand equilibrium** is a Nash equilibrium for the price-setting game.

Even though the Cournot and Bertrand equilibria are applications of the same **equilibrium concept,** they lead to very different outcomes. The Cournot equilibrium produces a price and a quantity that are intermediate between the monopoly and welfare-optimal levels, but the Bertrand equilibrium results in the welfare-optimal price and quantity. We can express the latter outcome as the Bertrand proposition: At the equilibrium of the simultaneous-move price-setting duopoly game, the price of the product is driven down to its marginal cost and the quantity sold in the market is the welfare-optimal quantity.

Proving the Bertrand Proposition

We can prove the Bertrand proposition without too much difficulty. As we do so, note that our proof illustrates the central idea of price competition in a free-market economy.

Let us say that, contrary to the Bertrand proposition, the equilibrium price is not the same as the marginal cost of the product for either firm. Indeed, we will assume that $p_i > p_j > c$, so that the price set by firm *i* is greater than the price set by firm *j*. In this case, firm *i* will sell no goods and receive no profit (assuming that the firm produces only after it knows its demand), but firm *j* will earn a positive profit because its price is above *c*. Clearly, firm *i*'s best response to this situation is to set a price for its product that is just below the price of firm *j*. To be more specific, let us say that firm *i* will set a price of $p_i = p_j - \epsilon > c$, where ϵ is some arbitrarily small decrease in firm *j*'s price, because, by slightly underpricing its competitor, firm *i* can capture the entire market and make a positive profit (since $p_i - \epsilon > c$). Firm *j* will then respond in a similar way by setting a still lower price, such as $p_j = p_i - \epsilon - \epsilon$, and thereby recapture the market from firm *i*. Hence, a pair of unequal prices above the marginal cost of the product cannot be a Bertrand equilibrium. The two firms involved will simply continue to make competitive price reductions until the price reaches the marginal cost.

Can a pair of equal prices above the marginal cost of the product be a Bertrand equilibrium? The answer to this question is no. At $p_i = p_j > c$, both firms will share the market. However, such an arrangement is not stable because if either firm merely reduces its price by ϵ, it will capture the entire market. An infinitesimal reduction in price can produce much higher profits. Hence, there is again an incentive for both firms to decrease their prices until they reach the marginal cost of the product.

Because we cannot have a Bertrand equilibrium if the two firms set different or equal prices that are above marginal cost, the only other arrangements left are for one firm to set a price at marginal cost while the other firm sets a price above it, or for both firms to set their prices at marginal cost. To prove that the former arrangement is impossible, let us assume that $p_i > p_j = c$. In this case, firm *j* will earn no profit because it is setting a price exactly at marginal cost and firm *i* will also earn no profit because it will have no customers. However, because $p_i > p_j$, firm *j* will have an incentive to raise its price (but still keep it below p_i). By using this strategy, firm *j* will be able to capture the entire market at a price above marginal cost, which will yield positive profits. Therefore, the only way that the market can arrive at a Bertrand equilibrium is if both firms set a price that is equal to the marginal cost of the product. At this outcome, the two firms will not earn positive profits but will be indifferent between staying in the market and exiting the market because normal or zero economic profits include an amount necessary to keep entrepreneurs in the market. If either firm increases its price above marginal cost (the equilibrium level), it will lose all sales to its competitor. If either firm decreases its price below marginal cost, it will incur losses.

By simply changing the basis of competition from quantity to price, the Bertrand model produces a dramatically different outcome for duopolistic markets than the Cournot model. At the Bertrand equilibrium, the prices in such markets are driven down to marginal cost, a level far below what we observed at the Cournot equilibrium.

Solved Problem 11.6

Question (Content Review: Bertrand and Cournot Equilibria)

Consider two oligopolists producing an identical product with identical cost functions $C = q^2$ (that is, $MC = 2q$) who face a demand curve $p = 1 - (q_1 + q_2)$.

a) What is the Cournot equilibrium in this market?
b) If firm 1 can choose its output first, what will the outcome be?
c) Suppose the two firms choose *price* instead of *quantity*. What will the market outcome be?

Answers

a) Denote firm 1's output by q_1 and firm 2's output by q_2. Firm 1's total revenue function is $R = p \cdot q_1 = (1 - (q_1 + q_2))q_1 = q_1 - q_1^2 - q_2q_1$. The associated marginal revenue curve is given by $MR_1 = 1 - q_2 - 2q_1$. Equating firms 1's marginal cost to its marginal revenue, we get its reaction function:

$$2q_1 = 1 - q_2 - 2q_1$$

That is,

$$q_1 = \frac{1 - q_2}{4}$$

Since firm 2 is identical to firm 1 in all respects, we can immediately deduce that firm 2's reaction function is

$$q_2 = \frac{1 - q_1}{4}$$

Substituting firm 1's reaction function into firm 2's we see that

$$4q_2 = 1 - q_1 = 1 - \frac{1 - q_2}{4} = \frac{3 + q_2}{4}$$

That is,

$$16q_2 = 3 + q_2 \rightarrow 15q_2 = 3$$

Therefore, $q_1 = q_2 = \frac{1}{5}$, $p = \frac{3}{5}$.

b) If firm 1 produces first, it will take firm 2's reaction to its own output into account. Therefore, we can rewrite the demand curve faced by firm 1 as:

$$p = 1 - q_1 - q_2 = 1 - q_1 - \frac{1 - q_1}{4} = \frac{3}{4} - \frac{3q_1}{4}$$

Firm 1's marginal revenue function is:

$$MR_1 = \frac{3}{4} - \frac{3q_1}{2}$$

Equating marginal cost to marginal revenue, we get

$$2q_1 = \frac{3}{4} - \frac{3q_1}{2} \rightarrow q_1 = \frac{3}{14},\quad q_2 = \frac{11}{56},\ p = \frac{33}{56}$$

c) No matter what price one firm sets, the other firm can always do better by setting a "slightly" lower price and capturing the whole market. Therefore, the only possible equilibrium is when both firms set a price equal to marginal cost. Since the two firms produce an identical product, consumers have no reason to discriminate between the two firms, so the two firms will split the market. Thus, the demand curve can be rewritten as $p = 1 - 2q$, where q is the amount produced by each firm. Then setting price equal to marginal cost, we get $q = \frac{1}{4}$ and $p = \frac{1}{2}$.

11.6 Collusive Duopoly

There is something that does not sound right about the results of the Bertrand model. With only two firms in a market, it is hard to believe that the price will be driven down to a level that will maximize the welfare of consumers but minimize the profits of the firms. The Cournot model seems more intuitively correct because it tells us that as more firms enter a market, the price will *gradually* drop from the monopoly level to the welfare-maximizing level. In the Bertrand model, the addition of just one firm brings about a dramatic change in price level.

One simple reason why we find it difficult to believe that price will decrease to marginal cost in a duopolistic market is that we expect the two firms involved to get together and work out a more favorable pricing arrangement between themselves. In other words, we expect the two firms to collude on price. In such a **collusive duopoly,** both firms agree to set the same price at some level above marginal cost and to split the market and its profits. The problem with arrangements of this type is that each firm has a great incentive to cheat and sell to some customers at a price below the agreed price of p. Hence, collusive arrangements are usually not stable. At some point, most firms involved in such arrangements will cheat; and once cheating starts, it usually continues until the price is driven down to marginal cost.

To understand this situation more clearly, let us consider the following simple matrix game between two Bertrand duopolists who have agreed to collude at a price

Table 11.1

Matrix of the Payoffs from a Game Involving a Collusive Pricing Arrangement

		Firm 2			
		Honor Agreement		Cheat	
Firm 1	Honor Agreement	$1,000,000	$1,000,000	$200,000	$1,200,000
	Cheat	$1,200,000	$ 200,000	$500,000	$ 500,000

above marginal cost. Once the agreement is in effect, each firm is tempted to cheat by offering secret deals to some customers at slightly lower prices in order to obtain their business.

The game matrix in Table 11.1 illustrates the situation that our colluding duopolists face. Each of them has two possible strategies: cheat or honor the agreement. If both firms honor the agreement, each will receive a payoff of $1 million. However, if one firm honors the agreement and the other firm cheats, the cheater will do relatively better. That firm will receive $1.2 million, but the firm that honors the agreement will receive only $200,000. Mutual cheating will yield a payoff of $500,000 for each firm.

Note that this game is nothing more than another example of the prisoner's dilemma game described in Chapter 7. Each firm has a dominant strategy, which is to cheat; consequently, cheating by both firms forms the only equilibrium for the game. However, as in all prisoner's dilemma games, the equilibrium is worse for both firms than is honoring the collusive agreement. The lesson that this game teaches us is that collusive agreements are inherently unstable and very vulnerable to cheating by all parties involved. In recent years, the failure of the OPEC cartel to control oil prices effectively because of disagreements among its members has provided an example of the instability of cartels. Widespread cheating on production quotas is just one of the many problems that OPEC has faced in trying to enforce its price-fixing rules. These problems are merely a reflection of the basic weakness of collusive arrangements as illustrated by the simple matrix game in Table 11.1.

Solved Problem 11.7

Question (Application and Extension: Collusive Duopoly)

The city of San Dimas offers franchises for street taco stands for bid. The demand for tacos in San Dimas is $q = 2{,}000 - 200p$, and the marginal cost of a taco is $1 (there are no fixed costs). The bids, which must be submitted in a sealed envelope, must state the price at which bidders intend to sell the tacos. Whoever submits the lowest price will win the franchises. There are only two prospective bidders, Bill and Tina. They both know each other very well and decide to meet and discuss how they should bid before their weekly round of golf at the local country club. They agree to enter the same bid so that they can each get half the stands and maximize their profits.

If Bill and Tina do collude, what price would both announce? Is the arrangement stable? If not, what is the equilibrium price each will bid?

Answer

If Bill and Tina do collude, they would each bid the monopoly price. The inverse demand curve is $p = 10 - q/200$, which means that the marginal revenue curve is $MR = 10 - q/100$. Setting marginal revenue equal to marginal cost, we see that

$$10 - \frac{q}{100} = 1$$

$$q = 900$$

Plugging this quantity back into the demand curve, we see that the monopoly price is \$5.50. This is what Bill and Tina would both bid if they colluded, and both would earn the following profit:

$$\pi = \frac{1}{2}[(5.50)(900) - (1)(900)] = \$2,025$$

Collusion here is not stable, though, because there is no way to enforce it. Imagine if Bill goes through with the plan, but Tina bids \$5.49 instead. Then Tina will win all of the franchises and will get the following profit:

$$\pi = (5.49)(902) - (1)(902) = \$4,049.98$$

Therefore, each would have incentive to cheat on the deal. The only equilibrium is for them to bid \$1, their marginal cost. At any higher bid there would be incentive for the other bidder to undercut. This situation is nothing more than the Bertrand oligopoly game.

11.7 Making Cartels Stable: The Kinked Demand Curve Conjecture

From our discussion in Section 11.6, it would appear that all cartels are doomed to failure because they are inherently unstable. However, that instability is predicated on certain assumptions; if these assumptions are relaxed, it may turn out that collusive arrangements are more viable than we thought. For example, our analysis of the prisoner's dilemma game at the end of Section 11.6 presents a collusive arrangement as a game that is to be played once and only once between the two firms involved. However, in the real world, we know that firms that enter into a collusive arrangement meet each other regularly in the marketplace and interact repeatedly. It is natural to expect that this repeated interaction will facilitate collusion because it permits firms to *punish* a cheater by lowering their price once they become aware of the cheater's defection from the collusive agreement. Consequently, if we treat a collusive arrangement as a repeated game and not a one-time game as we did previously, then we may find that such an arrangement can have a more stable outcome. (The appendix to this chapter presents a model of repeated interaction between Cournot-like firms and demonstrates that if a market has an infinite life, collusion is an equilibrium outcome. This discussion has been relegated to the appendix because it is more technical in nature than the rest of our analysis and is probably best suited to those students who have a taste for mathematics.)

Even without ascribing infinite life to markets, we can envision the emergence of stable collusive agreements if we relax the definition of what constitutes an equilibrium for these markets. The Cournot and Bertrand models define an equilibrium as a situation in which no firm or player has any incentive to change its behavior (either the quantity it is producing or the price it is charging), given the actions of its opponents and *given the assumption that its opponents will not respond to any action that it takes.* We earlier called this assumption the Cournot conjecture. Actually, it might make more sense for a firm to expect its opponents to react when it changes its strategy; such reactions, if taken into account before the players make their moves, might change the outcome of the game and make collusion more likely. This line of reasoning allows us to see how a stable collusive arrangement might emerge even if a game is not repeated an infinite number of times. For example, let us make the logical assumption that in a Bertrand game, any action by one firm to raise the prevailing price in the market will not be matched by its competitors, but any action by one firm to lower the prevailing price will be matched. Hence, a firm that raises its price will find that its demand will drop to zero. Because the firm's competitors will not match the price increase, they will be able to take away its market share. On the other hand, a firm that lowers its price will experience an increase in demand but will see its profits fall because its competitors will match the price reduction, pushing profits further from their joint maximum.

To make our example more precise, let us say that at present both firms in our duopolistic industry are charging a price of p, which is between the marginal cost and the monopoly price, $c < p < p^m$. At a price of p for the product, the demand facing firm i can be expressed as follows.

$$D(p_i) = \begin{cases} 0 & \text{if } p_i > p \\ \dfrac{D(p)}{2} & \text{if } p_i = p \\ \dfrac{D(p_i)}{2} & \text{if } p_i < p \end{cases}$$

Clearly, with this demand function, neither firm will want to raise or lower its price. According to our conjecture about behavior in a Bertrand game, any attempt by one firm to change the prevailing market price will be of no advantage to that firm. In fact, the firm will be worse off. If the firm raises its price, its competitor will not match the increase and it will lose all its sales. If the firm lowers its price, say from p to $p' < p$, then its competitor will react by matching the reduction. As a result, firm i's demand will rise from $D_i(p, p)/2$ to $D_i(p', p')/2$, but because p is already below the monopoly price of p^m, a further reduction will only serve to decrease the profits of both duopolists. Profits will fall because the increased demand will lead the two firms to expand production to units whose marginal cost is even further above marginal revenue. Therefore, neither firm will choose to make such a price reduction. The assumption that firms will match a reduction but not an increase in the prevailing price is called the **kinked demand curve conjecture** and is responsible for the stability of duopolistic and oligopolistic markets.

The kinked demand curve conjecture establishes any price between c and p^m as an equilibrium price as long as all firms choose it. This will be true even if the game is played only once as long as the firms behave according to the kinked demand curve conjecture. In short, the Cournot and Bertrand models exclude the possibility of a stable collusive arrangement because they use a conjectural assumption about the behavior of competing firms that is too restrictive.

11.8 The Edgeworth Model

To find a more profitable equilibrium for a duopolistic industry than the welfare-optimal prices and quantities in the Bertrand model, we have had to resort to the kinked demand curve conjecture or to the idea of markets with infinite horizons. Our gadget maker is not satisfied with this analysis or with our previous analysis of duopolistic markets, especially since the results differ so dramatically depending on whether we use a price version or a quantity version of the duopoly model. Our gadget maker therefore decides to obtain the views of one more consulting firm. This firm bases its opinions on the work of Francis Ysidro Edgeworth, whose name we encountered previously in our study of exchange in Chapter 4.

The logic behind the Edgeworth model is simple. Let us say that both firms in the gadgets industry do indeed charge the marginal cost price, and together, they have enough capacity to satisfy demand so that all consumers who want gadgets can obtain them. As a result, neither firm makes a profit. But what will happen if one firm, say firm 1, raises its price above marginal cost? Obviously, all consumers will attempt to buy their gadgets from firm 2. However, because firm 2 is capacity-constrained, it will not be able to serve everyone, so there will be some unsatisfied customers willing to pay more than marginal cost to buy gadgets. Firm 1 can now offer gadgets to these customers at a price above marginal cost and thereby make a profit. The marginal cost solution is not an equilibrium in this situation. The exact nature of the solution will depend on how we define the rationing rule that tells us who will obtain gadgets from firm 2 when it keeps its price at marginal cost after firm 1 has deviated from this price.

Although a full description of the pricing process in the Edgeworth model is beyond our needs here, the following example will provide an intuitive understanding of this process. Assume that both firms in the gadgets industry have enough capacity individually to satisfy demand for the monopoly quantity of q^m but not enough capacity to satisfy demand for the welfare-optimal quantity of q^c when the price is equal to the marginal cost. Further, assume that prices are set such that $p^m > p^1 > p^2 > p^c$, so that firm 1's price is above firm 2's price. In this case, all consumers will want to buy from firm 2. If firm 2 can satisfy the entire market at a price of p^2, then firm 1 will have no customers. As a result, firm 1 will surely lower its price from p^1 to $p^2 - \epsilon$ and attract all the demand. This price reduction by firm 1 will cause prices to fall until both firms are charging the marginal cost price. However, we already know that a situation in which both firms charge the marginal cost price is not an equilibrium in the Edgeworth model. Hence, one firm will raise its

Consulting Report 11.4

Using the Edgeworth Model to Describe Price Behavior in a Duopolistic Market

The consultants tell our gadget maker that the **Edgeworth model** presents both good and bad news about price behavior in a duopolistic market. The good news is that this model offers a solution to the problem of price competition in which prices do not fall to marginal cost. The bad news is that this model does not have an equilibrium of the type we expect to see. In other words, a game defined by the Edgeworth model will not have a pair of prices that constitute an equilibrium. The consultants explain that this lack of an equilibrium occurs because underlying the Edgeworth model is the assumption that the two firms in a duopolistic market are **capacity-constrained,** which means that neither firm has enough capacity to produce the quantity that would be demanded at the marginal cost price of c. This rather realistic assumption is all that is needed to establish a situation in which the prices set by the two firms do not inevitably fall to marginal cost and remain there.

Instead, the Edgeworth model describes a market in which prices move in cycles. As each firm attempts to maximize its profits, prices rise and then fall, but they never settle permanently at one level. If prices reach marginal cost, they always move back to a higher level.

price and the process of changing the price to maximize profits will start all over again. If, however, at the original price configuration, firm 2 cannot satisfy the entire demand, then firm 1 will receive some customers. There will then be an incentive for firm 2 to raise its price to $p^1 - \epsilon$ because, by doing so, it will increase its profits even though the higher price will drive some customers away. (We know that this is true because as the firm raises its price, p^2 comes closer to the monopoly price and the firm's profits increase.) The price configuration in the gadgets industry will now be firm 1 charging p^1 and firm 2 charging $p^1 - \epsilon$. Again, there is no equilibrium. After the two firms establish this pair of prices, firm 1 will want to lower its price to $p^1 - \epsilon - \epsilon$ because this small price reduction will bring a large increase in demand. As the low-cost firm in the market, firm 1 will now be able to capture sales from firm 2.

Prices will continue to fall until marginal cost is reached, and then they will rise again when one firm decides to increase its price. Thus, in the Edgeworth model, capacity constraints cause prices to cycle endlessly and never settle at any particular level. An industry will go through periods when prices fall ("price wars") and periods when prices rise.

11.9 Conclusions

Our primitive society will soon make the transition to markets that are composed of many competing firms. However, at the moment, its markets are still dominated by a few large firms. We would expect such firms, knowing that more competition is on the way, to try to develop strategies to keep other firms from entering their markets. Naturally, incumbent firms fear that the entry of new firms will lead to lower prices and lower profits. In the next chapter, we will see how the battle is waged between the new firms that want to enter an established market and the incumbent firms that are attempting to prevent such entry. We will also see what happens when the number of firms in a market goes to infinity. This next chapter, then, will lay the groundwork for our study of perfectly competitive markets, or markets inhabited by a great many small firms. We will examine such markets in detail in Chapter 13.

11.10 Summary

In this chapter, we saw how competition for profits affects quantity and price in duopolistic and oligopolistic markets. We studied various well-known models for such markets: the Cournot quantity-setting model, the Stackelberg leader-follower model, the Bertrand price-setting model, and the Edgeworth model. To describe the equilibria or lack of equilibria envisioned by these models, we have defined the concept of a reaction or best-response function. We observed that each model makes a very different prediction about how prices and quantities will behave. We also analyzed the welfare properties of duopolistic and oligopolistic markets and found that they varied according to the model used to describe the market.

We investigated collusive arrangements (cartels) in which firms agree to set certain prices or quantities in order to ensure profitability for each participant. Although such arrangements are normally considered unstable, we saw that it was possible to envision stable collusive arrangements by using the kinked demand curve conjecture or the idea of markets with infinite horizons.

Appendix A

Nash Equilibrium in Duopoly

In markets with few firms, each firm must take into account not only the parameters it faces, that is, the market demand and its costs, but also the anticipated actions of its competitors. When the anticipated actions of each firm are realized in the market, an equilibrium is established. To see this, assume the market consists of two firms that produce the same (homogenous) product—the two firms, labelled 1 and 2, produce, respectively, quantities q_1 and q_2. Hence, the aggregate quantity on the market is $Q = q_1 + q_2$. Let

$$\begin{aligned} P(Q) &= a - Q \qquad \text{for } Q < a \\ &= 0 \qquad\qquad \text{for } Q \geq a \end{aligned}$$

be the inverse demand function, which just indicates the market-clearing price when quantity Q is on the market.

Suppose the total cost to the firm i of producing the quantity q_i is $C_i(q_i) = c_i q_i$, $i = 1, 2$; that is, firm 1's marginal cost is c_1 and firm 2's marginal cost is c_2. The payoff is the same as profits, and

$$\pi_i(q_i, q_j) = q_i(P(q_i + q_j) - c_i) = q_i(a - (q_i + q_j) - c_i)$$

1. Best-Response Functions

The best-response functions describe the profit-maximizing output of firm *i*, given any output by that firm *j*. Hence, the best-response function for firm 1 is obtained by maximizing the profit function for firm 1 given that firm 2 is known to produce the (arbitrary) amount q_2; q_1^{BR} solves

$$\text{Max}_{\{0 \le q_1 < \infty\}}\ \pi_1(q_1, q_2)$$

$$\Rightarrow \text{Max}_{\{0 \le q_1 < \infty\}}\ q_1(a - (q_1 - q_2) - c_1)$$

The first-order condition can be written as

$$q_1^{BR} = R_1(q_2) = \frac{1}{2}(a - q_2 - c_1)$$

Similarly, the best response q_2^{BR} for firm 2 is obtained as the solution to

$$\text{Max}_{\{0 \le q_2 < \infty\}}\ \pi_2(q_1, q_2)$$

$$\Rightarrow \text{Max}_{\{0 \le q_2 < \infty\}}\ q_2(a - (q_1 + q_2) - c_2)$$

and the first-order conditions yield

$$q_2^{BR} = R_2(q_1) = \frac{1}{2}(a - q_1 - c_2)$$

$R_1(q_2)$ and $R_2(q_1)$ are the best-response functions. See Figure 11.7 in the text.

2. The Cournot Model

In the Cournot model, both firms make their production decisions simultaneously, and then the total quantity is brought to the market. Each firm chooses its output q_i from a set of nonnegative real numbers $[0, \infty)$; that is, $0 \le q_i < \infty$.

The quantity pair (q_1^*, q_2^*) is a Nash equilibrium if,

(i) for firm 1, q_1^* solves

$$\text{Max}_{\{0 \le q_1 < \infty\}}\ \pi_1(q_1, q_2^*)$$

or

$$\text{Max}_{\{0 \le q_1 < \infty\}}\ q_1(a - (q_1 + q_2^*) - c_1)$$

(ii) for firm 2, q_2^* solves

$$\text{Max}_{\{0 \le q_2 < \infty\}}\ \pi_1(q_1^*, q_2)$$

or

$$\text{Max}_{\{0 \le q_2 < \infty\}}\ q_2(a - (q_1^* + q_2) - c_2)$$

In other words, if both firms anticipate (q_1^*, q_2^*) in the market, then their best response to that anticipation is in fact to choose (q_1^*, q_2^*).

The first-order conditions yield the best response of firm 1 to firm 2's equilibrium output q_j^*, $j = 2, 1$. These best responses can be written as

$$q_1^* = \frac{1}{2}(a - q_2^* - c_1)$$

$$q_2^* = \frac{1}{2}(a - q_1^* - c_2)$$

The intersection of the reaction curves (derived in the previous sections) is the Nash equilibrium of the Cournot game (see Figure 11.7 in the text): clearly, at (q_1^*, q_2^*), the best reactions of the two firms match one another. To calculate the Nash equilibrium, we simply solve the pair of simultaneous equations for q_1^* and q_2^*. This procedure yields

$$q_1^* = \frac{1}{3}(a + c_2 - 2c_1)$$

$$q_2^* = \frac{1}{3}(a + c_1 - 2c_2)$$

Consider the symmetric case, when $c_1 = c_2 = c$. Then

$$q_1^* = q_2^* = \frac{1}{3}(a - c)$$

$$Q^* = q_1^* + q_2^* = \frac{2}{3}(a - c)$$

$$P(Q) = a - Q = \frac{1}{3}(a + 2c)$$

It is important to note that the total production under the Nash outcome is higher than that in the collusive outcome. In this case, the joint profits are maximized:

$$\text{Max}_{\{0 \le (q_1, q_2) < \infty\}}\ q_1(a - (q_1 + q_2^*) - c_1) + q_2(a - (q_1^* + q_2) - c_2)$$

$$\text{Max}_{\{0 \le (q_1, q_2) < \infty\}}\ (q_1 + q_2)a - (q_1 + q_2)^2 - (q_1 + q_2)c, \text{ assuming } c_1 = c_2 = c$$

The first-order conditions yield $Q = q_1 + q_2 = (\frac{1}{2})(a - c)$, which is lower than the output $(\frac{2}{3})(a - c)$ of the Cournot equilibrium and $P(Q) = (\frac{1}{2})(a + c)$, which is higher than the price $(\frac{1}{3})(a + 2c)$ associated with the Cournot equilibrium.

3. *The Stackelberg Model*

In the Stackelberg model, one of the firms (called the dominant firm) moves first and chooses output, and then the other firm makes its output decision; that is, (i) firm 1 chooses $q_1 \ge 0$, then (ii) firm 2 observes q_1 and chooses $q_2 \ge 0$. Assume that all costs, demands, and profits are identical to those in the Cournot case.

To compute the subgame perfect equilibrium of this game, we proceed backwards. Given that firm 1 has produced a quantity q_1, firm 2's decision problem is to

$$\text{Max}_{\{q_2 > 0\}}\ q_2(a - (q_1 + q_2) - c_2)$$

which yields a reaction function

$$R_2(q_1) = \frac{1}{2}(a - q_1 - c_2)$$

as before.

Firm 1 should anticipate that its quantity choice of q_1 will be met by the reaction $R_2(q_1)$. This implies that firm 1's problem is:

$$\text{Max}_{\{q_1>0\}}\ \pi_1(q_1, R_2(q_1))$$

or

$$\text{Max}_{\{q_1>0\}}\ q_1(a - (q_1 + R_2(q_1)) - c_1))$$

Substitution for $R_2(q_1)$ yields

$$\text{Max}_{\{q_1>0\}}\ q_1\left(\frac{a - q_1 + c_2 - 2c_1}{2}\right)$$

which yields

$$q_1^* = \frac{1}{2}(a + c_2 - 2c_1)$$

$$q_2^* = R_2(q_1^*) = \frac{1}{4}(a - 3c_2 + 2c_1)$$

In the symmetric case, where all costs are identical, $q_1^* = \frac{1}{2}(a - c)$ and $q_2^* = \frac{1}{4}(a - c)$.

Graphically, what this means is that firm 1 chooses its output such that firm 2's reaction curve $R_2(q_1)$ is tangent to firm 1's isoprofit curve. See Figure 11.9 in the text.

Appendix B

Implicit Collusion and Repeated Games

Most duopolistic situations are repeated over and over again and involve the same two firms. We cannot properly analyze these situations by using the Bertrand model (or even the Cournot model) because such models assume that duopoly games will be played only once and therefore provide a static view of these games. However, as we learned in Chapter 11, if duopolistic games are repeated, there may be more of a chance to establish stable collusive arrangements because there will be a greater opportunity to punish firms that cheat. Hence, the proper game to analyze is the supergame (see the appendix of Chapter 7), which we will define here as the one-time Bertrand game played repeatedly for an infinite number of periods. If the Bertrand game is played in this manner, then a strategy will involve a rule dictating behavior at each point in time, possibly as a function of what has happened in all periods in the past—history of the game.

A Strategy for Achieving a Collusive Equilibrium in the Bertrand Supergame

To prove that implicit collusion that is self-enforcing can occur in a market, let us consider the following strategy. We will assume that both firms in the gadgets industry use this strategy.

1. Choose the monopoly price of p^m in period 0.
2. Continue to choose the monopoly price of p^m in period t as long as one's opponent has chosen p^m or a higher price in every period from period 0 to period $t - 1$.
3. If one's opponent has deviated and chosen a price of $\overline{p} < p^m$ in period $t - 1$, then choose the marginal cost price of $p = c$ in period t and every period thereafter.

Our gadget maker claims that if both firms in the industry follow such a strategy and if the discount factor used by both firms is sufficiently large, then this strategy will provide an infinite stream of choices, in which each firm will select the monopoly price of p^m in each period. Neither firm has any incentive to cheat and choose a lower price. Perfect collusion at the monopoly price is a Nash equilibrium for the Bertrand supergame.

Proving the Collusive Equilibrium

Our proof of this proposition follows along the same lines as the proof given in Chapter 7 for the existence of a supergame equilibrium in a repeated prisoner's dilemma game. We will assume, as the one-time Bertrand model has us do, that when both firms set the monopoly price, they will share the market and receive the profit denoted by $\pi^m/2$. At this outcome, the price is p^m and each firm produces $D(p^m, p^m)/2$. Clearly, because p^m is greater than the marginal cost of production of c, both firms will make a positive profit. Now, let us call π^c the profit that is earned when both firms set a price equal to marginal cost, and let us call $\pi^i(p^m - \epsilon, p^m)$ the profit to firm i when it chooses a price of $p^m - \epsilon < p^m$ and firm j chooses a price of p^m. Obviously, the profit to firm i in the latter case is larger than either $\pi^m/2$ or π^c because $\pi^i(p^m - \epsilon, p^m)$ represents a situation where firm i is serving the entire market at a price only slightly below the monopoly price, which must be better than serving only half the market at p^m.

Let us say that our gadget maker's proposed strategy is used by both firms. As a result, they will receive the payoff of $\pi^m/2$ in each period. The present value of receiving this payoff forever is $(\pi^m/2)/(1 - \delta)$, where δ is the discount factor used for both firms. If perfect collusion at the monopoly price is to be an equilibrium for the game, it must be that neither firm has an incentive to deviate. We can demonstrate that this is so for the following reasons. Let us say that firm 1 contemplates cheating in period t by choosing a lower price than p^m. (We will assume that neither firm will want to cheat by choosing a higher price because such a deviation can never be beneficial. The other firm will not respond to a price increase, and the deviating firm will therefore lose all its sales.) The deviating firm's strategy can be summarized in the following way: "I will choose p^m for all periods until period t. In period t, I will deviate and choose $p^m - \epsilon < p^m$. From that point on, I know that my opponent will try to punish me forever by choosing $p = c$. My best response to such punishment is to choose $p = c$ also, which is what I will do starting in period $t + 1$."

Such a deviation strategy will yield firm 1 a payoff stream of $\pi^m/2, \pi^m/2, \ldots \pi^m/2, \pi^1_t(p^m - \epsilon, p^m), \pi^c, \ldots$. Thus, if firm 1 deviates in period t but firm 2 adheres to the original strategy, then firm 1 will receive a payoff of $\pi^m/2$ for all periods until period t. In period t, when firm 1 cheats by lowering its price to $p^m - \epsilon$, it will receive a one-period cheater's payoff of $\pi^1(p^m - \epsilon, p^m)$. From then on, both firms will choose a price that is equal to marginal cost and receive a payoff of π^c. Is such a deviation profitable for firm 1, given that firm 2 will not change its planned strategy? Put differently, is the one-period cheater's payoff sufficiently enticing to make firm 1 want to risk eternal marginal cost pricing?

To identify the conditions under which no deviation is profitable, we will let P_1 be the payoff to firm 1 in the supergame. If firm 1 deviates and firm 2 adheres to its strategy, then firm 1's supergame payoff when discounted to the beginning of time is as follows:

$$P_1 = \sum_{\rho=0}^{t-1} \frac{\delta^\rho \pi^m}{2} + \delta^t \pi_t^1(p^m - \epsilon, p^m) + \sum_{\rho=t+1}^{\infty} \delta^\rho \pi^c$$

The payoff for adhering to the proposed strategy is $P_1 = \Sigma_{\rho=0}^{\infty} \delta^\rho \pi^m/2$. Note that until period t, the two strategies yield the same payoff because they both dictate the same actions. In period t, however, the actions differ and so do the payoffs. The question then is whether a planned deviation in period t would be profitable when contemplated in period 0. Such a deviation is profitable under the following conditions:

$$\delta^t \pi_t^1(p^m - \epsilon, p^m) + \sum_{\rho=t+1}^{\infty} \delta^\rho \pi^c \geq \sum_{\rho=t}^{\infty} \frac{\delta^\rho \pi^m}{2}$$

Hence, it is profitable to deviate if the payoff stream from deviating in period t (the terms on the left side of the inequality) is greater than the payoff from not deviating (the term on the right side of the inequality). This inequality can be rewritten in the following manner:

$$\frac{\delta^t(\pi^m/2)}{(1-\delta)} \leq \delta^t \pi_t^1(p^m - \epsilon, p^m) + \frac{\delta^{t+1}\pi^c}{(1-\delta)}$$

After algebraic manipulation, we find that a deviation is profitable only under the following circumstance:

$$\delta < \frac{\pi_t^1(p^m - \epsilon, p^m) - \pi^m/2}{\pi_t^1(p^m - \epsilon, p^m) - \pi^c}$$

If this duopolistic situation is repeated over an infinite horizon and if the discount factors of the firms are large enough, then it will be possible to support an infinite history of monopoly prices, with no firm having any incentive to deviate. Infinite horizons plus high discount factors equal collusive behavior.

Exercises and Problems

1. Consider a duopolistic market with a demand function of $p = 10 - 2(q_A + q_B)$. Firm A has a cost function of $C^A = 4 - q^A + q_A^2$, while firm B has a cost function of $C_B = 5 - q_B + q_B^2$. Assume that these firms can choose only their output levels and that their choices are constrained in the following way: firm A can produce either $q_A = .92$ or .94, while firm B can produce either $q_B = .41$ or .74.

   a) Assuming that payoffs are identical to profits, supply the information that is missing from the matrix given below. In this matrix, Π_A and Π_B are the payoffs to firms A and B, respectively.

		q_B	
		.41	**.74**
q_A	**.92**	(Π_A, Π_B)	(. , .)
	.94	(. , .)	(. , .)

   b) Say that firms A and B are players in a game where they can choose only the output levels specified previously. Does the choice of .94 by firm A and the choice of .74 by firm B constitute a Nash equilibrium for the game?

2. Assume that two firms, A and B, compete with each other in the same market. They produce a commodity that has the following demand: $p = 1 - q_A - q_B$. Each firm must decide what fraction of the market to supply by choosing an output level between 0 and 1. There are no fixed costs of production, and the marginal costs are zero. The profit of firm A is $II_A = (1 - q_A - q_B)q_A$, and the profit of firm B is $II_B = (1 - q_A - q_B)q_B$.

   a) If firm B sets output levels of $q_B = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, and 1, what is the demand function facing firm A in each case?
   b) What is firm A's marginal revenue function for each output level of q_B chosen by firm B? (*Note:* The slope of the marginal revenue curve for a firm facing a linear downward-sloping demand curve is twice the slope of the demand curve.)
   c) Using the Cournot conjecture, assume that after firm B sets its output levels, firm A will consider these output levels to be fixed. What is the best response of firm A to firm B's choice of the output levels of $q_B = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, and 1? (*Hint:* Remember that firm A will set $MR = MC = 0$ for each level of output chosen by firm B.)
   d) What will the Cournot-Nash equilibrium be in this example? What will the corresponding equilibrium price be?
   e) On a graph, plot the market demand curve, the equilibrium output for the industry, and the consumer surplus generated at this equilibrium. Also, calculate the deadweight loss.

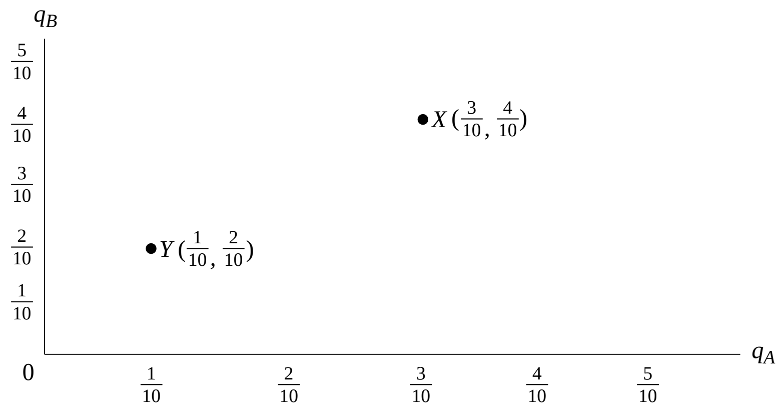
3. Consider two firms, A and B, that produce a commodity with the same demand and cost structure as in Problem 2. However, assume that instead of choosing a quantity

like Cournot duopolists, the firms choose a price like Bertrand duopolists. The game they play is as follows. If firm A's price is lower than firm B's price, firm A obtains all the customers in the market who are willing to pay its price (or more). Firm A will therefore sell $q_A = 1 - p_A$ units, and firm B will sell zero units. Firm A's profit will be $\pi_A = (1 - p_A)p_A$, while firm B's profit will be zero. The opposite happens if firm B sets a lower price than firm A. If the two firms set the same price, $p_A = p_B$, they will split the market demand equally and will receive profits of $\Pi_A = (\frac{1}{2})(1 - p_A)p_A = \pi_B = (\frac{1}{2})(1 - p_B)p_B$.

a) If firm A sets a price of $\frac{1}{3}$ and so does firm B, what profit will each firm make?
b) What output will each firm sell when they both set a price of $\frac{1}{3}$?
c) Is this pair of prices an equilibrium?
d) What is the only pair of prices that constitutes an equilibrium for this game?
e) Is this equilibrium the one that maximizes the sum of consumer welfare and producer welfare?

4. Consider a duopolistic market with two firms, A and B, facing a demand curve of $p = 1 - q_A - q_B$. Assume that initially each firm has access to the same technology with constant returns to scale and that the cost of production is $C_A = q_A/2$ for firm A and $C_B = q_B/2$ for firm B. Also assume that the two firms can only set output levels that are between 0 and $\frac{1}{3}$.

a) What is the profit function for each firm?
b) Assume that you are told that the reaction functions are $q_A = \frac{1}{4} - q_B/2$ for firm A and $q_B = \frac{1}{4} - q_B/2$ for firm B. Graph these reaction functions in a box like the one shown below.



c) What is the Nash equilibrium for this game?
d) Assume that the inital output levels of the two firms are given by points *X* and *Y* in the box illustrated here. Show the process of change in the output levels of the two firms and the point at which their output levels converge.
e) On the basis of the two paths, one leading from point *X* and the other leading from point *Y*, do you think that the Nash equilibrium of this game is stable?

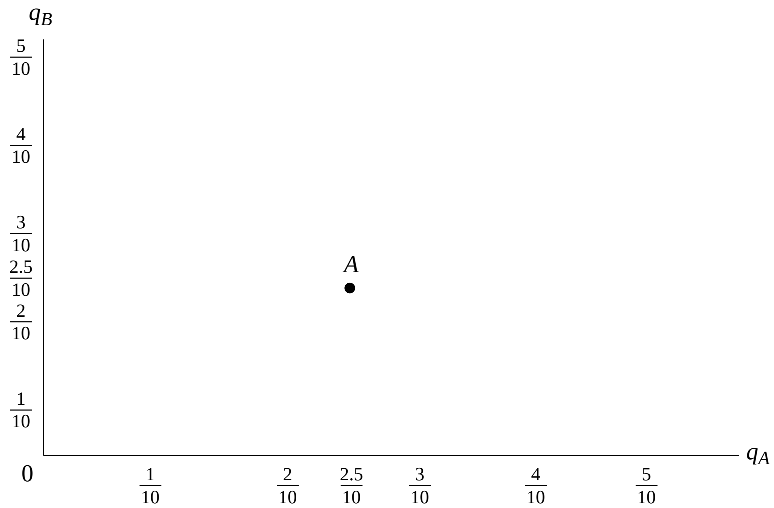
5. Assume that two firms, A and B, have the same demand function as in Problem 4, but their cost functions are: $C_A = (\frac{1}{2})q_A - (\frac{3}{4})q_A^2$ for firm A and $C_B = (\frac{1}{2})q_B - (\frac{3}{4})q_B^2$ for firm B.

   a) Do these firms have increasing, decreasing, or constant returns to scale in production?
   b) The reaction functions of the two firms are as follows:

$$q_A = \begin{cases} (1/2), & \text{if } 0 < q_B < (1/4) \\ 1 - 2q_B, & \text{if } (1/4) \leq q_B \leq (1/2) \end{cases}$$

$$q_B = \begin{cases} (1/2), & \text{if } 0 \leq q_A \leq (1/4) \\ 1 - 2q_A, & \text{if } (1/4) \leq q_A \leq (1/2) \end{cases}$$

   Graph these reaction functions in a box like the one given below. Assume that the output of each firm is restricted to levels between 0 and $\frac{1}{2}$.

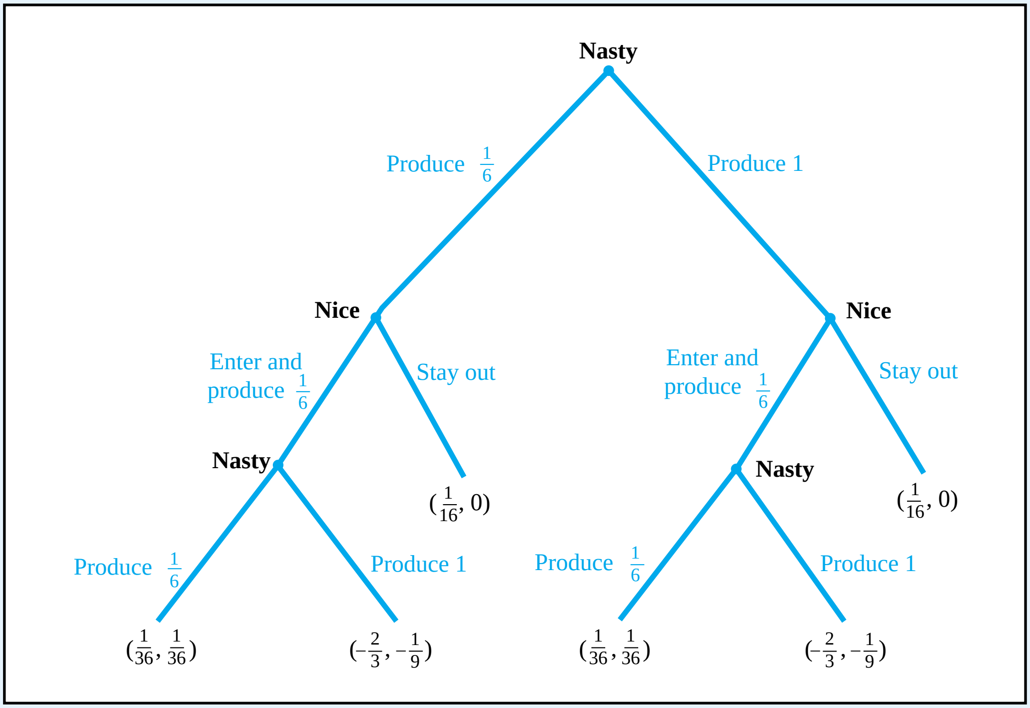


   c) What are the Nash equilibria for this game? How many are there? (*Hint:* Find the point where the reaction functions of the two firms intersect.)
   d) Assume that $(\frac{1}{3}, \frac{1}{3})$ is a Nash equilibrium. If we start at point $A = (\frac{1}{4}, \frac{1}{4})$, do the output levels of the duopolists converge at the equilibrium point $(\frac{1}{3}, \frac{1}{3})$? What might we conclude about the stability of the equilibrium point $(\frac{1}{3}, \frac{1}{3})$?

6. Consider an industry that consists of two firms: the Nice firm and the Nasty firm. The demand in this industry is $p = 1 - q_{Nice} - q_{Nasty}$, and the two firms have cost functions of $C_{Nice} = (\frac{1}{2})q_{Nice}$ and $C_{Nasty} = (\frac{1}{2})q_{Nasty}$. Assume that there is a Nash equilibrium of $(\frac{1}{6}, \frac{1}{6})$ in the industry. Then the Nasty firm announces that unless the Nice firm produces no more goods and leaves the market (thus allowing Nasty to be a monopolist and produce an output of $\frac{1}{4}$), it will "flood the market" by producing an output of 1 and will therefore drive the price to zero. The game tree in Figure 11.13 depicts this

Figure 11.13

The numbers in parentheses refer to profits. The first number is Nasty's profit, and the second number is Nice's profit.



situation. In the first stage of the game, Nasty announces its intention to produce either 1 or $\frac{1}{6}$. In the second stage, Nice decides whether to leave the market or not after hearing Nasty's announcement. In the third stage, Nasty chooses its output after observing Nice's decision. Note that Nasty's announcement at the first stage of the game is nonbinding because Nasty does not have to carry out its threat.

a) Is the threat of the Nasty firm to produce an output of 1 and flood the market credible? If not, why not? (*Hint:* Start at the end of the game tree, and work backward to find the subgame perfect equilibrium.)
b) Does Nasty's ability to announce an intended strategy increase its equilibrium payoff compared to the Cournot-Nash equilibrium payoff?
c) Now assume that the game is played by allowing Nasty to choose an output level first and then having Nice choose a response. In other words, Nasty is a Stackelberg leader. Find the Stackelberg equilibrium. Does it pay for Nasty to be a Stackelberg leader? (*Hint:* The best response of Nice is $\frac{1}{4} - (\frac{1}{2})q_{Nasty}$.)

7. Assume that there are two firms in a market, firms 1 and 2. The total demand for the identical product they make is $p = 200 - 2(q_1 + q_2)$, where q_1 is the output of firm 1 and q_2 is the output of firm 2. The production costs of firms 1 and 2 are $C_1 = q_1^2$ and $C_2 = q_2^2$, respectively.

   a) Assume that firm 2 decides to produce either 20, 40, 60, or 100 units of output. Show the demand curve and the marginal revenue curve facing firm 1 in each of these situations, assuming that the output levels will remain unchanged once they are chosen.
   b) Define the output that represents the best (the profit-maximizing) response of firm 1 to each of the output levels chosen by firm 2. (*Hint:* Given the output of firm 2, define the demand and marginal revenue functions. Then set the marginal revenue so that it is equal to the marginal cost, where the marginal cost of the two firms is $MC_1 = 2q_1$ and $MC_2 = 2q_2$.)

8. Consider a monopolist facing a demand curve of $p = 1 - q_M$, where q_M is the monopolist's output. The firm has no marginal cost, but it must bear a fixed cost of $\frac{1}{4}$ in order to produce. Thus, its cost function is $C(q_M) = \frac{1}{4}$.

   a) Determine the monopolist's profit-maximizing output.
   b) Suppose that another firm is thinking about entering the market. It also has a zero marginal cost, but its fixed cost is $\frac{1}{10}$. If the second firm does decide to enter the market, what will the Nash equilibrium profits of the two firms be after entry occurs?
   c) Suppose that the monopolist commits itself to a monopoly output (the output it was producing before there was any threat of competition), and it will produce this output no matter what the entrant does. Can the entrant make a positive profit by choosing the *best response* to the monopoly output?
   d) What is the smallest output that the monopolist can choose that will prevent entry by the rival firm? In other words, what is the smallest output that will deny the rival firm a positive profit if it does enter the market and the monopolist actually produces its chosen output?

9. Consider the following matrix, which shows the payoffs for a game between two firms in a duopolistic industry.

		Firm II	
		Low Price	**High Price**
Firm I	**Low Price**	0, 0	20, –8
	High Price	–8, 20	5, 5

   a) What is the only Nash equilibrium in pure strategies for this game?
   b) Are there dominant strategies for each firm?

c) Now suppose that the cost structure in the industry has changed so that the new payoffs for the game are as shown below. Is the Nash equilibrium determined in Part a of this problem still an equilibrium?

		Firm II	
		Low Price	**High Price**
Firm I	**Low Price**	0, 0	0, –10
	High Price	–10, 0	5, 5

d) Are there now any other equilibria?

e) If there are now several equilibria for the game, which one do you think is likely to be chosen? Why?