Preface

Economic analysis lends itself to mathematical frameworks. These mathematical frameworks provide a powerful set of analytic tools. While elementary economic concepts are typically taught with only a modicum of mathematics, more advanced concepts require a broader set of mathematical skills. This book presents the mathematics required for studying economics at the advanced undergraduate level and at the graduate level.

The focus of this book is to teach the mathematical tools that are of central importance in economics in the context of economic examples. This second edition includes many new applications, thus expanding the economic context used in the teaching of mathematics. The economic context is important for four reasons. First, the economic context motivates economics students to learn the mathematical material by explicitly showing the use of the mathematical tools and techniques in economics. To appeal to a wide audience, examples in this book are drawn from a range of fields in economics, including microeconomics, macroeconomics, economic growth, international trade, international finance, labor economics, environmental economics, industrial organization, devlopment economics, and finance. Second, this book provides an explicit link between mathematical tools and their uses in economics. This link is often not obvious to students who are approaching this material for the first time. Third, abstract mathematical concepts can be more easily understood and the intuition behind them becomes more accessible when they are presented in a concrete context. Mathematical concepts in this book are developed in the context of economic examples. Finally, students will hone their economic intuition through the study of the formal mathematical models presented in this text.

Pedagogically, many chapters introduce an economic example at the outset of the chapter and then reference it, or closely related examples, throughout the rest of the chapter to develop students' comprehension of both economics and mathematics. Repeated use of some important economic models, such as the Solow growth model, across chapters also serves to develop the students' knowledge of economics and reinforce the lessons learned in earlier chapters. Many chapters include separate applications sections, while in other chapters applications are included within sections that develop mathematical concepts and tools. Many of the applications are drawn directly from recent research published in leading journals. Each chapter includes a large number of exercises designed to reinforce the concepts of the chapter and to build students' ability to conduct economic analysis. This second edition includes more than twenty-five new exercises. Both mathematical exercises and word problems drawn from economics are included. Short answers to odd-numbered problems are provided in the back of the book. A solutions manual with more detailed answers to all exercises is available for instructors.

The book's modular structure affords instructors a good deal of flexibility in designing courses. Application sections often do not require students to cover the most

advanced material in chapters. Thus this book is appropriate for a wide range of audiences. An undergraduate course can make use of most of the applications in a chapter even if that course does not cover the more advanced material presented in the sections of that chapter. Undergraduate students will find that the material presented in this book provides them access to a wide range of economic research. Masters-level courses can include some of the more advanced topics in the text. Students who are studying for a Masters of Business Administration will benefit from the link between material covered elsewhere in their programs and applications on the pricing of bonds, the volatility of bond prices, the pricing of stocks, present value, advertising, exchange-rate determination, and strategic pricing. Students enrolled in masters programs in public policy or international affairs will find that many of the applications with a policy focus, such as the sustainability of deficits, the incidence of a tax, the inflation tax, the international pattern of women's work, measuring inequality, income and the environment, and the Golden Rule, are especially relevant. The most advanced material is appropriate for doctoral students. Such applications as elements of probability and statistics, the Golden Rule, wage gaps and international trade, sources of comparative advantage, rules versus discretion in monetary policy, the price of stocks, exchange-rate determination, exchange-rate overshooting, the constant elasticity of substitution production function, utility functions and risk aversion, optimal growth, and life-cycle consumption tie in with material covered by doctoral students in their other courses. These students will also benefit from the advanced material on dynamic analysis, including the material on systems of difference equations, systems of differential equations, and dynamic optimization.

Structure

The book consists of five parts. Part One, Introduction, consists of three chapters that should be read by all students. Chapter 1 sets the stage for the book by introducing reasons for the use of mathematical modeling in economics and providing some basic definitions. Chapter 2 introduces important concepts that are used throughout the rest of the book, including properties of functions, graphical representation of functions, necessary and sufficient conditions, and concavity. This chapter also has a "menu" of functions to introduce different functional forms used throughout the rest of the book. Chapter 3 focuses on exponential and logarithmic functions, which have a very wide range of uses in economic analysis. The discussion of exponential functions focuses on the important concepts of growth and present value. The section on logarithmic functions discusses the "Rule-of-70," a rule-of-thumb for calculating doubling times, and also shows how the logarithmic transformation is helpful for graphing time series of data. Students with more preparation may spend little time on these chapters, and for them the chapters serve as a review of some basic concepts. Even doctoral students with prior mathematical training, however, will find some of the applications in Chapter 3 useful.

Part Two, Matrix Algebra, consists of two chapters. Professors who want to review basic techniques of solving systems of equations (like demand and supply models or the basic macroeconomic IS/LM model) can cover only the beginning of Chapter 4.

The rest of Chapter 4 serves as an introduction to the concepts and basic results of matrix algebra. This chapter can stand alone as an introduction to the basic terminology, concepts, and tools of matrix algebra. Chapter 5 includes a more thorough presentation of some of the topics in Chapter 4 as well as the more advanced topic of characteristic roots. The material in Chapter 5 is important for the more advanced treatment of topics like optimization and dynamic analysis.

Part Three, Differential Calculus, consists of three chapters. Chapter 6 is a basic introduction of the concepts behind differential calculus. This chapter develops the central concepts of differential calculus in the context of economic examples, including the link between tax rates and tax revenues and the link between exchange rates and the volume of imports. Equations, numerical examples, and graphical analysis are all used to develop the concept of differentiation. Chapter 7 develops the rules of differentiation that are needed to put into practice the concepts introduced in the previous chapter. Applications motivate the study of these rules, show their importance in economic contexts, and provide concrete examples of the use of the rules. These applications include the analysis of the sustainability of government budget deficits and the mathematical framework for thinking about risk aversion. This chapter also introduces the important economic concept of an elasticity. Elasticities are used in two application sections in this chapter: the relationship between a nation's income per capita and its infant mortality rate and the incidence of a tax. Chapter 8 presents multivariate calculus. This chapter demonstrates that partial differentiation is the mathematical analogue to the concept used in economic analysis of *ceteris paribus*, that is, "holding all else equal." Application sections in this chapter focus on returns to education, the puzzling lack of international capital flows to developing countries, means and outliers, and the division of national income. For more advanced students, this chapter has a discussion of properties of homogeneous and homothetic functions. The correspondence between the mathematical concepts and important economic concepts like the effect of changes "at the margin'' are emphasized throughout.

A central economic paradigm is that people do the best they can while facing certain constraints. Part Four, Optimization, shows how the tools of differential calculus can be used to determine the optimal actions undertaken by consumers, producers, or governments. Each of the chapters in this section begins by developing the key mathematical techniques, often in the context of an economic problem. Each chapter then includes a section on economic applications, some of which are standard textbook applications and some of which are drawn from recent research. The concept of an extreme value of a univariate function that is briefly introduced in Chapter 7 is revisited in Chapters 9 and 10. Application sections in Chapter 9 include an analysis of an empirical study of the link between income and pollution levels, some standard microeconomic problems like factor demands and the behavior of a monopolist, and three macroeconomic applications, rules versus discretion in monetary policy, the tax raised by governments through printing money, and the optimal savings rate. Chapter 10 extends the discussion in the previous chapter to functions with several arguments. The second section of this chapter presents second-order conditions in the bivariate case to develop students' intuition. The more general case is presented in the third section. This part concludes with Chapter 11, which presents techniques for solving constrained

optimization problems. The chapter begins with a simple consumption problem that is referenced throughout the chapter in increasingly more advanced ways. In the development of the Lagrangian method, there is a discussion of the economic interpretation of the Lagrange multiplier and the envelope theorem. The third section of this chapter includes five economic applications. The chapter ends with the more advanced topic of optimization with inequality constraints. Throughout the chapter, analytical methods are illustrated with numerical results. Undergraduate, masters, and doctoral students will all cover these three chapters, although there are sections of Chapters 10 and 11 that may be omitted for undergraduate or masters students.

Time plays a central role in many areas of economics, including finance, macroeconomics, monetary economics, international economics, economic growth, and environmental economics. Part Five, Integration and Dynamic Analysis, presents tools and techniques of dynamic analysis, that is, the analysis of variables over time. Chapter 12, which presents integral calculus, begins with a discussion of the concept of integration and shows how it can be interpreted as an area under a curve. The economic example of valuing a stream of payments (for example, finding the price of a bond that offers a stream of payments) is used to illustrate this concept. Rules of integration are presented. The final section of this chapter includes five applications of integration: consumer's surplus, the relationship between the maturity of a bond and the volatility of its price, a discussion of the cost of inflation, a look at measuring inequality, and a discussion of some basic concepts of probability and statistics. Difference equations, the subject of Chapter 13, link the value of a variable in one period to its value in another period as well as to the value of other variables. This chapter presents techniques for analyzing difference equations and also introduces some important concepts in dynamic analysis. The first two sections of this chapter may serve as a useful introduction to dynamic analysis in undergraduate courses. Applications include a dynamic Keynesian macroeconomic model, a macroeconomic model of inflation, a determination of the half-life of exchange rate misalignments, and a model of stock prices. The third section of this chapter presents more advanced topics, including systems of difference equations and second-order difference equations. Chapter 14 presents an analysis of differential equations which are the continuous time analogue to difference equations. Undergraduate courses may study the first two sections of this chapter, which consist of an introduction to the simplest types of differential equations and a set of applications of these types of equations. More advanced courses may also study the final section that presents systems of differential equations. Applications include the determination of inflation, two models of exchange-rate determination, a mathematical presentation of an eighteenth century theory of the balance of payments, and a presentation of the well-known Dornbusch overshooting model of exchange rates. The book's concluding chapter presents dynamic optimization. This is a more advanced topic, and this chapter is most appropriate for doctoral students. The chapter begins by drawing a link between the constrained optimization methods presented in Chapter 11 and dynamic optimization. The chapter then presents the central results of dynamic optimization. The chapter includes a number of extensions of the basic results. It also includes a number of applications that students are likely to encounter in advanced theory courses, such as optimal growth, the life-cycle theory of consumption, and the optimal rate of depletion of a natural resource.

Some suggested course outlines, corresponding to courses taught at different levels, are presented below. Instructors may choose to include a subset of applications for class presentation.

Suggested Course Outlines

Undergraduate	Masters	Doctoral
Part One	Part One	Part One
Section 4.1	Chapter 4	Part Two
Part Three	Part Three	Part Three
Part Four, except section 10.3, part of section 11.2, and section 11.4	Part Four, except section 10.3 and part of section 11.2	Part Four
Sections 13.1 and 13.2 or sections 14.1 and 14.2	Sections 13.1 and 13.2 and sections 14.1 and 14.2	Part Five

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