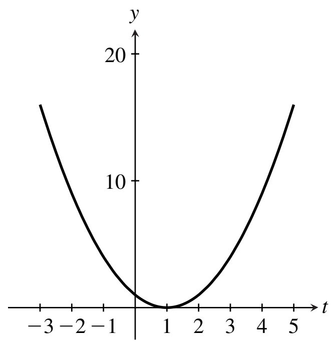
27. The graph of $f(x) = x^2 - 2x + 1 = (x-1)^2$ shown below fails the horizontal line test. In particular, we see that $f(0) = 1 = f(2)$. Hence, f is not one-to-one.



The above figure suggests that f is one-to-one on either $(-\infty, 1]$ or $[1, \infty)$. To find f^{-1} on $[1, \infty)$ we exchange the roles of x and y in $y = (x-1)^2$ and solve for y.

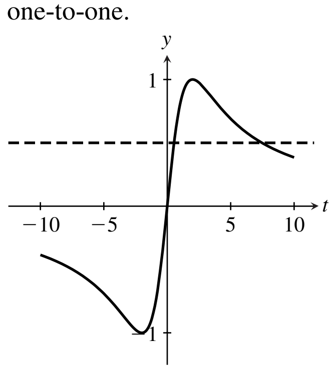
$x = (y-1)^2$
$\pm\sqrt{x} = y - 1$
$y = 1 \pm \sqrt{x}$

On the interval $[1, \infty)$, $f^{-1}(x) > 0$ and so we conclude that $f^{-1}(x) = 1 + \sqrt{x}$.

Thus, the inverse of f on $\left[\frac{7}{6}, \infty\right)$ is given by

$f^{-1}(x) = \dfrac{7 + \sqrt{49 - 12x}}{6}$.

29. The graph of $f(x) = \dfrac{4x}{x^2+4}$ shown below fails the horizontal line test. In particular, we see that $f(4 \pm 2\sqrt{3}) = \dfrac{1}{2}$. Hence, f is not one-to-one.



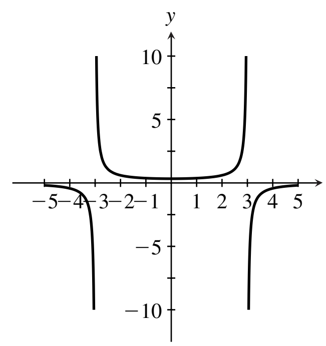
The above figure suggests that f is one-to-one on $[2, \infty)$. (We note $f(2)$ is a local maximum value.) To find f^{-1} on $[2, \infty)$ we exchange the roles of x and y in $y = \dfrac{4x}{x^2+4}$ and solve for y.

Applying the quadratic formula to the equation $xy^2 - 4y + 4x = 0$ yields $y = \dfrac{2 \pm 2\sqrt{1-x^2}}{x}$.

Thus, the inverse of f on $[2, \infty)$ is given by

$f^{-1}(x) = \dfrac{2 + 2\sqrt{1-x^2}}{x}$.

30. The graph of $f(x) = \dfrac{-3}{x^2-9}$ shown below fails the horizontal line test. In particular, we see that $f(-2) = \dfrac{3}{5} = f(2)$. Hence, f is not one-to-one.



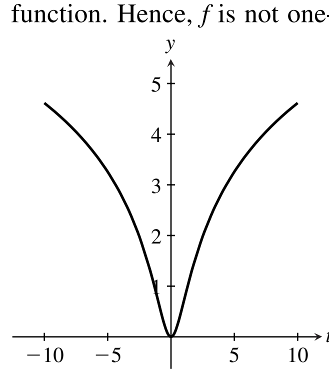
The above figure suggests that f is one-to-one on $(3, \infty)$. To find f^{-1} on $(3, \infty)$ we exchange the roles of x and y in $y = \dfrac{-3}{x^2-9}$ and solve for y. Applying the quadratic formula to the equation $xy^2 + (3 - 9x) = 0$ yields

$y = \pm\sqrt{\dfrac{9x-3}{x}}$.

Thus, the inverse of f on $(3, \infty)$ is given by

$f^{-1}(x) = \sqrt{\dfrac{9x-3}{x}}$.

31. The graph of $f(x) = \ln(x^2 + 1)$ shown below fails the horizontal line test. In particular, we see that $f(-2) = f(2)$ since f is an even function. Hence, f is not one-to-one.



The above figure suggests that f is one-to-one on either $(-\infty, 0]$ or $[0, \infty)$. To find f^{-1} on $(-\infty, 0]$ we exchange the roles of x and y in $y = \ln(x^2 + 1)$ and solve for y.

$\ln(y^2 + 1) = x$

$y^2 + 1 = e^x$

$y = \pm\sqrt{e^x - 1}$

Thus, the inverse of f on $(-\infty, 0]$ is given by $f'(x) = -\sqrt{e^x - 1}.$

33. Let $F(x) = f(x) + b$. Then solving $F(y) = x$ for y yields

$f(y) + b = x$

$f(y) = x - b$

$y = f^{-1}(x - b) = g(x - b).$

Thus, $F^{-1}(x) = g(x - b).$

35. Let $F(x) = f(cx)$ $(c \neq 0)$. Then solving $F(y) = x$ for y yields

$f(cy) = x$

$cy = f^{-1}(x) = g(x)$

$y = \frac{1}{c}g(x).$

Therefore, $F^{-1}(x) = \frac{1}{c}g(x).$

37. If the graphs of f and f^{-1} intersect at a point (a, b) then $f(a) = b$ and $f^{-1}(a) = b$. If $a = b$ then graphs intersect on the line $y = x$. Otherwise, since $f(a) = b$ we know that $f^{-1}(b) = a$. And if $f^{-1}(a) = b$, we get that $f(b) = a$. So (b, a) is also a point of intersection for the graphs of f and f^{-1}.

39. To show that the graphs of a function and its inverse are reflections over the line $y = x$ it is important that the scales on the x- and y-axes are the same so that the reflection is shown. If the scales differ then the line $y = x$ will not look precise (it will not have the expected slope) and the reflection won't be as obvious.

Section 2.9 Inverse Trigonometric Functions

1. $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $-\frac{\pi}{2} \le \frac{\pi}{3} \le \frac{\pi}{2}$.

3. $\arcsin(-1) = -\frac{\pi}{2}$ since $\sin\left(-\frac{\pi}{2}\right) = -1$ and $-\frac{\pi}{2} \le -\frac{\pi}{2} \le \frac{\pi}{2}$.

5. $\operatorname{arcsec} 2 = \arccos\frac{1}{2} = \frac{\pi}{3}$ since $\cos\frac{\pi}{3} = \frac{1}{2}$ and $0 \le \frac{\pi}{3} \le \pi$, $\frac{\pi}{3} \neq \frac{\pi}{2}$.

7. $\sin\left(\arcsin\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$

9. $\cos(\operatorname{arcsec} 10.6) = \cos\left(\arccos\frac{1}{10.6}\right) = \frac{1}{10.6}$

11. Let $\theta = \arcsin x$. Then $\sin\theta = x$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Hence, $\cos\theta = \sqrt{1 - x^2}$ since the cosine function is nonnegative on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\tan(\arcsin x) = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{\sqrt{1 - x^2}}$

13. Let $\theta = \operatorname{arcsec}(2x) = \arccos\left(\frac{1}{2x}\right)$. Then $\cos\theta = \frac{1}{2x}$ with $0 \le \theta \le \pi$. Hence

$\sin\theta = \sqrt{1 - \left(\frac{1}{2x}\right)^2} = \frac{\sqrt{4x^2 - 1}}{2x}$ since the sine function is nonnegative on $[0, \pi]$.

$\sin(\operatorname{arcsec}(2x)) = \sin\theta = \frac{\sqrt{4x^2 - 1}}{2x}$

15. Find $\frac{dy}{dx}$ where $y = \arcsin\,(2x)$.

$$\begin{aligned}\frac{dy}{dx} &= (\arcsin(2x))' \\ &= \frac{1}{\sqrt{1-(2x)^2}}(2x)' \\ &= \frac{2}{\sqrt{1-4x^2}}\end{aligned}$$

17. Find $\frac{dr}{d\theta}$ where $r = \arctan\left(\frac{1}{\theta}\right)$.

$$\begin{aligned}\frac{dr}{d\theta} &= (\arctan(\theta^{-1}))' \\ &= \frac{1}{1+(\theta^{-1})^2}(\theta^{-1})' \\ &= \frac{1}{1+(\theta^{-1})^2}(-1)\theta^{-2} \\ &= \frac{-1}{1+\theta^2}\end{aligned}$$

19. Find $g'(s)$ where $g(s) = \operatorname{arcsec}(3s-4)$.

$$\begin{aligned}g'(s) &= (\operatorname{arcsec}(3s-4)) \\ &= \frac{1}{|3s-4|\sqrt{(3s-4)^2-1}}\cdot(3s-4)' \\ &= \frac{3}{|3s-4|\sqrt{(3s-4)^2-1}}\end{aligned}$$

21. Find $\frac{dy}{dx}$ where $y = \arcsin\frac{1+x}{1-x}$.

$$\begin{aligned}\frac{dy}{dx} &= \left(\arcsin\frac{1+x}{1-x}\right)' \\ &= \frac{1}{\sqrt{1-\left(\frac{1+x}{1-x}\right)^2}}\left(\frac{1+x}{1-x}\right)' \\ &= \frac{1}{\sqrt{1-\left(\frac{1+x}{1-x}\right)^2}}\;\frac{(1+x)'(1-x)-(1+x)(1-x)'}{(1-x)^2} \\ &= \frac{1}{\sqrt{1-\left(\frac{1+x}{1-x}\right)^2}}\;\frac{(1)(1-x)-(1+x)(-1)}{(1-x)^2} \\ &= \frac{1}{\sqrt{1-\left(\frac{1+x}{1-x}\right)^2}}\;\frac{2}{(1-x)^2}\end{aligned}$$

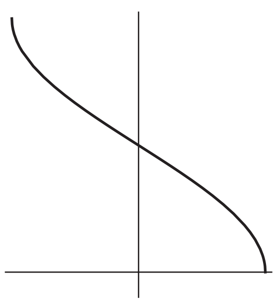
23. Find $f'(x)$ where $f(x) = e^{\arctan x^2}$.

$$\begin{aligned} f'(x) &= (e^{\arctan x^2})' \\ &= e^{\arctan x^2}(\arctan x^2)' \\ &= e^{\arctan x^2} \frac{1}{1+(x^2)^2}(x^2)' \\ &= e^{\arctan x^2} \frac{1}{1+x^4}(2x) \end{aligned}$$

25. Find $F'(w)$ where $F(w) = \dfrac{1+\arccos w}{1-\arccos w}$.

$$\begin{aligned} F'(w) &= \left(\frac{1+\arccos w}{1-\arccos w}\right)' \\ &= \frac{(1+\arccos w)'(1-\arccos w)-(1+\arccos w)(1-\arccos w)'}{(1-\arccos w)^2} \\ &= \frac{\left(-\frac{1}{\sqrt{1-w^2}}\right)(1-\arccos w)-(1+\arccos w)\left(\frac{1}{\sqrt{1-w}}\right)}{(1-\arccos w)^2} \\ &= \frac{-2}{\sqrt{1-w^2}\,(1-\arccos w)^2} \end{aligned}$$

27.



29. **a.** Since $\sin B = \dfrac{x}{1}$ and $\cos A = \dfrac{x}{1}$, we see that $\arcsin x = B$ and $\arccos x = A$.

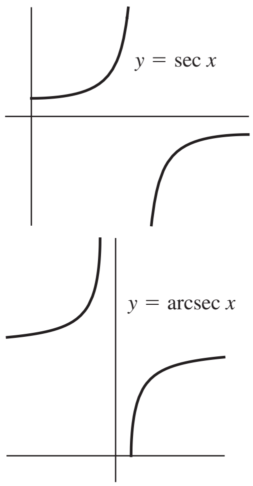
b. Because the sum of the measures of the angles of a triangle equals π and because the measure of angle C is $\dfrac{\pi}{2}$,

$$\arcsin x + \arccos x = \frac{\pi}{2} \quad (x \neq 0).$$

c. We take the derivative of the equation in **b**.

$$\begin{aligned} \frac{d}{dx}(\arcsin x + \arccos x) &= \frac{d}{dx}\left(\frac{\pi}{2}\right) \\ \frac{d}{dx}(\arcsin x) + \frac{d}{dx}(\arccos x) &= 0 \\ \frac{1}{\sqrt{1-x^2}} + \frac{d}{dx}(\arccos x) &= 0 \\ \frac{d}{dx}(\arccos x) &= -\frac{1}{\sqrt{1-x^2}} \end{aligned}$$

31.



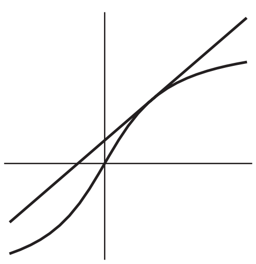
33. a. We differentiate $y = \arctan x$ at $\left(1, \frac{\pi}{4}\right)$ to find the slope.

$\frac{dy}{dx} = (\arctan x)' = \frac{1}{1+x^2}$

$(\arctan x)'\big|_{x=1} = \frac{1}{2}.$

So the equation of the tangent line at $\left(1, \frac{\pi}{4}\right)$ is $y - \frac{\pi}{4} = \frac{1}{2}(x-1).$

b.



c. $\arctan(0.95) \approx \frac{\pi}{4} + \frac{1}{2}(0.95-1)$
≈ 0.760398

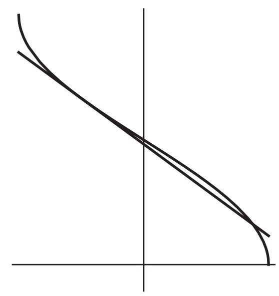
$\arctan(1.03) \approx \frac{\pi}{4} + \frac{1}{2}(1.03-1)$
≈ 0.800398

35. a. We differentiate $y = \arccos x$ at $\left(-\frac{1}{2}, \frac{2\pi}{3}\right)$ to find the slope.

$\frac{dy}{dx} = (\arccos x)' = -\frac{1}{\sqrt{1-x^2}},$

$(\arccos x)'\big|_{x=-\frac{1}{2}} = -\frac{2}{\sqrt{3}}$. So the equation of the tangent line is $y - \frac{2\pi}{3} = -\frac{2}{\sqrt{3}}\left(x + \frac{1}{2}\right).$

b.



c. $\arccos(-0.48) \approx -\frac{2}{\sqrt{3}}\left(-0.48 + \frac{1}{2}\right) + \frac{2\pi}{3}$
≈ 2.071

$\arccos(-0.56) \approx -\frac{2}{\sqrt{3}}\left(-0.56 + \frac{1}{2}\right) + \frac{2\pi}{3}$
≈ 2.16368

37. If $x < 0$ then the triangle and result of Example 4 are both still valid. If $x < 0$ then $-\frac{\pi}{2} < \theta < 0$ where both $\sin\theta$ and $\tan\theta$ are negative.

Section 2.10 Modeling: Translating the World into Mathematics

1. a. If $y = \frac{e^x}{x^3} = e^x x^{-3}$ then

$$\frac{dy}{dx} = \frac{d}{dx}(e^x x^{-3})$$
$$= \frac{d}{dx}\, e^x (x^{-3}) + (e^x)\, \frac{d}{dx}(x^{-3})$$
$$= \frac{e^x}{x^3}\left(1 - \frac{3}{x}\right).$$

We substitute for y and $\frac{dy}{dx}$ in

$$x\,\frac{dy}{dx}+3y=\frac{e^x}{x^2}.$$

$$x\left(\frac{e^x}{x^3}\left(1-\frac{3}{x}\right)\right)+3\left(\frac{e^x}{x^3}\right)=\frac{e^x}{x^2}$$

$$\frac{e^x}{x^2}-\frac{3e^x}{x^3}+\frac{3e^x}{x^3}=\frac{e^x}{x^2}$$

$$\frac{e^x}{x^2}=\frac{e^x}{x^2}$$

b. If $y=\frac{e^x}{x^3}+\frac{c}{x^3}$ then

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{e^x}{x^3}\right)+\frac{d}{dx}\left(\frac{c}{x^3}\right)$$

$$=\frac{e^x}{x^3}\left(1-\frac{3}{x}\right)-\frac{3c}{x^4}.$$

We substitute for y and $\frac{dy}{dx}$ in

$$x\frac{dy}{dx}+3y=\frac{e^x}{x^2}.$$

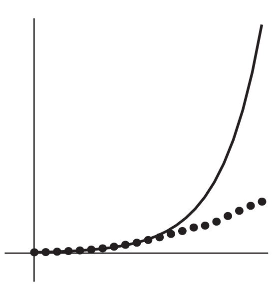
$$x\left(\frac{e^x}{x^3}\left(1-\frac{3}{x}\right)-\frac{3c}{x^4}\right)+3\left(\frac{e^x}{x^3}+\frac{c}{x^3}\right)=\frac{e^x}{x^2}$$

$$\frac{e^x}{x^2}-\frac{3e^x}{x^3}-\frac{3c}{x^3}+\frac{3e^x}{x^3}+\frac{3c}{x^3}=\frac{e^x}{x^2}$$

$$\frac{e^x}{x^2}=\frac{e^x}{x^2}$$

3. a. We seek to estimate the values of C and k in $P(t)=Ce^{kt}$ given that $P(0)=4\times10^6$ (we take t = 0 as 1790) and $P(90)=50\times10^6$. Since $P(0)=4\times10^6$, it follows that $C=4\times10^6$. Now, $P(90)=50\times10^6$ becomes $(4\times10^6)e^{90k}=50\times10^6$ and so solving for k we find that $k=\frac{1}{90}\ln\frac{25}{2}\approx0.0280637.$

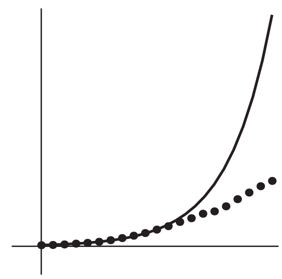
b.



The function fits the early data very well but eventually diverges in the later years.

c. We seek to estimate the value of C and k in $P(t)=Ce^{kt}$ given that $P(0)=13\times10^6$ (we take t = 0 as 1830) and $P(60)=63\times10^6$. Since $P(0)=13\times10^6$, it follows that $C=13\times10^6$.
Now, $P(60)=63\times10^6$ becomes $(13\times10^6)e^{60k}=63\times10^6$ and so solving for k we find that $k=\frac{1}{60}\ln\frac{63}{13}\approx0.0263031.$

d.



The function fits the early data very well but eventually diverges in the later years.

5. The graph of $\frac{dP}{dt}=kP-cP^2$ where k = 0.025 and $c=8.33\times10^{-11}$ is that of a downward opening parabola with vertex at $(1.5006\times10^8, 1.87575\times10^6)$ and x-intercepts of (0, 0) and $(3.0012\times10^8, 0)$.

a. Since $\left.\frac{dP}{dt}\right|_{P=2.5\times10^8}>0,$ the population is increasing when P = 250 million. Since $\left.\frac{dP}{dt}\right|_{P=4.0\times10^8}<0,$ the population is decreasing when P = 400 million.

b. The population is in equilibrium (the number of births = the number of deaths) when $\frac{dP}{dt}=0,$ which occurs when $P\approx3\times10^8.$

7. **a.** If $R(t) = (2 - 0.4 \cdot L)\ e^{-t/2T}$ where L is constant, then

$$\frac{dR}{dt} = \frac{d}{dt}\left((2 - 0.4 \cdot L)\ e^{-t/2T}\right)$$
$$= \left(\frac{d}{dt}(2 - 0.4 \cdot L)\right)(e^{-t/2T}) + (2 - 0.4L)\ \frac{d}{dt}(e^{-t/2T})$$
$$= 0 + (2 - 0.4L)\left(-\frac{1}{2T}e^{-t/2T}\right)$$
$$= -\frac{1}{2T}(2 - 0.4 \cdot L)\ e^{-t/2T}$$
$$= -\frac{1}{2T}(R(t)).$$

Verifying the initial condition, $R(0) = (2 - 0.4 \cdot L)e^{-0/2T} = (2 - 0.4 \cdot L)(1) = 2 - 0.4 \cdot L.$

b. If $L = 0$ then $R(T) = 2e^{-T/2T} = 2e^{-1/2} \approx 1.21$ liters. Since the initial amount was 2 liters, the loss is approximately 0.79 liters of red blood cells.

c. If $L = 1$ then $R(T) = 1.6e^{-T/2T} = 1.6e^{-1/2} \approx 0.97$ liters. One liter of blood was removed, this means that 0.4 liters of red blood cells were removed. So there was a loss of approximately $1.6 - 0.97 = 0.63$ liters of red blood cells during the surgery. This is 0.16 liters less than the amount than in **b**.

9. $C'(t) = -kC(t)\ \ (k > 0)$. We note that $k > 0$ since the concentration of the drug dissipates with the passage of time.

11. $T'(t) = -k(T(t) - T_0)\ \ (k > 0)$. We note that $k > 0$ since the temperature of the object must decline with the passage of time.

13. **i.** For positive constants c, d, f, g we have

$$\frac{dA}{dt} = cA(t) - dA(t)\ B(t) \text{ and}$$
$$\frac{dB}{dt} = fB(t) - gA(t)\ B(t).$$

In $\frac{dA}{dt} = cA(t) - dA(t)\ B(t)$ the term $cA(t)$ denotes the change in the population that arises from the assumption that the number of offspring produced is proportional to $A(t)$ and the term $dA(t)\ B(t)$ is the change in population due to competition. Similar statements hold for $\frac{dB}{dt} = fB(t) - gA(t)\ B(t)$. We note that it is likely that $c \neq f$ and $d \neq g$.

ii. Solving the system

$$cA(t) - dA(t)\ B(t) = 0$$
$$fB(t) - gA(t)\ B(t) = 0$$

for $A(t)$ and $B(t)$ we have either

$A(t) = 0 = B(t)$ or $\left(A(t) = \frac{f}{g} \text{ and } B(t) = \frac{c}{d}\right)$. Under these conditions, the populations are in "equilibrium" and will remain stable for all future time. Under almost all initial conditions one species will eventually die out while the other will grow.

15. **i.** For positive constants a, b, c, and d we have

$$\frac{dF}{dt} = aR(t)\ F(t) - bF(t)$$

and

$$\frac{dR}{dt} = cR(t) - dR(t)\ F(t).$$

$aR(t)\ F(t)$ means that the number of new foxes is proportional to the number of encounters between rabbits and foxes. $-bF(t)$ is the effect of population pressure on the fox population. $cR(t)$ means new rabbits are born at a rate proportional to the rabbit population. $-dR(t)\ F(t)$ means rabbits are killed by foxes at a rate proportional to the number of encounters between rabbits and foxes.

ii. The equilibrium points for the system

$$aRF - bF = 0$$
$$cR - dRF = 0$$

are $(R, F) = (0, 0)$ and $(R, F) = \left(\frac{b}{a}, \frac{c}{d}\right)$. Under these conditions the populations of both rabbits and foxes are stable and unchanging.

Chapter 2 Review Exercises

1. $y = -4x^5 + 7x^4 - \sqrt{2}x^2 + 3x - \pi$

$$\frac{dy}{dx} = -20x^4 + 28x^3 - 2\sqrt{2}x + 3$$

3. $r = \left(\theta^2 - \frac{3}{\theta}\right)\sin\theta = (\theta^2 - 3\theta^{-1})\sin\theta$

$$\frac{dr}{d\theta} = \left(\frac{d}{d\theta}(\theta^2 - 3\theta^{-1})\right)(\sin\theta) + (\theta^2 - 3\theta^{-1})\left(\frac{d}{d\theta}\sin\theta\right)$$
$$= (2\theta + 3\theta^{-2})(\sin\theta) + (\theta^2 - 3\theta^{-1})(\cos\theta)$$

5. $y = \ln(4x^3 - 2x + 3)$

$$\frac{dy}{dx} = \frac{1}{(4x^3 - 2x + 3)} \cdot \frac{d}{dx}(4x^3 - 2x + 3)$$
$$= \frac{12x^2 - 2}{4x^3 - 2x + 3}$$

7. $k(p) = \frac{p}{\sqrt{p^2 + 3}} = \frac{p}{(p^2 + 3)^{1/2}}$

$$k'(p) = \frac{p'(p^2 + 3)^{1/2} - p((p^2 + 3)^{1/2})'}{((p^2 + 3)^{1/2})^2}$$
$$= \frac{(p^2 + 3)^{1/2} - p\left(\frac{1}{2}(p^2 + 3)^{-1/2}\right) \cdot 2p}{p^2 + 3}$$
$$= \frac{(p^2 + 3)^{1/2} - p^2(p^2 + 3)^{-1/2}}{p^2 + 3}$$
$$= \frac{3}{(p^2 + 3)^{3/2}}$$

9. $g(x)=\ln\left(\dfrac{2x-3}{4x+5}\right)$

$$g'(x)=\frac{1}{\left(\frac{2x-3}{4x+5}\right)}\cdot\left(\frac{(2x-3)'(4x+5)-(2x-3)(4x+5)'}{(4x+5)^2}\right)$$
$$=\left(\frac{4x+5}{2x-3}\right)\left(\frac{2(4x+5)-(2x-3)(4)}{(4x+5)^2}\right)$$
$$=\frac{22}{8x^2-2x-15}$$

11. $y=\sqrt{t}\arcsin t^2=t^{1/2}\arcsin(t^2)$

$$\frac{dy}{dt}=(t^{1/2})'\arcsin(t^2)+(t^{1/2})(\arcsin(t^2))'$$
$$=\frac{\arcsin(t^2)}{2\sqrt{t}}+\sqrt{t}\cdot\frac{1}{\sqrt{1-(t^2)^2}}\cdot 2t$$
$$=\frac{\arcsin(t^2)}{2\sqrt{t}}+\frac{2t^{3/2}}{\sqrt{1-t^4}}$$

13. $s(t)=\ln(\tan 2t)$

$$s'(t)=\frac{1}{\tan 2t}\cdot(\tan 2t)'$$
$$=\cot(2t)\cdot\sec^2(2t)\cdot 2$$
$$=2\csc(2t)\sec(2t)$$

15. $y=\dfrac{1}{\arctan\left(1+\frac{1}{x}\right)}=\left(\arctan\left(1+\frac{1}{x}\right)\right)^{-1}$

$$\frac{dy}{dx}=-\left(\arctan\left(1+\frac{1}{x}\right)\right)^{-2}\cdot\frac{d}{dx}\left(\arctan\left(1+\frac{1}{x}\right)\right)$$
$$=-\left(\arctan\left(1+\tfrac{1}{x}\right)\right)^{-2}\cdot\left(\frac{1}{1+\left(1+\frac{1}{x}\right)^2}\right)\left(-\frac{1}{x^2}\right)$$
$$=\frac{1}{\left(\arctan\left(1+\frac{1}{x}\right)\right)^2\left(1+1+\frac{2}{x}+\frac{1}{x^2}\right)(x^2)}$$
$$=\frac{1}{\left(\arctan\left(1+\frac{1}{x}\right)\right)^2(2x^2+2x+1)}$$

17. $r=\sin(\cos(1+\tan 2t))$

$$\frac{dr}{dt}=\cos(\cos(1+\tan 2t))\cdot(\cos(1+\tan 2t))'$$
$$=\cos(\cos(1+\tan 2t))'\cdot(-\sin(1+\tan 2t))\cdot(\sec^2(2t))\cdot 2$$
$$=-2\sec^2(2t)(\sin(1+\tan 2t))(\cos(\cos(1+\tan 2t)))$$

19. $H(s)=e^{as^2+bs+c}$

$$H'(s)=(2as+b)\ e^{as^2+bs+c}$$

21. $y^3 + xy - 3x^2 = 0$

$$\frac{d}{dx}(y)^3 + \frac{d}{dx}(xy) - \frac{d}{dx}(3x^2) = \frac{d}{dx}(0)$$

$$3y^2\frac{dy}{dx} + y + x\frac{dy}{dx} - 6x = 0$$

$$\frac{dy}{dx} = \frac{6x - y}{3y^2 + x}$$

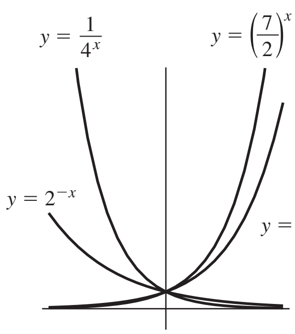
23. $\sin(x + y) = x - y$

$$\frac{d}{dx}(\sin(x + y)) = \frac{d}{dx}(x - y)$$

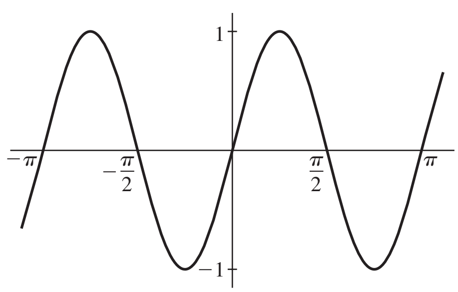
$$\cos(x + y)\left(1 + \frac{dy}{dx}\right) = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 - \cos(x + y)}{1 + \cos(x + y)}$$

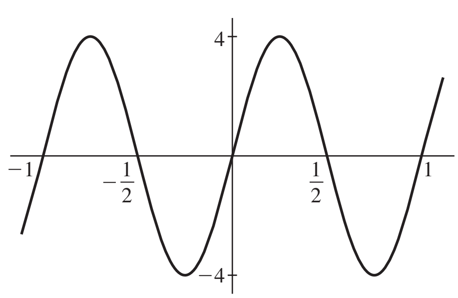
25.



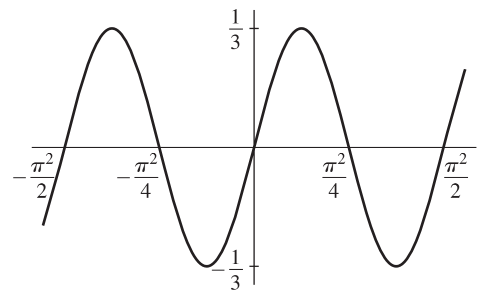
27. a.



b.

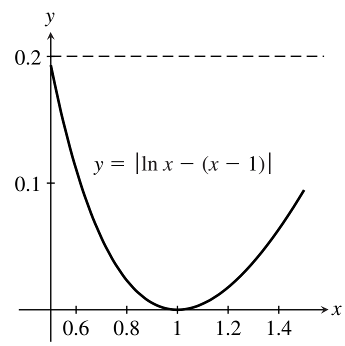


c.



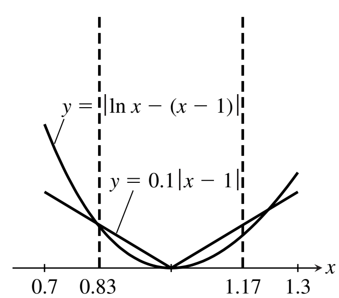
29. a. $g(x) = \ln x$, so $g'(x) = \frac{1}{x}$ and $g'(1) = 1.$
The equation for the tangent line at (1, 0) is $y - 0 = 1(x - 1)$ or $y = x - 1$.

b. The graph of $y = |\ln x - (x - 1)|$ for $0.5 \le x \le 1.5$ is shown.



The maximum possible absolute error is approximately 0.2.

c.



Any $r \le 0.17$ works.

31. a. Since $\tan\theta = \frac{h}{300}$ we get $\theta = \arctan\left(\frac{h}{300}\right).$

b. We assume the rocket cannot go below ground, so $50 \le h < \infty$ and $\arctan\left(\frac{1}{6}\right) \le \theta < \frac{\pi}{2}.$

c. $\dfrac{d}{dh}(\theta)=\dfrac{d}{dh}\left(\arctan\left(\dfrac{h}{300}\right)\right)$

$$\frac{d\theta}{dh}=\frac{1}{1+\left(\frac{h}{300}\right)^2}\cdot\frac{1}{300}$$
$$=\frac{300}{90{,}000+h^2}$$

d. The units for $\dfrac{d\theta}{dh}$ are radians/feet. This means that the number of radians that θ increases for each foot of increase in the height of the rocket is $\dfrac{300}{90{,}000+h^2}$.

33. a. We let β be the boy's angle of vision from the front of the plane to the ground, and θ is the boy's angle of vision from the tail of the plane to the ground. So $\theta=\beta+\phi$, or $\phi=\theta-\beta$.

Now we get $\theta=\arcsin\left(\dfrac{500}{y}\right)$ and $\beta=\arctan\left(\dfrac{500}{150+\sqrt{y^2-500^2}}\right)$.

Therefore $\phi=\arcsin\left(\dfrac{500}{y}\right)-\arctan\left(\dfrac{500}{150+\sqrt{y^2-500^2}}\right)$.

b. The height of the plane is 500 feet and y is the distance from the plane to the boy, $y\ge 500$.

ϕ is largest when the plane is directly overhead; $\phi=\dfrac{\pi}{2}-\arctan\left(\dfrac{500}{150}\right)\approx 0.291$ radians so the possible values for ϕ are $0<\phi\le 0.291$ radians.

c. $\dfrac{d\phi}{dy}=\dfrac{d}{dy}\left(\arcsin\left(\dfrac{500}{y}\right)-\arctan\left(\dfrac{500}{150+\sqrt{y^2-500^2}}\right)\right)$

$$=\frac{1}{\sqrt{1-\left(\frac{500}{y}\right)^2}}\cdot\left(-\frac{500}{y^2}\right)-\left(\frac{1}{1+\left(\frac{500}{150+\sqrt{y^2-500^2}}\right)^2}\right)\frac{-500}{\left(150+\sqrt{y^2-500^2}\right)^2}\cdot\frac{1}{2\sqrt{y^2-500^2}}\cdot 2y$$
$$=\frac{-500}{y\sqrt{y^2-500^2}}+\frac{500\cdot y}{((150+\sqrt{y^2-500^2})^2+500^2)\sqrt{y^2-500^2}}$$

d. The units are radians/foot. The derivative tells how much the angle ϕ subtended by the plane decreases per foot that the plane is away from the boy.

35. a. $a(40)$ = –57.3 arcsin(–0.53 + 0.33 cos(0.017·40)) ≈ 15.9 so $a(t)\approx 15.9°$ on January 31.
$a(313)$ = –57.3 arcsin(–0.53 + 0.33 cos(0.017 · 313)) ≈ 20.0 so $a(t)\approx 20.0°$ on October 31.

b. $30=-57.3\arcsin(-0.53+0.33\cos(0.017t))$
$$0.091010=\cos(0.017t)$$
$$\frac{\cos^{-1}(0.091010)}{0.017}\approx 87$$
Since we want a date after June 21 we choose t = 365 – 87 = 278 so our date is near September 25.

c. $45 = -57.3 \arcsin(-0.53 + 0.33\cos(0.017t))$

$-0.536563 = \cos(0.017t)$

$\frac{\cos^{-1}(-0.536563)}{0.017} \approx 126$

We want dates for both $t = 126$ and $t = 365 - 126 = 239$. We get dates near April 27 and August 17.

d. $\frac{da}{dt} = -57.3 \cdot \frac{0.33 \cdot (-\sin(0.017t)) \cdot 0.017}{\sqrt{1-(-0.53+0.33\cos(0.017t))^2}}$

$\approx \frac{0.32\sin(0.017t)}{\sqrt{1-(0.53-0.33\cos(0.017t))^2}}$

$\frac{da}{dt}$ gives the rate of increase of maximum angle with respect to time.

37. a. We use the cosine function for the position of the shadow. Assuming that when $t = 0$ the car is at the bottom and when $x = 0$ the shadow is below the center of the wheel, we get $x = 60\cos\left(\frac{2\pi}{40}t - \frac{\pi}{2}\right)$ if the wheel turns counter-clockwise.

b. $\frac{dx}{dt} = \frac{d}{dt}\left(60\cos\left(\frac{2\pi}{40}t - \frac{\pi}{2}\right)\right)$

$= -60\sin\left(\frac{2\pi}{40}t - \frac{\pi}{2}\right)\left(\frac{2\pi}{40}\right)$

$= -3\pi\sin\left(\frac{2\pi}{40}t - \frac{\pi}{2}\right)$

This gives the speed of the shadow with respect to time t.

c. Finding the maximum of $-3\pi\sin\left(\frac{2\pi}{40}t - \frac{\pi}{2}\right)$ we get $t = 0, 20, 40, 60, \ldots$ At these times the rate of change is $\pm 3\pi$ and the shadow is below the center of the wheel. Finding the minimum of $-3\pi\sin\left(\frac{2\pi}{40}t - \frac{\pi}{2}\right)$ we get $t = 10, 30, 50, 70, \ldots$ At these times the rate of change is 0 and the shadow is at $x = \pm 60$.

39. a. $A(x) = x \cdot \left(\frac{3+(4x+3)}{2}\right) = 2x^2 + 3x$

b. $A'(x) = 4x + 3$. $A'(x)$ gives the rate of change of the area with respect to x.