11. Find $g'(t)$ if $g(t)=\dfrac{\cot(3t)}{1+\csc^2(3t)}$. If $f(t)=\cot(3t)$ and $h(t)=1+\csc^2(3t)$ then $f'(t)=-3\csc^2(3t)$ and

$$h'(t)=2\csc(3t)\cdot(-\csc(3t)\cot(3t))(3)$$
$$=-6\csc^2(3t)\cot(3t).$$

Because $g(t)=\dfrac{f(t)}{h(t)}$ we can find $g'(t)$ by the quotient rule.

$$g'(t)=\frac{f'(t)h(t)-f(t)h'(t)}{(h(t))^2}$$
$$=\frac{-3\csc^2(3t)(1+\csc^2(3t))-\cot(3t)(-6\csc^2(3t)\cot(3t))}{(1+\csc^2(3t))^2}$$
$$=\frac{-3\csc^2(3t)(1+\csc^2(3t))+6\cot^2(3t)\csc^2(3t)}{(1+\csc^2(3t))^2}$$

13. Determine $\dfrac{dz}{dr}$ if $z = \sin r \cos r$. By the product rule,

$$\frac{dz}{dr}=(\sin r\cos r)'$$
$$=(\sin r)'(\cos r)+(\sin r)(\cos r)'$$
$$=(\cos r)(\cos r)+(\sin r)(-\sin r)$$
$$=\cos^2 r-\sin^2 r$$
$$=\cos(2r).$$

15. Find $\dfrac{dy}{dx}$ where $y = \sin(x \cos x)$. We exhibit
$y = \sin(x \cos x)$ as a composition chain:
$y = \sin u$
$u = x \cos x$.
We then have

$$\frac{dy}{du}=(\sin u)'=\cos u$$
$$\frac{du}{dx}=(x\cos x)'$$
$$=(x)'(\cos x)+(x)(\cos x)'$$
$$=(1)(\cos x)+(x)(-\sin x)$$
$$=\cos x-x\sin x.$$

By the chain rule,

$$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$$
$$=(\cos u)(\cos x-x\sin x).$$

Substitute $u = x \cos x$ into the last expression to obtain $\dfrac{dy}{dx}=(\cos(x\cos x))(\cos x-x\sin x)$.

17. Compute $f'(w)$ where $y = f(x) = 1 + \tan^2(2x)$. We show $y = f(x) = 1 + \tan^2(2x)$ as a composition chain:

$y = 1 + u^2$
$u = \tan v$
$v = 2x$

We then have

$$\frac{dy}{du} = (1 + u^2)' = 2u$$
$$\frac{du}{dv} = (\tan v)' = \sec^2 v$$
$$\frac{dv}{dx} = (2x)' = 2.$$

By the chain rule,

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}\\ &= (2u)(\sec^2 v)(2)\\ &= (2\tan v)(\sec^2 v)(2)\\ &= (2\tan 2x)(\sec^2 2x)(2)\\ &= 4\tan 2x \sec^2 2x.\end{aligned}$$

19. Compute y' where $y = q(\cot t)$ and $q(t) = t\sin t$. First we compute

$$\begin{aligned}q'(t) &= (t\sin t)'\\ &= (t')\sin t + t(\sin t)'\\ &= \sin t + t\cos t.\end{aligned}$$

Using the chain rule we find

$$\begin{aligned}y' &= (q(\cot t))'\\ &= (q'(\cot t))(\cot t)'\\ &= (\sin(\cot t) + \cot t\cos(\cot t))(-\csc^2 t).\end{aligned}$$

21. Determine $\left.\frac{dy}{dx}\right|_{x=\frac{\sqrt{2}}{2},\ y=\frac{3\pi}{4}}$ where $\sin y = x$ and $y = y(x)$. We differentiate both sides of $\sin y = x$ with respect to x using the chain rule on the left-hand-side of the equation.

$$(\sin y)' = (x)'$$
$$(\cos y)\frac{dy}{dx} = (1)$$
$$\frac{dy}{dx} = \frac{1}{\cos y}$$

Hence,

$$\left.\frac{dy}{dx}\right|_{x=\frac{\sqrt{2}}{2},\ y=\frac{3\pi}{4}} = \frac{1}{\cos\frac{3\pi}{4}} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}.$$

23. Determine $\left.\frac{dy}{dx}\right|_{x=\pi,\ y=\pi}$ where $xy\cos(x+y)=\pi^2$ and $y=y(x)$. We differentiate both sides of $xy\cos(x+y)=\pi^2$ with respect to x using the product rule and the chain rule on the left hand side of the equation.

$(xy\cos(x+y))'=(\pi^2)'$

$(xy)'\cos(x+y)+(xy)(\cos(x+y))'=(\pi^2)'$

$(x'y+xy')\cos(x+y)+(xy)(\cos(x+y))'=(\pi^2)'$

$\left(y+x\frac{dy}{dx}\right)\cos(x+y)+(xy)(-\sin(x+y))\left(1+\frac{dy}{dx}\right)=0$

Solving for $\frac{dy}{dx}$ we get $\frac{dy}{dx}=\frac{y(\cos(x+y)-x\sin(x+y))}{x(y\sin(x+y)-\cos(x+y))}$.

Hence

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=\pi,\ y=\pi} &= \frac{\pi(\cos 2\pi-\pi\sin 2\pi)}{\pi(\pi\sin 2\pi-\cos 2\pi)}\\ &= \frac{1-0}{0-1}\\ &= -1.\end{aligned}$$

25. Determine $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6},\ y=\frac{\pi}{3}}$ where

sin x + cos y = 1 and $y=y(x)$. We differentiate both sides of $\sin x+\cos y=1$ with respect to x.

$(\sin x+\cos y)'=1'$

$(\sin x)'+(\cos y)'=1'$

$\cos x-(\sin y)\frac{dy}{dx}=0$

Solving for $\frac{dy}{dx}$ we get

$\frac{dy}{dx}=\frac{\cos x}{\sin y}$. So

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6},\ y=\frac{\pi}{3}}=\frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{3}}=1.$$

27. First we find $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}}$ for $y = \tan x$.

$\frac{dy}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x$

So $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \sec^2\left(\frac{\pi}{4}\right) = 2.$

Then the equation of the tangent of $y = \tan x$ at $\left(\frac{\pi}{4}, 1\right)$ is $y - 1 = 2\left(x - \frac{\pi}{4}\right)$ or

$y = 1 + 2\left(x - \frac{\pi}{4}\right).$

Using the tangent line to approximate a value of $\tan\left(\frac{\pi}{4} + 0.03\right)$ we find the y-value of the tangent line when $x = \frac{\pi}{4} + 0.03.$

$y = 1 + 2\left(\left(\frac{\pi}{4} + 0.03\right) - \frac{\pi}{4}\right) = 1 + 2(0.03) = 1.06$

The calculator value for $\tan\left(\frac{\pi}{4} + 0.03\right)$ is ≈ 1.0619. So the error in the approximation is ≈ 0.0019 (approximations may vary depending upon your calculator).

29. First we find $\left.\frac{dy}{dx}\right|_{x=\frac{1}{4}}$ for $y = \cos(\pi x)$

$\frac{dy}{dx} = \frac{d}{dx}(\cos(\pi x)) = -\pi \sin(\pi x)$

So $\left.\frac{dy}{dx}\right|_{x=\frac{1}{4}} = -\pi \sin\left(\frac{\pi}{4}\right) = \frac{-\pi}{\sqrt{2}}.$

Then the equation of the tangent line of $y = \cos(\pi x)$ at $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ is

$y - \frac{1}{\sqrt{2}} = \frac{-\pi}{\sqrt{2}}\left(x - \frac{1}{4}\right)$ or $y = \frac{1}{\sqrt{2}} - \frac{\pi}{\sqrt{2}}\left(x - \frac{1}{4}\right).$

Using the tangent line to approximate a value for $\cos(\pi(0.265))$ we find the y-value of the tangent line when $x = 0.265$.

$$\begin{aligned} y &= \frac{1}{\sqrt{2}} - \frac{\pi}{\sqrt{2}}\left(0.265 - \frac{1}{4}\right) \\ &= \frac{1}{\sqrt{2}} - \frac{\pi}{\sqrt{2}}(0.015) \\ &\approx 0.6738 \end{aligned}$$

The calculator value for $\cos(\pi(0.265)) \approx 0.6730$. So the error in approximation is $\approx 8 \times 10^{-4}$ (approximations may vary depending upon your calculator).

31. If $f(x) = \sin x$ then $f'(x) = \cos x$. Since $f(a) = \sin a = 0.6$ and $\cos^2(a) = 1 - \sin^2(a) = 0.64$ we get $f'(a) = \cos(a) = 0.8$ or -0.8.

33. If $F(\theta) = \sin(2\theta)$ then $F'(\theta) = 2\cos(2\theta)$. Since $F(a) = \sin(2a) = -0.5$ and $\cos^2(2a) = 1 - \sin^2(2a) = 0.75$ we get $F'(a) = 2\cos(2a) = \pm 2\sqrt{0.75} = \sqrt{3}$ or $-\sqrt{3}$.

35. If $h(r) = 4\tan(2r + 1)$ then $h'(r) = (4\tan(2r+1))' = 8\sec^2(2r+1)$. Since $h(a) = 4\tan(2a+1) = -2$ and $\sec^2(2a+1) = \tan^2(2a+1) + 1 = \frac{1}{4} + 1 = \frac{5}{4}$ we get $h'(a) = 8\sec^2(2r+1) = 10$.

37. The curve traced is a horizontal shift to the right by $\frac{\pi}{2}$ units and so the equation of the curve is given by $y = \sin\left(x - \frac{\pi}{2}\right) = -\cos x$.

39. We use the trigonometric identity $2\sin x \cos x = \sin(2x)$ to show that $\frac{d}{dx}\sin^2 x = \sin(2x)$. First we differentiate and use the chain rule.

$$\begin{aligned} \frac{d}{dx}(\sin^2 x) &= 2\sin x\left(\frac{d}{dx}\sin x\right) \\ &= 2\sin x \cos x \\ &= \sin(2x) \end{aligned}$$

Next we differentiate.

$$\begin{aligned} \frac{d}{dx}\cos^2 x &= 2\cos x\left(\frac{d}{dx}\cos x\right) \\ &= -2\cos x \sin x \\ &= -\sin(2x) \end{aligned}$$

41. First we find $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}}$ for $y = \sin x$.

$\frac{dy}{dx} = \frac{d}{dx}\sin x = \cos x$ so $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

Next we find $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}}$ for $y = \cos x$.

$\frac{dy}{dx} = \frac{d}{dx}\cos x = -\sin x$ so

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

A line with a slope of $\frac{\sqrt{2}}{2}$ has an angle of $\theta = \arctan\left(\frac{\sqrt{2}}{2}\right)$ with respect to the horizon a line with a slope of $-\frac{\sqrt{2}}{2}$ has an angle of

$\beta = \arctan\left(-\frac{\sqrt{2}}{2}\right) = -\arctan\left(\frac{\sqrt{2}}{2}\right)$.

So the horizontal angle between these lines is $\theta - \beta = 2\arctan\left(\frac{\sqrt{2}}{2}\right)$ and the vertical angle between these lines is

$\pi - (\theta - \beta) = \pi - 2\arctan\left(\frac{\sqrt{2}}{2}\right)$.

43. We establish that $(\sin\theta)' = \cos\theta$.

$(\sin\theta)'$

$= \lim_{h\to 0}\frac{\sin(\theta+h) - \sin\theta}{h}$

(definition of the derivative)

$= \lim_{h\to 0}\frac{(\sin\theta\cos h + \cos\theta\sin h) - \sin\theta}{h}$

(trigonometric identity)

$= \lim_{h\to 0}\left(\sin\theta\frac{\cos h - 1}{h}\right) + \lim_{h\to 0}\left(\cos\theta\frac{\sin h}{h}\right)$

(basic algebra and limit of a sum = sum of the limits provide the limits exist)

$= \sin\theta\lim_{h\to 0}\left(\frac{\cos h - 1}{h}\right) + \cos\theta\lim_{h\to 0}\left(\frac{\sin h}{h}\right)$

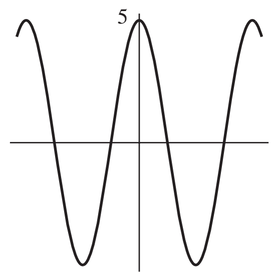
(limit of a constant times function equal the constant times the limit of the function)

$= \sin\theta(0) + \cos\theta(1)$

(substitution of special limit values)

$= \cos\theta$.

45. a. t = time is in minutes. At $t = 0$ cork is at a high point, labeled as height 5. Height $h = 0$ is midway between the high and low points of the cork. (Other answers are possible.)



b. We assume we can model the motion using a trigonometric function and assume height is 5 m at $t = 0$. Since frequency is 6 times a minute we will use $6 \cdot 2\pi \cdot t$ as the argument for the trigonometric function. Using the assumptions above we get $h(t) = 5\cos(12\pi t)$.

c. $h'(t) = 5\cos(12\pi t)' = -60\pi\sin(12\pi t)$. This tells us the velocity of the cork.

47. a. If $A = (\cos a, \sin a)$ and $B = (\cos b, \sin b)$ then the slope of segment AB is $\frac{\sin b - \sin a}{\cos b - \cos a}$.

b. The equation for the line through $P = (1, 0)$ and parallel to $\overline{AB}$ is given by $y = \frac{\sin b - \sin a}{\cos b - \cos a}(x - 1)$.

c. To find $Q \neq (1, 0)$ find the intersection of $y = \frac{\sin b - \sin a}{\cos b - \cos a}(x-1)$ and $x^2 + y^2 = 1$.

Substitution yields $x^2 + \left(\left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)(x-1)\right)^2 - 1 = 0$

$$\left(\left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)^2 + 1\right)x^2 - 2\left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)^2 x + \left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)^2 - 1 = 0$$

Using the quadratic formula we get

$x = \frac{\left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)^2 \pm 1}{\left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)^2 + 1}$. Since $x \neq 1$, we get $x = \frac{\left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)^2 - 1}{\left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)^2 + 1} = -\frac{\cos 2b + \cos 2a - 2\cos(b+a)}{2 - 2\cos(b-a)}$

so $Q = \left(-\frac{\cos 2b + \cos 2a - 2\cos(b+a)}{2 - 2\cos(b-a)}, \left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)\left(-\frac{\cos 2b + \cos 2a - 2\cos(b+a)}{2 - 2\cos(b-a)} - 1\right)\right)$.

d. Because segments AB and PQ are parallel, arcs $\overset{\frown}{AP}$ and $\overset{\frown}{BQ}$ are equal.
Therefore angles $\measuredangle AOP$ and $\measuredangle BOQ$ are equal.
So $\measuredangle BOQ = a$, and $\measuredangle POQ = \measuredangle BOQ + \measuredangle BOP = a + b$.
Hence $Q = (\cos(a+b), \sin(a+b))$.

e. Combining **c** and **d** above we get

$\cos(a+b) = -\frac{\cos 2b + \cos 2a - 2\cos(b+a)}{2 - 2\cos(b-a)}$ and

$\sin(a+b) = \left(\frac{\sin b - \sin a}{\cos b - \cos a}\right)\left(-\frac{\cos 2b + \cos 2a - 2\cos(b+a)}{2 - 2\cos(b-a)} - 1\right)$.

49. Use the results from Exercise 48. By basic right triangle trigonometry, we have $OB = \sec(\theta + h)$, $OA = \sec\theta$, and $OA = OC \cos h$. So, for h close to zero we have $OA \approx OC$. Hence, for small h we see that

$$\begin{aligned}\sec(\theta+h) - \sec\theta &= OB - OA \\ &= OB - OC\cos h \\ &\approx OB - OC \\ &= BC.\end{aligned}$$

Since $\theta = \measuredangle BAC$, it follows that

$$\begin{aligned}\tan\theta &= \frac{BC}{AC} \\ &\approx \frac{\sec(\theta+h) - \sec\theta}{h\sec\theta}.\end{aligned}$$

Thus $\frac{\sec(\theta+h) - \sec\theta}{h} \approx \tan\theta\sec\theta$.

Section 2.6 Exponential Functions

1. Solve $2^{-3x} = 16$ for x.

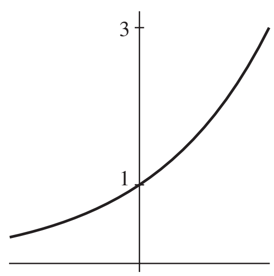
$$\begin{aligned}2^{-3x} &= 2^4 \\ -3x &= 4 \\ x &= \frac{-4}{3}\end{aligned}$$

3. Solve $e^{\theta} = e$ for θ.
$e^{\theta} = e^1$
$\theta = 1$

5. Solve $e^{2x} - 3e^x + 2 = 0$ for x. Set $w = e^x$. Then $(e^x)^2 - 3ex + 2 = 0$ becomes $w^2 - 3w + 2 = 0$.
Solving the equation in w yields
$(w-2)(w-1) = 0$
$w = 1, 2.$
In terms of x, we have the two equations $e^x = 1$ and $e^x = 2$. For $e^x = 1$ the solution is $x = 0$ and for $e^x = 2$ we have that $x = \log_e 2 = \ln 2 \approx 0.693147$.

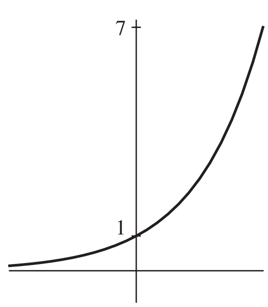
7. Solve $\dfrac{e^u + e^{-u}}{2} = 5$ for u.
$$e^u + e^{-u} = 10$$
$$e^{2u} + 1 = 10e^u$$
$$(e^u)^2 - 10e^u + 1 = 0.$$
Set $w = e^u$. Then $w^2 - 10w + 1 = 0$ and
$$w = \frac{10 \pm \sqrt{96}}{2} = 5 \pm 2\sqrt{6}.$$
This gives $e^u = 5 \pm 2\sqrt{6}$ so $u = \ln\left(5 + 2\sqrt{6}\right)$ or $u = \ln\left(5 - 2\sqrt{6}\right)$.

9.



$$\frac{dy}{dx} = \frac{d}{dx}(3^x) = C_3 3^x \approx 1.09861 \cdot 3^x$$
The slope of the tangent line to $y = 3^x$ at $x = 0.5$ is $C_3 \cdot 3^{0.5}$ so the equation of the tangent line is $y = \sqrt{3} + C_3\sqrt{3}\left(x - \frac{1}{2}\right)$.

11.



$$\frac{dy}{dx} = \frac{d}{dx}(7^x) = C_7 7^x \approx 1.94591 \cdot 7^x$$
The slope of the tangent line to $y = 7^x$ at $x = 0.5$ is $C_7 \cdot 7^{0.5}$ so the equation of the tangent line is $y = \sqrt{7} + C_7\sqrt{7}\left(x - \frac{1}{2}\right)$.

13. Find $\dfrac{dy}{dx}$ where $y = e^{x/7}$. We write $y = e^{x/7}$ as a composition chain:
$y = e^u$
$u = \dfrac{x}{7}.$
We then have
$$\frac{dy}{du} = (e^u)' = e^u$$
$$\frac{du}{dx} = \left(\frac{x}{7}\right)' = \frac{1}{7}.$$
Hence, by the chain rule,
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot \frac{1}{7} = \frac{1}{7}e^{x/7}.$$

15. Find $\dfrac{du}{dt}$ where $u = e^{-t/2}$. We write $u = e^{-t/2}$ as a composition chain:
$u = e^w$ and $w = -\dfrac{t}{2}.$
We then have
$$\frac{du}{dw} = e^w \text{ and } \frac{dw}{dt} = -\frac{1}{2}.$$
Hence, by the chain rule,
$$\frac{du}{dt} = \frac{du}{dw} \cdot \frac{dw}{dt} = e^w \cdot -\frac{1}{2} = -\frac{e^{-t/2}}{2}.$$

17. Find $f'(x)$ where $f(x) = \cos(2e^x)$.
If $g(x) = \cos(2x)$ and $h(x) = e^x$ then $f(x) = g(h(x))$.
Next we find $g'(x)$ and $h'(x)$:
$g'(x) = -\sin(2x) \cdot 2 = -2\sin(2x)$
$h'(x) = e^x.$

Hence, by the chain rule,
$$f'(x) = g'(h(x))h'(x)$$
$$= -2\sin(2(h(x)))e^x$$
$$= -2e^x \sin(2e^x).$$

19. Find $h'(t)$ where $h(t) = e^{-t^2}$.
If $f(t) = e^t$ and $g(t) = -t^2$ then $h(t) = f(g(t))$.
Next we find $f'(t)$ and $g'(t)$:
$$f'(t) = e^t \text{ and } g'(t) = -2t$$
Hence, by the chain rule,
$$h'(t) = f'(g(t))g'(t)$$
$$= e^{g(t)}(-2t)$$
$$= -2te^{-t^2}.$$

21. Find $\frac{dw}{dx}$ where $w = \sqrt{(e^x)^2 - e^x + 3}$. We write a composition chain:
$$w = \sqrt{a}$$
$$a = b^2 - b + 3$$
$$b = e^x.$$
We than have
$$\frac{dw}{da} = (\sqrt{a})' = \frac{1}{2\sqrt{a}}$$
$$\frac{da}{db} = (b^2 - b + 3)' = 2b - 1$$
$$\frac{db}{dx} = (e^x)' = e^x.$$
By the chain rule,
$$\frac{dw}{dx} = \frac{dw}{da}\frac{da}{db}\frac{db}{dx}$$
$$= \left(\frac{1}{2\sqrt{a}}\right)(2b-1)(e^x)$$
$$= \left(\frac{1}{2\sqrt{b^2 - b + 3}}\right)(2b-1)(e^x)$$
$$= \left(\frac{1}{2\sqrt{(e^x)^2 - e^x + 3}}\right)(2e^x - 1)(e^x)$$
$$= \frac{2e^{2x} - e^x}{2\sqrt{e^{2x} - e^x + 3}}.$$

23. Find $q'(s)$ where $q(s) = 2^s e^{4s}$.
$$q'(s) = (2^s e^{4s})$$
$$= (2^s)'(e^{4s}) + (2^s)(e^{4s})'$$
$$= (C_2 2^s)(e^{4s}) + (2^s)(e^{4s})(4s)'$$
$$= (C_2 2^s)(e^{4s}) + (2^s)(e^{4s})(4)$$
$$= C_2 2^s e^{4s} + 2^s 4e^{4s}$$

25. Find $R'(t)$ where $R(t) = e^{-t}\sin(2t)$.
$$R'(t) = (e^{-t}\sin(2t))'$$
$$= (e^{-t})'\sin(2t) + (e^{-t})(\sin(2t))'$$
$$= (e^{-t}(-1))\sin(2t) + e^{-t}(\cos(2t))(2)$$
$$= -e^{-t}\sin(2t) + 2e^{-t}\cos(2t)$$

27. Find $\frac{dv}{du}$ where $v = \frac{1}{e^u - e^{2u}} = (e^u - e^{2u})^{-1}$.
$$\frac{dv}{du} = ((e^u - e^{2u})^{-1})'$$
$$= -1(e^u - e^{2u})^{-2}(e^u - e^{2u})'$$
$$= -1(e^u - e^{2u})^{-2}(e^u - 2e^{2u})$$
$$= \frac{2e^{2u} - e^u}{(e^u - e^{2u})^2}$$

29. Find $\frac{dy}{dx}$ if $y = \sec(e^x + 1)$.
We write a composition chain:
$$y = \sec(u)$$
$$u = e^x + 1.$$
We then have
$$\frac{dy}{du} = \sec(u)\tan(u)$$
$$\frac{du}{dx} = e^x.$$
By the chain rule
$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$
$$= \sec(u)\tan(u)e^x$$
$$= e^x\sec(e^x + 1)\tan(e^x + 1).$$

31. Find $s'(t)$ where $s(t) = \frac{e^t - e^{-t}}{2}$.
$$s'(t) = \left(\frac{e^t - e^{-t}}{2}\right)'$$
$$= \frac{1}{2}[(e^t)' - (e^{-t})']$$
$$= \frac{1}{2}[e^t - e^{-t}(-t)']$$
$$= \frac{1}{2}[e^t - e^{-t}(-1)]$$
$$= \frac{e^t + e^{-t}}{2}$$

33. Determine $y'(x)$ where $ye^{x+y} = 1$ and $y = y(x)$. We differentiate both sides of the given relation with respect to x and simplify:
$(ye^{x+y})' = (1)'$
$(y)'e^{x+y} + y(e^{x+y})' = (1)'$
$y'e^{x+y} + ye^{x+y}(x+y)' = 0$
$y'e^{x+y} + ye^{x+y}(1+y') = 0.$
We solve the last equation for y' obtaining
$y' = \frac{-y}{y+1}.$

35. Determine $y'(x)$ where $e^{y^2} = xy$ and $y = y(x)$. We differentiate both sides of the given relation with respect to x and simplify:
$(e^{y^2})' = (xy)'$
$e^{y^2}(y^2)' = x'y + xy'$
$e^{y^2}2yy' = y + xy'.$
We solve the last equation for y' obtaining
$y' = \frac{y}{2ye^{y^2} - x}.$

37. Sample answer (other answers are possible): First find an approximation to $\sqrt{5}$. This can be done by letting $x = \sqrt{5}$, so $x^2 = 5$. Since $2^2 = 4 < 5 < 9 = 3^2$, start with the estimate of $x \approx 2.1$. Using the binomial expansion $(a+b)^2 = a^2 + 2ab + b^2$, we see that
$$\begin{aligned} 2.1^2 &= (2+0.1)^2 \\ &= 2^2 + 2(2)(0.1) + 0.1^2 \\ &= 4 + 0.4 + 0.01 \\ &= 4.41. \end{aligned}$$
Increase the estimates for x by 0.1 until the squares exceed 5, then reduce the estimates by 0.01 until the squares are less than 5, then increase the estimates by 0.001, etc. Continue in this fashion until you have a good estimate of $\sqrt{5}$, for instance $2.23606 < \sqrt{5} < 2.23607$.
Since $2.23606 = \frac{223,606}{100,000}$, now compute the 100,000th root of 11 to many decimal places. Raise this value to the 223,606th and 223,607th powers. If these two results are within 0.001 of each other, their average will be within 0.001 of $11^{\sqrt{5}}$.

39. We seek the value of b for which $(b^x)' = 2b^x$. That is, we seek b so that
$C_b = 2 = \lim_{h \to 0} \frac{b^h - 1}{h}$. Table 2.4 page 134 suggests that $7 < b < 10$. We set $h = 0.00001$ and consider the values of $y = \frac{b^h - 1}{h}$ for $7 < b$. Trial and error gets $b \approx 7.4$. Another method is to differentiate: Since $(e^{dx})' = b \cdot e^{dx} = b(e^d)^x$, we see that if $d = 2$, then $b = e^2 \approx 7.389056$.

41. We evaluate $A(T) = A_0 e^{0.01rT}$ when $T = 1000$, $A_0 = 1$, and $r = 4$ obtaining
$$\begin{aligned} A(1000) &= 1e^{(0.01)(4)(1000)} \\ &= 2.35385 \times 10^{17}. \end{aligned}$$

43. Because the half-life of C^{14} is 5730 years, a test tube containing 20 g of C^{14} will contain only 10 g after 5730 years, 5 g after 11,460 = 2(5730) years and 2.5 g after 17,190 = 3(5730) years.
The amount contained after t years can be expressed by $A(t) = 20\left(\frac{1}{2}\right)^{t/5730} = 20(2)^{-t/5730}$.
The rate of change of the 20 g of C^{14} after 5730 years is found by differentiating the equation for $A(t)$ and then substituting 5730 for t.
$$A'(t) = (20(2)^{-t/5730})' = 20C_2(2)^{-t/5730} \cdot \frac{-1}{5730}$$
$$A'(5730) = -C_2\left(\frac{20}{5730}\right)(2)^{-1} = -\frac{C_2}{573}$$

45. **a.** We suppose that a magnitude m star is k times as bright as a magnitude $m + 1$ star. Since a magnitude 1 star is 100 times brighter than a magnitude 6 star, $k^5 = 100$ or $k = \sqrt[5]{100} \approx 2.511886$.

b. $\frac{b_1}{b_2} = \left(\sqrt[5]{100}\right)^{5.3-1.2} \approx 43.652$

47. Since $w = e^x$ is continuous, $\lim_{x \to -\infty} e^x = 0$, and $\lim_{x \to \infty} e^x = \infty$, we conclude that the range of the natural exponential function is $\{w | w > 0\}$. On the interval $(0, \infty)$ the function $y = 3w^3 + w$ has the following properties:
(i) $y = 3w^3 + w$ is continuous
(ii) $\lim_{w \to 0} 3w^3 + w = 0$

(iii) $\lim_{w\to\infty} 3w^3 + w = \infty$.

We conclude that the range of $y = 3w^3 + w$ is $\{y|y > 0\}$ for $w \in \{w|w > 0\}$. If follows that $y = 3e^{3x} + e^x$ assumes all positive values of y.

49. Since $y = b^x$ is continuous,

$\lim_{x\to-\infty} b^x = \begin{cases} 0 & b > 1 \\ \infty & 0 < b < 1 \end{cases}$, and

$\lim_{x\to\infty} b^x = \begin{cases} \infty & b > 1 \\ 0 & 0 < b < 1 \end{cases}$, we conclude that the range of $y = b^x$ is $\{y|y > 0\}$. Thus, the equation $b^x = r$ has at least one solution for each $r > 0$.

Because $y = b^x$ is strictly increasing if $b > 1$, or strictly decreasing if $0 < b < 1$, each range value is assumed at most once. Thus, $b^x = r$ $(r > 0)$ has exactly one solution.

51. We seek the value of e^{C_b}. We consider

$e^{C_2} \approx e^{0.693147} \approx 1.99999963888$

$e^{C_3} \approx e^{1.09861} \approx 2.999993134$

$e^{C_{10}} \approx e^{2.30259} \approx 10.0000490702.$

We conjecture that $e^{C_b} = b$.

53. The number e is defined to be the value of b for which $c_b = \lim_{h\to 0} \frac{b^h - 1}{h} = 1$. So, for values of h close to 0, but distinct from zero, we have that $\frac{e^h - 1}{h} \approx 1$.

Solving $\frac{e^h - 1}{h} \approx 1$ for e we get

$e^h - 1 \approx h$

$e^h \approx 1 + h$

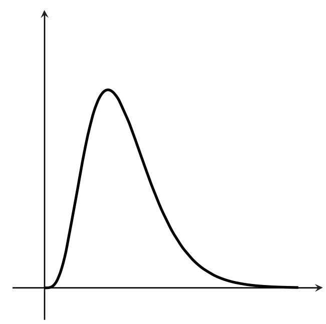
$(e^h)^{1/h} \approx (1 + h)^{1/h}$

$e \approx (1 + h)^{1/h}$.

We conclude that $\lim_{h\to 0} (1 + h)^{1/h} \approx e$.

55. Sample answer (other answers are possible): The functions $f(x) = \frac{1}{3}e^{3x} + \pi$ and $g(x) = \frac{1}{3}e^{3x} - \sqrt{2}$ have derivative e^{3x}.

57. a. The graph of $y = \frac{x^n}{e^x}$ (some n) on $(0, \infty)$ is shown below.



b. The figure suggests that $\lim_{x\to\infty} \frac{x^n}{e^x} = 0$.

59. We assume that the amount of radioactive material is modeled by the equation

$A(t) = A_0 e^{-kt}$

where A_0 is the amount present at time $t = 0$ and k is a positive constant. The rate of change of radioactive material is given by

$A'(t) = (A_0 e^{-kt})'$

$= A_0 e^{-kt}(-kt)'$

$= -kA_0 e^{-kt}$.

We observe that $A'(t) < 0$ for $t > 0$. Hence, the amount of radioactive material present decays with the passage of time. It is more dangerous to be near a large amount of radioactive material than a small amount as the radiation given off by the material is proportional to the amount present.

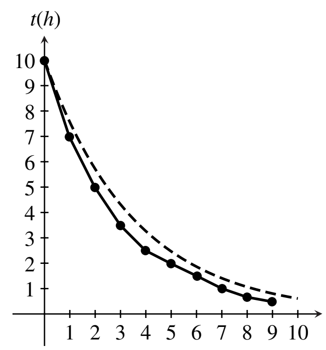
61. a. The concentration of the drug at time $n + 1$ is roughly 70% of the concentration at time n.

b. By computing $\frac{t(n+1)-t(n)}{(n+1)-n} = t(n+1)-t(n)$ we obtain estimates for $t'(n)$. The table below shows these estimates for $t'(n)$ as well as values for the ratio $\frac{t'(n)}{t(n)}$.

n		$t(n+1)-t(n$
0	–3	–0.30
1	–2	–0.29
2	–1.5	–0.30
3	–1	–0.29
4	–0.5	–0.20
5	–0.5	–0.25
6	–0.5	–0.33
7	–0.3	–0.30
8	–0.2	–0.29

We conclude that $t'(n) \approx -0.28t(n)$.

c. A function model for given situation is $t(h) = 10e^{-0.28t}$. The figure below shows the actual data (with the data points connected with line segments) with a solid polygonal curve and the modeling function $t(h) = 10e^{-0.28t}$ with a dashed curve.



Hence, the model tends to slightly overestimate the concentration of the drug.

Section 2.7 Logarithms

1. If $s = \log_3 81$, then $3^s = 81 = 3^4$. We conclude that $s = 4$.

3. If $s = \log_{0.23} 0.012167$, then $0.23^s = 0.012167 = 0.23^3$. We conclude that $s = 3$.

5. If $s = \log_{\sqrt{6}}(36\sqrt{6})$, then $(\sqrt{6})^s = 36\sqrt{6}$. Since $(\sqrt{6})^s = 6^{(1/2)s}$ and $36\sqrt{6} = 6^2 6^{1/2} = 6^{5/2}$, we have $\frac{1}{2}s = \frac{5}{2}$ and so $s = 5$.

7. Solve $2^{3x} = 5$ for x.

$2^{3x} = 2^{\log_2 5}$

$3x = \log_2 5$

$x = \frac{1}{3}\log_2 5 \approx 0.773976$

9. Solve $2e^{6x} - 5e^{3x} + 6 = 0$ for x. Set $w = e^{3x}$. Then $2e^{6x} - 5e^{3x} + 6 = 0$ becomes the quadratic equation $2w^2 - 5w + 6 = 0$. Using the quadratic equation we find that

$w = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(6)}}{2(2)}$.

Since $(-5)^2 - 4(2)(6) = -23$, the equation $2w^2 - 5w + 6 = 0$ has no real solutions in w. Hence, the equation $2e^{6x} - 5e^{3x} + 6 = 0$ has no real solutions.

11. Solve $\log_2 3x = 5$ for x.

$3x = 2^5 = 32$

$x = \frac{32}{3}$

13. Solve $\log_2 x + \log_5 x = \log_{10} x$ for x using the fact that $\log_b x = \frac{\ln x}{\ln b}$.

$\frac{\ln x}{\ln 2} + \frac{\ln x}{\ln 5} = \frac{\ln x}{\ln 10}$

$(\ln x)\left(\frac{1}{\ln 2} + \frac{1}{\ln 5} - \frac{1}{\ln 10}\right) = 0$

Since $\frac{1}{\ln 2} + \frac{1}{\ln 5} - \frac{1}{\ln 10} \neq 0$, $\ln x = 0$ and so $x = 1$.

15. Find $\frac{dy}{dx}$ if $y = \ln(x + 2)$.

$$\begin{aligned}\frac{dy}{dx} &= (\ln(x+2))' \\ &= \frac{1}{x+2}(x+2)' \\ &= \frac{1}{x+2}\end{aligned}$$

17. Compute $\frac{dr}{d\theta}$ if $r = \ln(\sin\theta)$.

$$\begin{aligned}\frac{dr}{d\theta} &= (\ln(\sin\theta))' \\ &= \frac{1}{\sin\theta}(\sin\theta)' \\ &= \frac{1}{\sin\theta}\cos\theta \\ &= \cot\theta\end{aligned}$$

19. Determine $r'(t)$ if $r(t) = (1 + \ln 2t)^{10}$.

$$\begin{aligned}r'(t) &= ((1+\ln 2t)^{10})' \\ &= 10(1+\ln 2t)^9(1+\ln 2t)' \\ &= 10(1+\ln 2t)^9(1' + (\ln 2t)') \\ &= 10(1+\ln 2t)^9\left(0 + \frac{1}{2t}(2t)'\right) \\ &= 10(1+\ln 2t)^9\left(\frac{1}{2t}2\right) \\ &= \frac{10}{t}(1+\ln 2t)^9\end{aligned}$$

21. Find $q'(z)$ if $q(z) = \ln\left(\frac{1-2z+z^3}{z\sin z}\right)$.

$$\begin{aligned}
q'(z) &= \left[\ln\left(\frac{1-2z+z^3}{z\sin z}\right)\right]' \\
&= \frac{1}{\frac{1-2z+z^3}{z\sin z}}\left(\frac{1-2z+z^3}{z\sin z}\right)' \\
&= \frac{z\sin z}{1-2z+z^3}\cdot\frac{(1-2z+z^3)'(z\sin z)-(1-2z+z^3)(z\sin z)'}{(z\sin z)^2} \\
&= \frac{z\sin z}{1-2z+z^3}\cdot\frac{(3z^2-2)(z\sin z)-(1-2z+z^3)(z'\sin z+z(\sin z)')}{(z\sin z)^2} \\
&= \frac{z\sin z}{1-2z+z^3}\cdot\frac{(3z^2-2)(z\sin z)-(1-2z+z^3)(\sin z+z\cos z)}{(z\sin z)^2} \\
&= \frac{3z^2-2}{1-2z+z^3}-\frac{\sin z+z\cos z}{z\sin z}
\end{aligned}$$

23. Compute $\frac{dr}{d\theta}$ if $r = e^{(\ln\theta)^2}$.

$$\begin{aligned}
\frac{dr}{d\theta} &= (e^{(\ln\theta)^2})' \\
&= e^{(\ln\theta)^2}((\ln\theta)^2)' \\
&= e^{(\ln\theta)^2}2(\ln\theta)(\ln\theta)' \\
&= e^{(\ln\theta)^2}2(\ln\theta)\frac{1}{\theta} \\
&= \frac{2\ln\theta}{\theta}e^{(\ln\theta)^2}
\end{aligned}$$

25. Find $\frac{dy}{dx}$ via logarithmic differentiation if $y = \frac{x^2}{2x+1}$. Taking the (natural) logarithm of each side of $y = \frac{x^2}{2x+1}$ produces the equation

$$\begin{aligned}
\ln y &= \ln\left(\frac{x^2}{2x+1}\right) \\
&= \ln x^2 - \ln(2x+1) \\
&= 2\ln x - \ln(2x+1).
\end{aligned}$$

Differentiating both sides of the equation $\ln y = 2\ln x - \ln(2x+1)$ with respect to x yields

$$\begin{aligned}
\frac{1}{y}\frac{dy}{dx} &= 2\frac{1}{x}-\frac{1}{2x+1}(2x+1)' \\
&= \frac{2}{x}-\frac{2}{2x+1}.
\end{aligned}$$

Solving the above for $\frac{dy}{dx}$ gives

$$\begin{aligned}
\frac{dy}{dx} &= \frac{2(1+x)}{x(1+2x)}y \\
&= \frac{2(1+x)}{x(1+2x)}\cdot\frac{x^2}{2x+1} \\
&= \frac{2x(1+x)}{(2x+1)^2}.
\end{aligned}$$

27. Find $\frac{dy}{dx}$ via logarithmic differentiation if $y = \sqrt{x\tan x} = (x\tan x)^{1/2}$. We take the (natural) logarithm of each side of $y = (x\tan x)^{1/2}$ obtaining the equation

$$\begin{aligned}
\ln y &= \ln(x\tan x)^{1/2} \\
&= \frac{1}{2}\ln(x\tan x) \\
&= \frac{1}{2}(\ln x + \ln\tan x).
\end{aligned}$$

Differentiating both sides of $2\ln y = \ln x + \ln\tan x$ with respect to x yields

$$\begin{aligned}
2\frac{1}{y}\frac{dy}{dx} &= \frac{1}{x}+\frac{1}{\tan x}(\tan x)' \\
&= \frac{1}{x}+\frac{\sec^2 x}{\tan x}.
\end{aligned}$$

Solving the above for $\frac{dy}{dx}$ produces

$$\begin{aligned}
\frac{dy}{dx} &= \frac{1}{2}y\left(\frac{1}{x}+\frac{\sec^2 x}{\tan x}\right) \\
&= \frac{1}{2}\sqrt{x\tan x}\left(\frac{1}{x}+\frac{\sec^2 x}{\tan x}\right).
\end{aligned}$$

29. To show that $\log_b xy = \log_b x + \log_b y$ we observe that $xy = b^{\log_b xy}$ and
$xy = b^{\log_b x} b^{\log_b y}$. It follows that
$b^{\log_b xy} = b^{\log_b x} b^{\log_b y} = b^{\log_b x + \log_b y}$
and so
$\log_b xy = \log_b x + \log_b y.$

31. For $b > 0$ $(b \neq 1)$ we have the relationship $u = b^v$ if and only if $v = \log_b u.$ To extend this to $b = 1$ would mean that $v = \log_1 u$ precisely when $1^v = u.$ Since $1^v = u$ has no solution for $u \neq 1,$ the equation $v = \log_1 u$ has little meaning.

33. Show that $(\ln|x|)' = \frac{1}{x}$ $(x \neq 0)$. We first assume that $x > 0$. In this case, we have
$(\ln|x|)' = (\ln x)' = \frac{1}{x}.$
We now assume that $x < 0$. In this case, we have

$$\begin{aligned}(\ln|x|)' &= (\ln(-x))' \\ &= \frac{1}{-x}(-x)' \\ &= \frac{1}{-x}(-1) \\ &= \frac{1}{x}.\end{aligned}$$

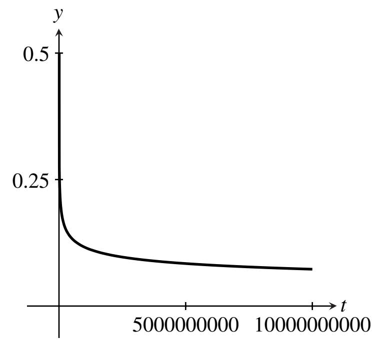
We conclude that $(\ln|x|)' = \frac{1}{x}$ $(x \neq 0)$.

35. Sample answer (other answers are possible): The two functions $y = \ln(x - 4)$ and $y = \ln(x - 4) + 45$ both have derivative $\frac{1}{x-4}$ $(x > 4)$.

37.a., b. Given the relation that $b = e^a$ if and only if $a = \ln b$, it follows that the point (a, b) is on the graph of $y = e^x$ precisely when the point (b, a) is on the graph of $y = \ln x$.

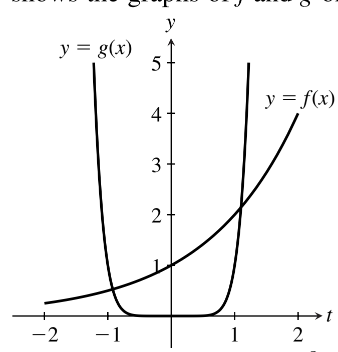
c. This relationship means that the graphs of $y = e^x$ and $y = \ln x$ are symmetric with respect to the line $y = x$.

39. The graph of $y = \frac{\ln x}{\sqrt[4]{x}}$ (shown below) suggests that $\lim_{x \to +\infty} \frac{\ln x}{\sqrt[4]{x}} = 0.$

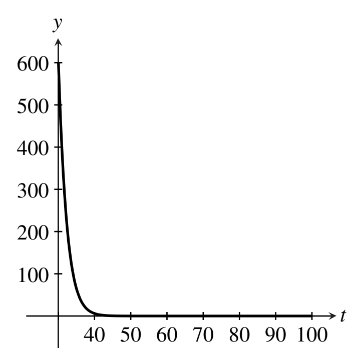


41. The function $\ln\ f(x) + 4\ln(-f(x))$ has an empty domain for all functions $f(x)$. To see this one need only observe that $f(x)$ and $-f(x)$ are oppositely signed and the domain of $\ln x$ is $(0, \infty)$.

43. Let $f(x) = 2^x$ and $g(x) = x^8$. The figure below shows the graphs of f and g on [–2, 2].



We see that $f(x) = 2^x > x^8 = g(x)$ for x in $(0, 1.1)$. Below is the graph of $y = \frac{x^8}{2^x}$.



Since $y = \frac{x^8}{2^x}$ approaches 0 as x get large, we conclude that for large values of x $f(x)=2^x > x^8 = g(x)$. Numerical exploration (i.e., substitution of a variety of values of x into the inequality $2^x > x^8$) shows that we have $f(x)=2^x > x^8 = g(x)$ on $(43.6, \infty)$.

45. Logarithmic differentiation is most useful for simplifying the differentiation of expressions involving products, quotients, and powers as these are converted to sums, differences, and products (respectively). The given function has "addition" as its domain operation and so logarithmic differentiation won't be of much use.

47. We are given that the time needed to factor an integer N is proportional to the expression $e^{\sqrt{(\ln N)(\ln\ln N)}}$. That is, the time needed to factor an integer N is given by $t(N)=ke^{\sqrt{(\ln N)(\ln\ln N)}}$ for some value k. To estimate k we observe that RSA 129 $\approx 1.14382 \times 10^{128}$ and so we have that

t(RSA 129)

$\approx ke^{\sqrt{(\ln 1.14382\times 10^{128})(\ln\ln 1.14382\times 10^{128})}}$

$\approx k6.07556\times 10^{17}$

≈ 8 months.

(If your calculator can't handle $\ln(1.14382\times 10^{128})$ then rewrite it as ln(1.14382) + 128 ln(10).)

Solving the above for k yields $k \approx 1.31675\times 10^{-17}$. So, $t(1\times 10^{149}) \approx 360$ months $= 30$ years and

$t(1\times 10^{199}) \approx 1.35368\times 10^{6}$ months

$\approx 110{,}000$ years.

49. a. If a certain substance has a pH of 5 then its $[H^+]$ can be found by $\text{pH} = \log_{10}\left(\frac{1}{[H^+]}\right)$.

Solving $5 = \log_{10}\left(\frac{1}{[H^+]}\right)$ we get $[H^+] = 10^{-5}$. This is 100 times greater than 10^{-7}, the $[H^+]$ for distilled water. If a substance has a pH of 10, then we get $[H^+] = 10^{-10}$ which is $\frac{1}{1000}$ the $[H^+]$ for distilled water.

b. Let $x = [H^+]$ for solution B, then $2x = [H^+]$ for solution A. So, for solution B the $\text{pH} = \log_{10}\left(\frac{1}{x}\right) = \log_{10} 1 - \log_{10} x$, and for solution A the

$\text{pH} = \log_{10}\left(\frac{1}{2x}\right)$

$= \log_{10} 1 - \log_{10} x - \log_{10} 2$.

So the pH's differ by $\log_{10} 2$.

51. a. We use $R = \log_{10}\left(\frac{I}{S}\right)$ to compare the intensities of the earthquakes. For $R = 7.1$ we get a value of $I = 10^{7.1}(S)$. For $R = 8.6$ we get a value of $I = 10^{8.6}(S)$. These intensities differ by $10^{1.5} \approx 31.6$.

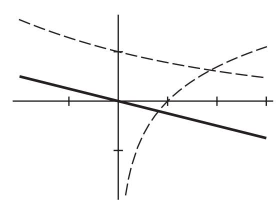
b. Since the 1906 San Francisco earthquake had an intensity 16 times that of the Loma Prieta quake, we can represent the San Francisco earthquake's intensity as $I = 16\cdot(10^{7.1})S$. So

$R = \log_{10}\left(\frac{16(10^{7.1})S}{S}\right)$

$= \log_{10}(16(10^{7.1}))$

≈ 8.3.

53. Since $y = \ln e^{kx} = kx$, we sketch the graph of $y = \ln e^{kx}$ using the techniques of Section 1.2.



Estimating the slope of the line $y = kx$ we find that $k = -\frac{1}{4}$.

Section 2.8 Inverse Functions

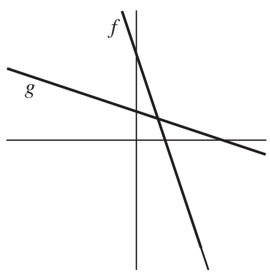
1. Since $f(x) = -3x + 8$,
$x = f(g(x)) = -3(g(x)) + 8$. So $x = -3(g(x)) + 8$, or $g(x) = -\frac{x+8}{3}$.

3. Since $f(x) = 3(x-2)^3 + 7$,
$x = f(g(x)) = 3(g(x)-2)^3 + 7$. So
$x = 3(g(x)-2)^3 + 7$ or $g(x) = \left(\frac{x-7}{3}\right)^{1/3} + 2$.

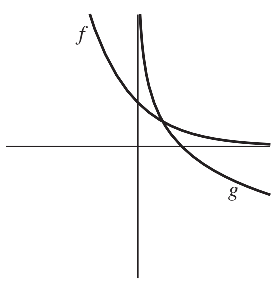
5. Since $f(x)=2\ln(3x+4)$,
$x=f(g(x))=2\ln(3(g(x))+4)$. So
$x=2\ln(3(g(x))+4)$, $e^{x/2}=3(g(x))+4$, or
$g(x)=\dfrac{e^{x/2}-4}{3}$.

7. Since $f(x)=3e^{2x-5}+6$,
$x=f(g(x))=3e^{2(g(x))-5}+6$. So
$x=3e^{2(g(x))-5}+6$, $e^{2(g(x))-5}=\dfrac{x-6}{3}$,
$g(x)=\dfrac{1}{2}\ln\left(\dfrac{x-6}{3}\right)+\dfrac{5}{2}$.

9.



11.



13. Suppose that u and v belong to the domain of $f(x)=\dfrac{2-x}{3+2x}$ with $f(u)=f(v)$. Then

$$\frac{2-u}{3+2u}=\frac{2-v}{3+2v}$$
$$\Rightarrow (2-u)(3+2v)=(2-v)(3+2u)$$
$$\Rightarrow 6+4v-3u-2uv=6+4u-3v-2uv$$
$$\Rightarrow 4v-3u=4u-3v$$
$$\Rightarrow u=v.$$

Hence, f is one-to-one on its domain.
To find f^{-1} we exchange the roles of x and y in $y=\dfrac{2-x}{3+2x}$ and solve for y.

$$x=\frac{2-y}{3+2y}$$
$$(3+2y)x=2-y$$
$$3x+2xy=2-y$$
$$2xy+y=2-3x$$
$$(2x+1)y=2-3x$$
$$f^{-1}(x)=y=\frac{2-3x}{2x+1}$$

The domain of f^{-1} is seen to be all real numbers except $x=-\dfrac{1}{2}$.

15. Show that $G(t)=\dfrac{1}{2}\ln 7t$ and $F(t)=\dfrac{1}{7}e^{2t}$, are inverse functions. It is sufficient to show that either $(G\circ F)(t)=G(F(t))=t$ or $(F\circ G)(t)=F(G(t))=t$. We will show that $(G\circ F)(t)=G(F(t))=t$.

$$(G\circ F)(t)=\frac{1}{2}\ln\left(7\left(\frac{1}{7}e^{2t}\right)\right)$$
$$=\frac{1}{2}\ln e^{2t}$$
$$=\frac{1}{2}(2t)$$
$$=t$$

17. Since $g = f^{-1}$ and $f(2) = g(15)$,
$g'(15) = \dfrac{1}{f'(2)}$. If $f(x) = x^3 + 2x + 3$ then
$f'(x) = 3x^2 + 2$ and $f'(2) = 3(2)^2 + 2 = 14$.
So $g'(15) = \dfrac{1}{14}$.

19. Since $k = h^{-1}$ and $h(1) = 2$, $k'(2) = \dfrac{1}{h'(1)}$. If
$h(x) = x^2 + 4x - 3$ then $h'(x) = 2x + 4$ and
$h'(1) = 6$. So $k'(2) = \dfrac{1}{6}$.

21. Since $g = f^{-1}$ and $f\left(-\dfrac{\pi}{4}\right) = -1$,
$g'(-1) = \dfrac{1}{f'\left(-\frac{\pi}{4}\right)}$. If $f(x) = \tan x$ then
$f'(x) = \sec^2 x$ and $f'\left(-\dfrac{\pi}{4}\right) = 2$. So
$g'(-1) = \dfrac{1}{2}$.

23. If $y = f(x)$ then $\dfrac{dy}{dx} = f'(x)$. Similarly, if
$x = g(y)$ then $\dfrac{dx}{dy} = g'(y)$. Since f and g are
inverses, $g'(y) = \dfrac{1}{f'(x)}$ and $g(f(x)) = x$.
So

$$\begin{aligned} \frac{d}{dx}(g(f(x))) &= \frac{d}{dx}(x), \\ g'(f(x)) \cdot f'(x) &= 1 \\ g'(y) \cdot f'(x) &= 1 \\ \frac{dx}{dy} \cdot \frac{dy}{dx} &= 1. \end{aligned}$$

25. January 31 is 40 days past December 22 and so we see that

$$\begin{aligned} A(40) &= -13.5 + \frac{70.5}{1 + 0.00006(40 - 182.5)^2} \\ &\approx 18.3. \end{aligned}$$

October 31 is 313 days past December 22 and so we have that

$$\begin{aligned} A(313) &= -13.5 + \frac{70.5}{1 + 0.00006(313 - 182.5)^2} \\ &\approx 21.4. \end{aligned}$$