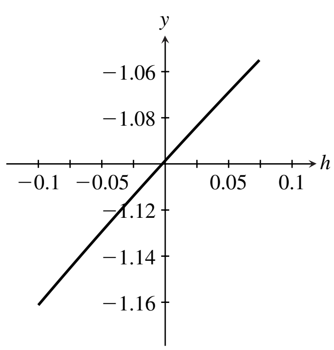
h	$\dfrac{\left(\frac{1}{2}\right)^h - 1}{h}$
0.001	–0.692907
–0.001	–0.693387
10^{-5}	–0.693145
-10^{-5}	–0.693150
$\pm 10^{-7}$	–0.693147
$\pm 10^{-9}$	–0.693147

Based on the evidence, it appears that $\lim_{h\to 0} \dfrac{\left(\frac{1}{2}\right)^h - 1}{h} \approx -0.693$.

d. The graph of $\dfrac{\left(\frac{1}{3}\right)^h - 1}{h}$ is shown.



The table shows approximate values of $\dfrac{\left(\frac{1}{3}\right)^h - 1}{h}$ for values of h close to 0.

h	$\dfrac{\left(\frac{1}{3}\right)^h - 1}{h}$
0.001	−1.098009
–0.001	−1.099216
10^{-5}	−1.098606
-10^{-5}	−1.098618
$\pm 10^{-7}$	−1.098612
$\pm 10^{-9}$	–1.09861

Based on the evidence, it appears that $\lim_{h\to 0} \dfrac{\left(\frac{1}{3}\right)^h - 1}{h} \approx -1.099$.

23. Sample answers (other answers are possible):

a. If $\dfrac{a}{b} = 3$, then $a = 3b$. One possibility is $a = 0.003$ and $b = 0.001$.

b. If $\dfrac{a}{b} = -500$, then $a = -500b$. One possibility is $a = -0.005$, $b = 0.00001$.

c. If $\dfrac{a}{b} = 10^9$, then $a = 10^9 b$. One possibility is $a = 10^{-2}$, $b = 10^{-11}$.

d. If $\dfrac{a}{b} = -10^{-9}$, then $a = -10^{-9} b$ or $-10^9 a = b$. One possibility is $a = -10^{-11}$, $b = 10^{-2}$.

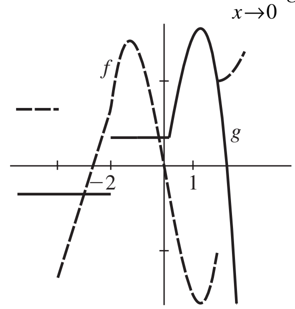
25. Sample answers (other answers are possible):

a. $f(x) = 5(x-3)$, $g(x) = x-3$.

b. $f(x) = -\sqrt{3}(x-3)$, $g(x) = x-3$.

c. $f(x) = x-3$, $g(x) = (x-3)^2$.

27. Since $\lim_{x\to -2} f(x)$ exists, $g(x)$ must be constructed so that $\lim_{x\to -2} g(x)$ does not exist. Also, since $\lim_{x\to 0} f(x) = 0$, $g(x)$ must be constructed so that $\lim_{x\to 0} g(x) = 1$.



29. Using $\lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$ with $y = f(x) = -2x + 3$ and $a = 4$, we have

$$\lim_{h\to 0} \frac{[-2(4+h)+3] - [-2(4)+3]}{h} = \lim_{h\to 0} \frac{-2h}{h} = -2$$

The rate of change of $y = -2x + 3$ with respect to x at $x = 4$ is –2.

31. Using $\lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$ with $y = f(x) = \dfrac{1}{x+1}$ and $a = 1$, we have

$$\lim_{h\to 0}\frac{\frac{1}{1+h+1}-\frac{1}{1+1}}{h}=\lim_{h\to 0}\left(\frac{1}{2+h}-\frac{1}{2}\right)\cdot\frac{1}{h}$$
$$=\lim_{h\to 0}\frac{2-(2+h)}{2h(2+h)}$$
$$=\lim_{h\to 0}\frac{-h}{2h(2+h)}$$
$$=\lim_{h\to 0}\frac{-1}{2(2+h)}$$
$$=-\frac{1}{4}$$

The rate of change of $y=\frac{1}{x+1}$ with respect to x at $x = 1$ is $-\frac{1}{4}$.

33. Using $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ with $y=f(x)=x^3$ and $a=0$, we have

$$\lim_{h\to 0}\frac{(0+h)^3-0^3}{h}=\lim_{h\to 0}\frac{h^3}{h}$$
$$=\lim_{h\to 0}h^2$$
$$=0$$

The rate of change of $y=x^3$ with respect to x at $x = 0$ is 0.

35. Using $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ with $y=f(x)=\frac{2}{x-2}$ and $a = 0$, we have

$$\lim_{h\to 0}\frac{\frac{2}{0+h-2}-\frac{2}{0-2}}{h}=\lim_{h\to 0}\left(\frac{2}{h-2}+1\right)\cdot\frac{1}{h}$$
$$=\lim_{h\to 0}\frac{2+h-2}{h(h-2)}$$
$$=\lim_{h\to 0}\frac{1}{h-2}$$
$$=-\frac{1}{2}$$

The rate of change of $y=\frac{2}{x-2}$ with respect to x at $x = 0$ is $-\frac{1}{2}$.

37. $\lim_{x\to a}g(x)=M$. The given limit says that $|g(x)-M|$ gets close to 0 as x gets close to a. This implies that $g(x)-M$ gets close to 0 as x gets close to a, which in turn implies that $g(x)$ gets close to M as x gets close to a.

39. **a.** $\lim_{x\to 1}f(x)=1^2+2(1)+2=1+2+2=5$

b. If x is within 0.05 of 1, then $0.95\le x\le 1.05$ and, from a graph (using technology), $4.8025\le f(x)\le 5.2025$. Thus, $|f(x)-5|<0.203$.

c. If $|f(x)-5|<0.1$, then $4.9<f(x)<5.1$ and, from a graph (using technology), $0.975<x<1.025$. Thus, $r\le 0.02$.

41. **a.** $\lim_{\theta\to 0}r(\theta)=\lim_{\theta\to 0}\frac{\sin 2\theta}{\theta}$

$$=\lim_{\theta\to 0}2\cdot\frac{\sin 2\theta}{2\theta}$$
$$=2\cdot\lim_{\theta\to 0}\frac{\sin 2\theta}{2\theta}$$
$$=2\cdot\lim_{2\theta\to 0}\frac{\sin 2\theta}{2\theta}$$
$$=2\cdot 1$$
$$=2$$

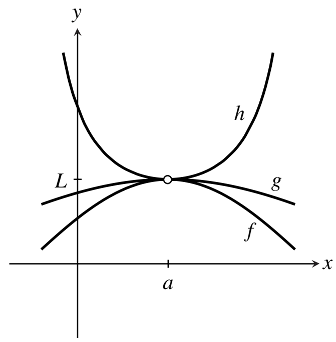
Note that since $2\theta\to 0$ as $\theta\to 0$, $\lim_{\theta\to 0}\frac{\sin 2\theta}{2\theta}=\lim_{2\theta\to 0}\frac{\sin 2\theta}{2\theta}$.

b. If θ is within 0.1 of 0, then $-0.1\le\theta\le 0.1$ and, from a graph (using technology), $1.98669<r(\theta)<2.0$. Thus $|r(\theta)-2|<0.01331$.

c. If $|r(\theta)-2|<0.05$, then $1.95<r(\theta)<2.05$ and, from a graph (using technology), r can be any positive number less than or equal to approximately 0.15.

43. The arc PS has length h and the line segment PR has length $\sin h$. Allowing for $h < 0$, the lengths are $|h|$ and $|\sin h|$, respectively. From Figure 1.49, it is clear that the arc is longer than the line segment, i.e., $|\sin h|\le|h|$.

45. a. Sample answer (other answers are possible):



b. Because $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} h(x) = L$, when x is close to a, $f(x)$ is close to L and $h(x)$ is close to L. Because $f(x) \le g(x) \le h(x)$, $g(x)$ is also close to L. This suggests that $g(x) \to L$ as $x \to a$.

c. We applied to squeeze theorem to the functions described in (23). cos h and $\frac{1}{\cos h}$ played the role of functions f and h in the squeeze theorem, where $\lim_{h\to 0} \cos h = \lim_{h\to 0} \frac{1}{\cos h} = 1$. Note that the function h in the squeeze theorem is not to be confused with the variable h in (23). The function $\frac{\sin h}{h}$, which played the role of function g, was trapped between cos h and $\frac{1}{\cos h}$.

47. a. From Exercise 46, we know that $0 \le |\sin\theta - \sin a| \le |\theta - a|$. We also have that $\lim_{\theta\to a} |\theta - a| = 0$ and $\lim_{\theta\to a} 0 = 0$. Thus, by the squeeze theorem, $\lim_{\theta\to a} |\sin\theta - \sin a| = 0$, hence $\lim_{\theta\to a} \sin\theta = \sin a$ (note that sin a is constant with respect to θ).

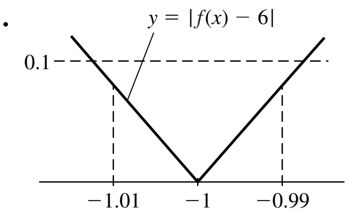
b. From Exercise 46, we know that $0 \le |\cos\theta - \cos a| \le |\theta - a|$. We also have that $\lim_{\theta\to a} |\theta - a| = 0$ and $\lim_{\theta\to a} 0 = 0$. Thus, by the squeeze theorem, $\lim_{\theta\to a} |\cos\theta - \cos a| = 0$, hence $\lim_{\theta\to a} \cos\theta = \cos a$ (note that cos a is constant with respect to θ).
These results are identical to (17) with x replaced by θ.

Section 1.6 More Work with Limits

In Exercises 1–13, sample answers are given for part **b**. Other answers are possible.

1. a. $\lim_{x\to -1} \left(3x^2 - 2x + 1\right) = 3(-1)^2 - 2(-1) + 1 = 6$

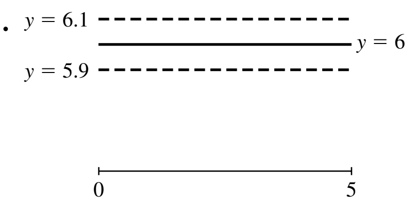
b.



δ can be any positive number < 0.01.

3. a. $\lim_{\theta\to\sqrt{5}} 6 = 6$

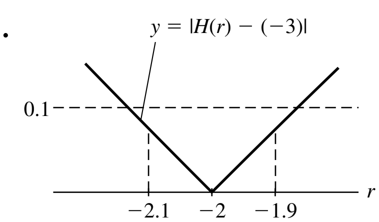
b.



δ can be any positive number since $r(\theta)$ is constant.

5. a.
$$\lim_{r\to -2} \frac{r^3+8}{r^2-4} = \lim_{r\to -2} \frac{(r+2)\left(r^2-2r+4\right)}{(r+2)(r-2)} = \lim_{r\to -2} \frac{r^2-2r+4}{r-2} = \frac{(-2)^2-2(-2)+4}{-2-2} = -3$$

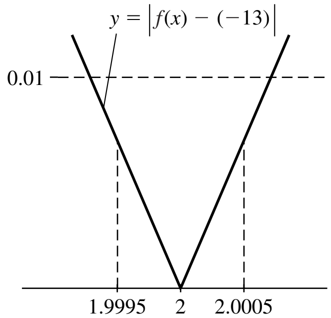
b.



δ can be any positive number < 0.1.

7. a. $\lim_{x\to 2}\left(-4x^2+2x-1\right) = -4(2)^2+2(2)-1$
$= -13$

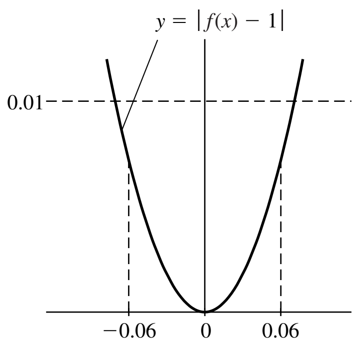
b.



$1.9995 < x < 2.0005$

9. a. $\lim_{x\to 0}\cos(2x) = \cos(2\cdot 0) = \cos 0 = 1$

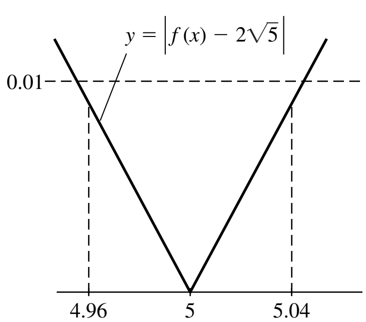
b.



$-0.06 < x < 0.06$

11. a. $\lim_{x\to 5}\frac{x-5}{\sqrt{x}-\sqrt{5}} = \lim_{x\to 5}\frac{x-5}{\sqrt{x}-\sqrt{5}}\cdot\frac{\sqrt{x}+\sqrt{5}}{\sqrt{x}+\sqrt{5}}$
$= \lim_{x\to 5}\frac{(x-5)\left(\sqrt{x}+\sqrt{5}\right)}{x-5}$
$= \lim_{x\to 5}\left(\sqrt{x}+\sqrt{5}\right)$
$= 2\sqrt{5}$

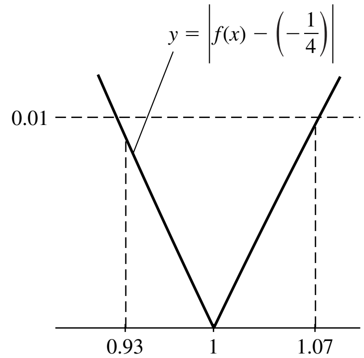
b.



$4.96 < x < 5.04$

13. a. $\lim_{x\to 1}\frac{\frac{1}{x+1}-\frac{1}{2}}{x-1} = \lim_{x\to 1}\left[\frac{1}{x-1}\cdot\left(\frac{2-(x+1)}{2(x+1)}\right)\right]$
$= \lim_{x\to 1}\frac{1-x}{2(x-1)(x+1)}$
$= \lim_{x\to 1}\frac{-1}{2(x+1)}$
$= -\frac{1}{2(2)}$
$= -\frac{1}{4}$

b.



$0.93 < x < 1.07$

Exercises 15–17, sample answers are given for part **a**. Other answers are possible.

15. a. $\lim_{r\to -2}\frac{\sqrt{2r^2+1}-3}{r+2}$
$= \lim_{r\to -2}\frac{\sqrt{2r^2+1}-3}{r+2}\cdot\frac{\sqrt{2r^2+1}+3}{\sqrt{2r^2+1}+3}$
$= \lim_{r\to -2}\frac{2r^2+1-9}{(r+2)\left(\sqrt{2r^2+1}+3\right)}$
$= \lim_{r\to -2}\frac{2(r+2)(r-2)}{(r+2)\left(\sqrt{2r^2+1}+3\right)}$
$= \lim_{r\to -2}\frac{2(r-2)}{\sqrt{2r^2+1}+3}$
$= \frac{2(-4)}{\sqrt{9}+3}$
$= -\frac{8}{6}$
$= -\frac{4}{3}$

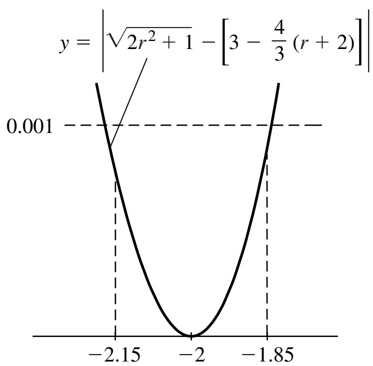
b. When r is close to –2,

$$\frac{\sqrt{2r^2+1}-3}{r+2}-\left(-\frac{4}{3}\right)=E(x),\text{ so}$$

$$\sqrt{2r^2+1}=3-\frac{4}{3}(r+2)+E(x)(r+2)$$

$$\approx 3-\frac{4}{3}(r+2)$$

c.



$-2.15 < x < -1.85$

d. $\left|\sqrt{2(-1.98)^2+1}-\left[3-\frac{4}{3}(-1.98+2)\right]\right|$

$\approx 1.495\times 10^{-5}$.

The error is about 1.495×10^{-5}.

17. a. To find the limit analytically, we let

$t = x-\frac{\pi}{4}$, so $x = t+\frac{\pi}{4}$.

Then as $x\to\frac{\pi}{4}$, $t\to 0$.

$$\lim_{x\to\pi/4}\frac{\tan x-1}{x-\frac{\pi}{4}}$$

$$=\lim_{t\to 0}\frac{\tan\left(t+\frac{\pi}{4}\right)-1}{t}$$

$$=\lim_{t\to 0}\left[\frac{1}{t}\cdot\left(\frac{\tan t+\tan\frac{\pi}{4}}{1-\tan t\tan\frac{\pi}{4}}-1\right)\right]$$

$$=\lim_{t\to 0}\left[\frac{1}{t}\cdot\left(\frac{\tan t+1}{1-\tan t}-1\right)\right]$$

$$=\lim_{t\to 0}\left[\frac{1}{t}\cdot\left(\frac{\tan t+1-(1-\tan t)}{1-\tan t}\right)\right]$$

$$=\lim_{t\to 0}\frac{2\tan t}{t(1-\tan t)}$$

$$=\lim_{t\to 0}\frac{2\sin t}{t\cos t(1-\tan t)}$$

$$=\left(\lim_{t\to 0}\frac{\sin t}{t}\right)\cdot\left(\lim_{t\to 0}\frac{2}{\cos t(1-\tan t)}\right)$$

$$=1\cdot\frac{2}{1(1)}$$

$$=2$$

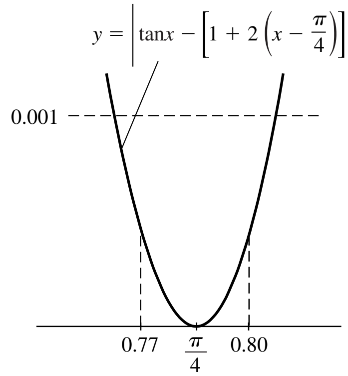
An estimate obtained by graphical or numerical means may be slightly different (but not wrong).

b. When x is close to $\frac{\pi}{4}$, $\frac{\tan x-1}{x-\frac{\pi}{4}}-2=E(x)$,

so $\tan x = 1+2\left(x-\frac{\pi}{4}\right)+E(x)\left(x-\frac{\pi}{4}\right)$

$$\approx 1+2\left(x-\frac{\pi}{4}\right)$$

c.



$0.77 < x < 0.80$

d. $\left|\tan 0.75 - \left[1 + 2\left(0.75 - \frac{\pi}{4}\right)\right]\right| \approx 0.00239$

The error is about 0.00239.

19. $|t| = \begin{cases} t & t \geq 0 \\ -t & t < 0 \end{cases}$.

a. $\lim_{t\to 0^+} \frac{t}{|t|} = \lim_{t\to 0^+} \frac{t}{t} = 1$

$\lim_{t\to 0^-} \frac{t}{|t|} = \lim_{t\to 0^-} \frac{t}{-t} = -1$

b. $\lim_{t\to 0} h(t)$ does not exist.

c. No, because $\lim_{t\to 0} h(t)$ does not exist.

21. a. $\lim_{x\to -1^+} f(x) = 1$

$\lim_{x\to -1^-} f(x) = 1$

b. $\lim_{x\to 0^+} f(x) = 1$

$\lim_{x\to 0^-} f(x) = 1$

c. $\lim_{x\to 1^+} f(x) = 2$

$\lim_{x\to 1^-} f(x) = 0$

d. $f(x)$ is discontinuous at $x = -1$ and $x = 1$ since it is not defined at either point.

23. Let $h(x)$ and $g(x)$ be functions. Suppose that $\lim_{x\to a} g(x) = L$ and that $h(x)$ is *continuous* at $x = L$. Then

$\lim_{x\to a} h(g(x)) = h\left(\lim_{x\to a} g(x)\right) = h(L)$.

25. Let a and L be real numbers, and let g be a function whose domain includes the interval $a - r < x < a$ for some $r > 0$.
We say that $g(x)$ has left-hand limit L as x approaches a if for each $\varepsilon > 0$ there is a corresponding $\delta > 0$ such that if $0 < a - x < \delta$ then $|g(x) - L| < \varepsilon$. We denote this by $\lim_{x\to a^-} g(x) = L$.
If there is no real number L for which the above is true, then we say $g(x)$ has no left-hand limit as x approaches a, or that $\lim_{x\to a^-} g(x)$ is undefined.

27. a. 0.05 corresponds to ε in the definitions of $\lim_{x\to a} g(x)$ and $\lim_{x\to a} h(x)$.

b. $|(g(x) + h(x)) - 8| = |(g(x) - 3) + (h(x) - 5)|$
$\leq |g(x) - 3| + |h(x) - 5|$
$< 0.05 + 0.05 = 0.1 = \varepsilon$.

29. a. $(x-1)(x^2 + x - 3) = x^3 - 4x + 3$
$= (x^3 - 4x + 6) - 3$
$= p(x) - p(1)$.

b. For $0 < x < 2$, $x^2 + x - 3 < 2^2 + 2 - 3 < 9$, and $x^2 + x - 3 > 0^2 + 0 - 3 > -9$.

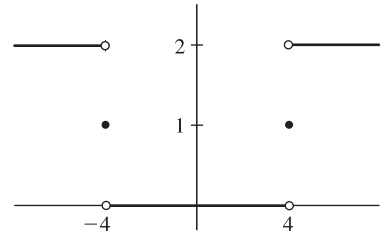
c. $|p(x) - 3| = |x^2 + x - 3| \cdot |x - 1| \leq 9|x - 1|$

d. Since $\lim_{x\to 1} 9|x - 1| = 0$, $\lim_{x\to 1} |p(x) - 3| = 0$.
Thus, $\lim_{x\to 1} p(x) = 3 = p(1)$.

Thus, this proves the rule for the limit of a polynomial from Section 1.5 in the specific case $p(x) = x^3 - 4x + 6$ and $a = 1$.

31. Just as in Exercise 29, we note that $p(x) - p(a)$ takes the value of 0 when $x = a$, so $p(x) - p(a) = (x - a) \cdot q(x)$, where $q(x)$ is another polynomial. It can be shown that for $|x - a| < 1$, $|q(x)| < M$ for some finite real number M. Thus, for $|x - a| < 1$, $|p(x) - p(a)| \leq M \cdot |x - a|$. Using the squeeze theorem we get the desired result.

33. The roots of a quadratic polynomial $x^2 + bx + 4$ are given by
$\frac{-b \pm \sqrt{b^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{-b \pm \sqrt{b^2 - 16}}{2}$.
Thus, $r(b) = 2$ when $b^2 - 16 > 0$ or $b < -4$, $b > 4$; $r(b) = 1$ when $b^2 - 16 = 0$, or $b = \pm 4$; and $r(b) = 0$ when $b^2 - 16 < 0$, or $-4 < b < 4$. The values of b for which r is not continuous are $b = \pm 4$.



35. $\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}=\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{\left(\sqrt[3]{x}-2\right)\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)}$

$=\lim_{x\to 8}\frac{1}{x^{2/3}+2x^{1/3}+4}$

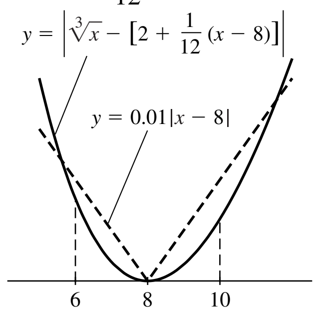
$=\frac{1}{4+2(2)+4}$

$=\frac{1}{12}$

When x is close to 8, $\frac{\sqrt[3]{x}-2}{x-8}-\frac{1}{12}=E(x)$, so

$\sqrt[3]{x}=2+\frac{1}{12}(x-8)+E(x)(x-8)$

$\approx 2+\frac{1}{12}(x-8)$



$\left|\sqrt[3]{x}-\left[2+\frac{1}{12}(x-8)\right]\right|<0.01|x-8|$ when $|x-8|<2$ (sample answer; other answers are possible).

37. $\lim_{x\to -2}\frac{\frac{1}{\sqrt{x^2+5}}-\frac{1}{3}}{x+2}$

$=\lim_{x\to -2}\frac{3-\sqrt{x^2+5}}{3\sqrt{x^2+5}}\cdot\frac{1}{x+2}$

$=\lim_{x\to -2}\frac{3-\sqrt{x^2+5}}{3(x+2)\sqrt{x^2+5}}\cdot\frac{3+\sqrt{x^2+5}}{3+\sqrt{x^2+5}}$

$=\lim_{x\to -2}\frac{9-\left(x^2+5\right)}{3(x+2)\left(3\sqrt{x^2+5}+x^2+5\right)}$

$=\lim_{x\to -2}\frac{-\left(x^2-4\right)}{3(x+2)\left(3\sqrt{x^2+5}+x^2+5\right)}$

$=\lim_{x\to -2}\frac{-(x-2)}{9\sqrt{x^2+5}+3x^2+15}$

$=\frac{4}{27+12+15}$

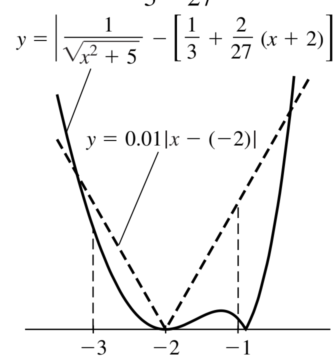
$=\frac{2}{27}$

When x is close to –2,

$\frac{\frac{1}{\sqrt{x^2+5}}-\frac{1}{3}}{x+2}-\frac{2}{27}=E(x)$, so

$\frac{1}{\sqrt{x^2+5}}=\frac{1}{3}+\frac{2}{27}(x+2)+E(x)(x+2)$

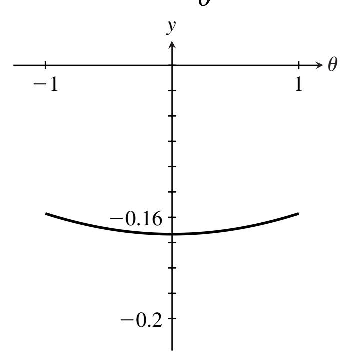
$\approx\frac{1}{3}+\frac{2}{27}(x+2)$



$\left|\frac{1}{\sqrt{x^2+5}}-\left[\frac{1}{3}+\frac{2}{27}(x+2)\right]\right|<0.01|x-(-2)|$

when $|x-(-2)|=|x+2|<1$ (sample answer; other answers are possible).

39. The graph of $\frac{\sin\theta-\theta}{\theta^3}$ is shown.



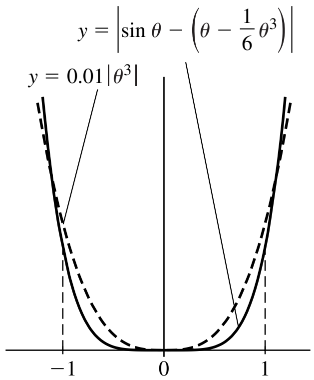
The table shows approximate values of $\frac{\sin\theta-\theta}{\theta^3}$ for values of θ close to 0.

θ	$\frac{\sin\theta-\theta}{\theta^3}$
0.1	–0.166583
–0.1	–0.166583
0.01	–0.166666
–0.01	–0.166666
0.001	–0.166667
–0.001	–0.166667

The evidence indicates that $\lim_{\theta \to 0} \frac{\sin\theta - \theta}{\theta^3} = -0.1\bar{6} = -\frac{1}{6}$.

When θ is close to 0, $\frac{\sin\theta - \theta}{\theta^3} - \left(-\frac{1}{6}\right) = E(\theta)$, so

$$\begin{aligned}\sin\theta &= \theta - \frac{1}{6}\theta^3 + E(\theta)\theta^3 \\ &\approx \theta - \frac{1}{6}\theta^3\end{aligned}$$



$\left|\sin\theta - \left(\theta - \frac{1}{6}\theta^3\right)\right| < 0.01\left|\theta^3\right|$ when $|\theta| < 1$

(sample answer; other answers are possible).x

41. $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$ since if either limit did not exist or had a value other than L, the statement $\lim_{x \to c} f(x) = L$ could not be true.

Section 1.7 The Derivative

1. $f(x) = x^2 - 3$

$$\begin{aligned}f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{\left((x+h)^2 - 3\right) - \left(x^2 - 3\right)}{h} \\ &= \lim_{h \to 0} \frac{2hx + h^2}{h} \\ &= \lim_{h \to 0} (2x + h) \\ &= 2x\end{aligned}$$

$f'(x) = 2x$

3. $h(t) = -3t^2 + 4t - \sqrt{2}$

$$\begin{aligned}h'(a) &= \lim_{t \to a} \frac{h(t) - h(a)}{t - a} \\ &= \lim_{t \to a} \frac{\left(-3t^2 + 4t - \sqrt{2}\right) - \left(-3a^2 + 4a - \sqrt{2}\right)}{t - a} \\ &= \lim_{t \to a} \frac{-3\left(t^2 - a^2\right) + 4(t - a)}{t - a} \\ &= \lim_{t \to a} \left(-3(t + a) + 4\right) \\ &= -6a + 4\end{aligned}$$

$h'(t) = -6t + 4$

5. $y = f(x) = \frac{4}{2 - x}$

$$\begin{aligned}f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{\frac{4}{2-(x+h)} - \frac{4}{2-x}}{h} \\ &= \lim_{h \to 0} \frac{4(2 - x) - 4(2 - (x + h))}{h(2 - x)(2 - (x + h))} \\ &= \lim_{h \to 0} \frac{(8 - 4x) - (8 - 4x - 4h)}{h(2 - x)(2 - (x + h))} \\ &= \lim_{h \to 0} \frac{4h}{h(2 - x)(2 - (x + h))} \\ &= \lim_{h \to 0} \frac{4}{(2 - x)(2 - (x + h))} \\ &= \frac{4}{(2 - x)^2}\end{aligned}$$

$f'(x) = \frac{4}{(2 - x)^2}$

7. $y = g(t) = 2t + 3\sqrt{t+9}$

$$
\begin{aligned}
&g'(a) \\
&= \lim_{t\to a} \frac{g(t)-g(a)}{t-a} \\
&= \lim_{t\to a} \frac{\left(2t+3\sqrt{t+9}\right)-\left(2a+3\sqrt{a+9}\right)}{t-a} \\
&= \lim_{t\to a} \frac{2(t-a)+3\left(\sqrt{t+9}-\sqrt{a+9}\right)}{t-a} \\
&= \lim_{t\to a}\left(2+\frac{3\left(\sqrt{t+9}-\sqrt{a+9}\right)\left(\sqrt{t+9}+\sqrt{a+9}\right)}{(t-a)\left(\sqrt{t+9}+\sqrt{a+9}\right)}\right) \\
&= \lim_{t\to a}\left(2+\frac{3((t+9)-(a+9))}{(t-a)\left(\sqrt{t+9}+\sqrt{a+9}\right)}\right) \\
&= \lim_{t\to a}\left(2+\frac{3}{\sqrt{t+9}+\sqrt{a+9}}\right) \\
&= 2+\frac{3}{2\sqrt{a+9}}
\end{aligned}
$$

$$g'(t) = 2+\frac{3}{2\sqrt{t+9}}$$

9. $y = g(x) = \dfrac{1}{ax+b}$

$$
\begin{aligned}
g'(x) &= \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0}\frac{\frac{1}{a(x+h)+b}-\frac{1}{ax+b}}{h} \\
&= \lim_{h\to 0}\frac{(ax+b)-(a(x+h)+b)}{h(ax+b)(a(x+h)+b)} \\
&= \lim_{h\to 0}\frac{ax+b-ax-ah-b}{h(ax+b)(a(x+h)+b)} \\
&= \lim_{h\to 0}\frac{-ah}{h(ax+b)(a(x+h)+b)} \\
&= \lim_{h\to 0}\frac{-a}{(ax+b)(a(x+h)+b)} \\
&= \frac{-a}{(ax+b)^2}
\end{aligned}
$$

$$y' = -\frac{a}{(ax+b)^2}$$

11. We seek the equation of the tangent line to the graph of $y = f(x) = x^2 - 3x$ through the point (1, –2). The equation for the tangent line is $y-(-2) = f'(1)(x-1)$. We compute $f'(1)$:

$$
\begin{aligned}
f'(1) &= \lim_{x\to 1}\frac{f(x)-f(1)}{x-1} \\
&= \lim_{x\to 1}\frac{\left(x^2-3x\right)-(-2)}{x-1} \\
&= \lim_{x\to 1}\frac{(x-2)(x-1)}{x-1} \\
&= \lim_{x\to 1}(x-2) \\
&= -1
\end{aligned}
$$

The tangent line is given by $y-(-2) = -1(x-1)$ or $y = -x-1$.

13. We seek the equation of the tangent line to the graph of $h(r) = 4r+2$ through the point (1, 6). Since $h(r) = 4r+2$ is a linear function, its rate of change is equal to its slope. Thus, the equation of the tangent line is given by $y-6 = 4(r-1)$ or $y = 4r+2$. (Note that the tangent line to a linear function at any point on the line will be the linear function itself.)

15. We seek the equation of the tangent line to the graph of $g(u) = \dfrac{2}{u^2-1}$ through the point $\left(2, \dfrac{2}{3}\right)$. The equation for the tangent line is $y - \dfrac{2}{3} = g'(2)(u-2)$. We compute $g'(2)$:

$$
\begin{aligned}
g'(2) &= \lim_{u\to 2}\frac{g(u)-g(2)}{u-2} \\
&= \lim_{u\to 2}\frac{\frac{2}{u^2-1}-\frac{2}{3}}{u-2} \\
&= \lim_{u\to 2}\left(\frac{2}{u-2}\cdot\frac{3-\left(u^2-1\right)}{3\left(u^2-1\right)}\right) \\
&= \lim_{u\to 2}\left(\frac{-2}{u-2}\cdot\frac{u^2-4}{3\left(u^2-1\right)}\right) \\
&= \lim_{u\to 2}\frac{-2(u+2)}{3\left(u^2-1\right)} \\
&= -\frac{8}{9}
\end{aligned}
$$

The tangent line is $y - \dfrac{2}{3} = -\dfrac{8}{9}(x-2)$ or $y = -\dfrac{8}{9}x + \dfrac{22}{9}$.

17. We seek the equation of the tangent line to the graph of $f(x) = 2|x|$ through the point (2, 4). The equation for the tangent line is $y - 4 = f'(2)(x-2)$. Using properties of absolute value we have

$$f(x) = \begin{cases} 2x & x \geq 0 \\ -2x & x < 0 \end{cases}.$$

Therefore, we conclude that $f'(2) = 2$ and so the equation of the tangent line is $y - 4 = 2(x-2)$ or $y = 2x$.

19. If $y = f(x) = \frac{1}{x}$, then the equation for the tangent line through the point $\left(3, \frac{1}{3}\right)$ is given by $y - \frac{1}{3} = f'(3)(x-3)$. We compute $f'(3)$:

$$\begin{aligned} f'(3) &= \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \\ &= \lim_{h\to 0} \frac{3-(3+h)}{3h(3+h)} \\ &= \lim_{h\to 0} \frac{-h}{3h(3+h)} \\ &= \lim_{h\to 0} -\frac{1}{3(3+h)} \\ &= -\frac{1}{9}. \end{aligned}$$

Hence, the equation for the tangent line is $y = -\frac{1}{9}x + \frac{2}{3}$. For values of x near 3, the tangent line approximates the function $y = f(x) = \frac{1}{x}$. In particular, if x = 3.15, then $\frac{1}{3.15} \approx -\frac{1}{9}(3.15) + \frac{2}{3} = \frac{19}{60} = 0.31\overline{6}$.

Since the actual value of $\frac{1}{3.15}$ is $\frac{20}{63} = 0.\overline{317460}$, the (absolute) error in using the tangent line is $\frac{1}{1260} = 0.00\overline{079365}$. If x = 2.85, then $\frac{1}{2.85} \approx -\frac{1}{9}(2.85) + \frac{2}{3} = \frac{7}{20} = 0.35$. In this case, the (absolute) error induced by approximating $\frac{1}{2.85}$ via the tangent line through $\left(3, \frac{1}{3}\right)$ is $\frac{1}{1140} \approx 0.000877$.

21. **a.** If $y = h(x) = \tan x$, then the equation for the tangent line through the point $\left(\frac{\pi}{4}, 1\right)$ is given by $y - 1 = h'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$. Since $h'(x) = \sec^2 x$, $h'\left(\frac{\pi}{4}\right) = \sec^2 \frac{\pi}{4} = 2$ and the tangent line is $y - 1 = 2\left(x - \frac{\pi}{4}\right)$ or $y = 2\left(x - \frac{\pi}{4}\right) + 1$.

b.
$$\begin{aligned} \tan\left(\frac{\pi}{4} - 0.05\right) &\approx 2\left(\frac{\pi}{4} - 0.05 - \frac{\pi}{4}\right) + 1 \\ &= -0.1 + 1 \\ &= 0.9 \end{aligned}$$

Check: $\tan\left(\frac{\pi}{4} - 0.05\right) \approx 0.904686$

23. In Example 3 it is noted that by Newton's law of gravitation a 100 kg object r meters above the surface of a spherical body (e.g., planet, moon, etc.…) is subject to a gravitational force of $F(r) = \frac{100GM}{(R+r)^2}$ newtons where M, is the mass of the body, R is the radius of the planet, and G is the gravitational constant. It is established in Example 3 that $F'(r) = \frac{-200GM}{(R+r)^3}$ N/m.

In the case where the body is the moon $\left(M \approx 7.34 \times 10^{22},\ R \approx 1.74 \times 10^6\right)$, the rate of change of the gravitational force on an object 10^7 meters above the surface of the moon is

$$\begin{aligned} F'\left(10^7\right) &\approx \frac{-200\left(6.67 \times 10^{-11}\right)\left(7.34 \times 10^{22}\right)}{\left(1.74 \times 10^6 + 10^7\right)^3} \text{ N/m} \\ &\approx -6.051 \times 10^{-7} \text{ N/m} \end{aligned}$$

The derivative tells us that the force on the object decreases by about 6.051×10^{-7} N if the object moves 1 meter farther from the moon.

25. **a.** The tangent line to the graph of $y = f(x)$ at the point (0, 2), given that $f'(x) = \frac{1}{x^2+1}$, is $y - 2 = \frac{1}{0^2+1}(x-0)$ or $y = x + 2$.

b. The tangent line estimate for $f(0.05)$ is 0.05 + 2 or 2.05. Similarly, the tangent line estimate for $f(-0.1, 1)$ is –0.1 + 2 or 1.9.

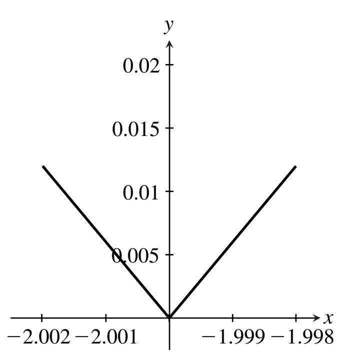
27. Since $y = 5x - 4$ is tangent to the graph of $y = g(x)$ at the point (–1, –9) and since the tangent line to the graph of a function has the same rate of change as the function at the point of tangency, we conclude that $g'(-1) = 5$. Using the tangent line to estimate $g(-0.88)$, we have $g(-0.88) \approx 5(-0.88) - 4 = -8.4$.

29. a. We seek the equation for the tangent line to the graph of $f(x) = x^3$ at the point (–2, –8). First, we compute $f'(-2)$:

$$\begin{aligned} f'(-2) &= \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h} \\ &= \lim_{h\to 0} \frac{(-2+h)^3 - (-2)^3}{h} \\ &= \lim_{h\to 0} \frac{12h - 6h^2 + h^3}{h} \\ &= \lim_{h\to 0}\left(12 - 6h + h^2\right) \\ &= 12. \end{aligned}$$

The tangent line is given by $y - (-8) = 12(x - (-2))$ or $y = 12x + 16$.

b. We wish to find $r > 0$ so that for values of x within r units of –2, we have that $\left|x^3 - (12x + 16)\right| < 0.01|x - (-2)|$. The figure shows the graph of $\dfrac{\left|x^3 - (12x+16)\right|}{|x - (-2)|}$ near $x = -2$.



The graph suggests that choosing $r = 0.001$ produces the desired results. (We note that other values of r also yield the desired result. In fact, values of r as large as 0.001666 also yield the desired result.)

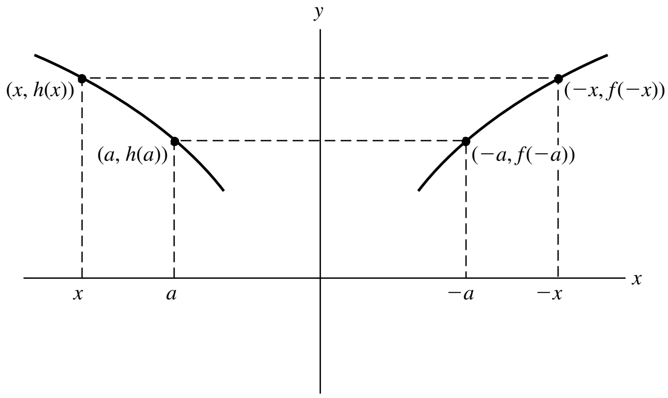
31. a. Since $h(x) = f(-x)$ and $f(4) = 3$, it follows that $h(-4) = f(4) = 3$. Thus, the point (–4, 3) is on the graph of $y = h(x)$. More generally, if (x, y) is on the graph of $y = f(x)$, then $(-x, y)$ is on the graph of $y = h(x)$. From this fact, we conclude that the graphs of $y = f(x)$ and $y = h(x)$ are symmetric with respect to the y-axis.

b. To determine $h'(-4)$ we consider the difference quotient $\dfrac{h(x) - h(-4)}{x - (-4)} = \dfrac{f(-x) - f(4)}{x - (-4)}$. Substituting $t = -x$ we have

$$\begin{aligned} \frac{f(-x) - f(4)}{x - (-4)} &= \frac{f(t) - f(4)}{-t + 4} \\ &= -1 \cdot \frac{f(t) - f(4)}{t - 4}. \end{aligned}$$

We conclude that $h'(-4) = -1 \cdot f'(4) = -7$.

c. Consider the graph shown of $y = h(x)$ and $y = f(x)$.



By considering the difference quotient for $h(x)$ at $x = a$ and the figure we have

$$\frac{h(x)-h(a)}{x-a} = \frac{f(-x)-f(-a)}{x-a}$$
$$= -1 \cdot \frac{f(-x)-f(-a)}{(-x)-(-a)}$$
$$= -1 \cdot \frac{f(t)-f(-a)}{t-(-a)} \quad (t = -x).$$

Letting x approach a (or, equivalently, letting t approach $-a$) we see that $h'(a) = -1 \cdot f'(-a)$.

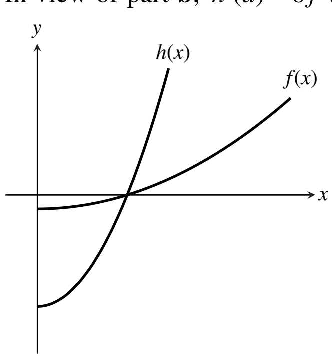
33. a. Since $h(x) = 8f(x)$ and $f(4) = 3$, it follows that $h(4) = 24$. Thus, the point (4, 24) is on the graph of $y = h(x)$.

b. Since

$$f'(4) = \lim_{x\to 4} \frac{f(x)-f(4)}{x-4} = 7,$$
$$h'(4) = \lim_{x\to 4} \frac{h(x)-h(4)}{x-4}$$
$$= \lim_{x\to 4} \frac{8f(x)-8f(4)}{x-4}$$
$$= 8 \cdot \lim_{x\to 4} \frac{f(x)-f(4)}{x-4}$$
$$= 8(7)$$
$$= 56.$$

The graph of $y = h(x)$ is everywhere a vertical stretch (by a factor of 8) of the graph of $y = f(x)$. Hence, the "rise/run" will be 8 times as large for $y = h(x)$ relative to $y = f(x)$ near a given value of x.

c. In view of part **b**, $h'(a) = 8f'(a)$.

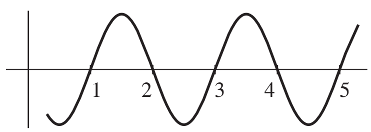


The graph of $h(x)$ is everywhere 8 times as steep as the graph of $f(x)$.

35. The derivative for $f(x)$ when $x = a$ is given by $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ provided this limit exists. If $f(x) = x^9 - 4x^7 + 3x - 2$ and $a = 2$, then $f(2) = 4$ and

$$f'(2) = \lim_{x \to 2} \frac{\left(x^9 - 4x^7 + 3x - 2\right) - 4}{x - 2} = 515.$$

37. The graph shown below is of a function satisfying the conditions
$f(1) = f(2) = f(3) = f(4) = f(5) = 0,$
$f'(1) = f'(3) = f'(5) = 1$ and $f'(2) = f'(4) = -1.$



$\left(\text{The function shown is } f(x) = -\frac{\cos \pi\left(x - \frac{1}{2}\right)}{\pi}.\right)$

Chapter 1 Review Exercises

1. The domain of f is {0, 1, 2, 3, 4, 5}.
The domain of g is {–3, –0.5, 0, 2, 3}.

3. The domain of $f + g$ is {0, 2, 3}.
$(f + g)(0) = f(0) + g(0) = -3 + 4 = 1$
$(f + g)(2) = f(2) + g(2) = (\sin 1) - \pi = -\pi + \sin 1$
$(f + g)(3) = f(3) + g(3) = 0 + 3 = 3$

x	$(f+g)(x)$
0	1
2	$-\pi + \sin 1$
3	3

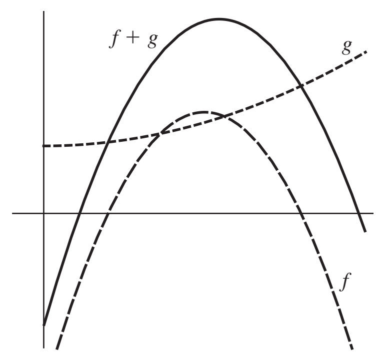
5. The domain of $\frac{f}{g}$ is {0, 2, 3}.

$$\frac{f}{g}(0) = \frac{f(0)}{g(0)} = \frac{-3}{4} = -\frac{3}{4}$$
$$\frac{f}{g}(2) = \frac{f(2)}{g(2)} = \frac{\sin 1}{-\pi} = -\frac{\sin 1}{\pi}$$
$$\frac{f}{g}(3) = \frac{f(3)}{g(3)} = \frac{0}{3} = 0$$

x	$\left(\frac{f}{g}(x)\right)$
0	$-\frac{3}{4}$
2	$-\frac{\sin 1}{\pi}$
3	0

7. If we take the graphs as showing the full domain of each function, the domain of f is $\{x : 0 < x < 5\}$ and the domain of g is $\{x : 0 \le x \le 6\}$.

9. The domain of $f + g$ is $\{x : 0 < x < 5\}$.



11. The domain of $\frac{g}{f}$ is $\{x : 0 < x < 5 \text{ with } x \ne 1, 4\}$.

13. Drawing a line tangent to the graph of $f(x)$ at (1, 0), it appears that the line passes through (0.75, –1) and (1.25, 1) so the rate of change of f with respect to x at $x = 1$ is approximately $\frac{1 - (-1)}{1.25 - 0.75} = \frac{2}{0.5} = 4.$

15. The units of the rate of change are meters per centimeters/sec or m/(cm/s).
The rate of change tells by how many meters the length of the object changes when the velocity increases by 1 centimeter per second.

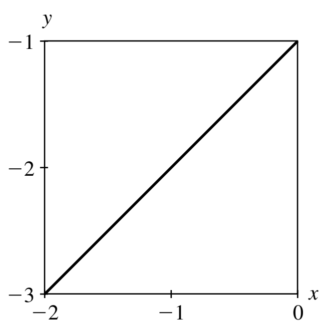
17. Sample answer (other answers are possible): The annual inflation rate of 3.5 percent in March 1994 indicates that the cost of an item increased by $\frac{1}{12}$ of 3.5 percent over the course of $\frac{1}{12}$ of the year $\left(\text{rate} = \frac{\frac{1}{12} \times 3.5\%}{\frac{1}{12} \times 1 \text{ year}} = 3.5\% / \text{year}\right)$. But of course the amount of price change can vary from month to month.

19. **a.** The table shows values of $\frac{x^2-1}{x+1}$ for values of x close to –1.

x	$\frac{x^2-1}{x+1}$
–1.1	–2.1
–0.9	–1.9
–1.01	–2.01
–0.99	–1.99
–1.001	–2.001
–0.999	–1.999

Based on the evidence, it appears that $\lim_{x\to -1} \frac{x^2-1}{x+1} = -2$.

b. The graph of $\frac{x^2-1}{x+1}$ is shown.



It appears that $\lim_{x\to -1} \frac{x^2-1}{x+1} = -2$.

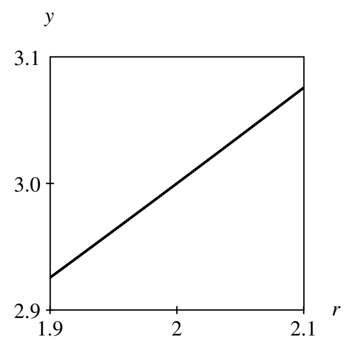
c.
$$\begin{aligned}\lim_{x\to -1} \frac{x^2-1}{x+1} &= \lim_{x\to -1} \frac{(x+1)(x-1)}{x+1}\\ &= \lim_{x\to -1} (x-1)\\ &= -1-1\\ &= -2\end{aligned}$$

21. **a.** The table shows values of $\frac{r^3-8}{r^2-4}$ for values of r close to 2.

r	$\frac{r^3-8}{r^2-4}$
1.9	2.925641
2.1	3.075610
1.99	2.992506
2.01	3.007506
1.999	2.999250
2.001	3.000750

Based on the evidence, it appears that $\lim_{r\to 2} \frac{r^3-8}{r^2-4} = 3$.

b. The graph of $\frac{r^3-8}{r^2-4}$ is shown



It appears that $\lim_{r\to 2} \frac{r^3-8}{r^2-4} = 3$.

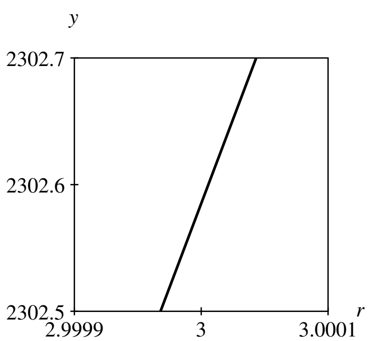
c.
$$\begin{aligned}\lim_{r\to 2} \frac{r^3-8}{r^2-4} &= \lim_{r\to 2} \frac{(r-2)\left(r^2+2r+4\right)}{(r-2)(r+2)}\\ &= \lim_{r\to 2} \frac{r^2+2r+4}{r+2}\\ &= \frac{2^2+2(2)+4}{2+2}\\ &= 3\end{aligned}$$

23. a. The table shows approximate values of $\frac{10^r - 1000}{r-3}$ for values of r close to 3.

r	$\frac{10^r - 1000}{r-3}$
2.999	2299.9362
3.001	2305.2381
2.99999	2302.5586
3.00001	2302.6116
2.9999999	2302.5848
3.0000001	2302.5850
2.999999999	2302.5900

Based on the evidence, it appears that $\lim_{r\to 3} \frac{10^r - 1000}{r-3} \approx 2302.59$. Note that $1000 \ln 10 \approx 2302.59$.

b. The graph of $\frac{10^r - 1000}{r-3}$ is shown.



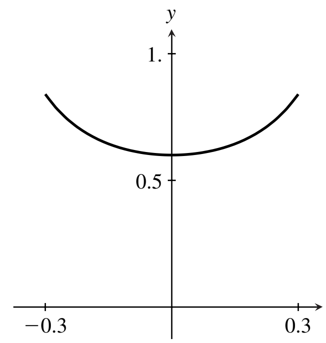
It appears that $\lim_{r\to 3} \frac{10^r - 1000}{r-3} \approx 2302.59$.

25. a. The table shows approximate values of $\frac{\tan 3t}{5t}$ for values of x close to 0.

t	$\frac{\tan 3t}{5t}$
0.1	0.618672
–0.1	0.618672
0.01	0.600180
–0.01	0.600180
0.001	0.600002
–0.001	0.600002

Based on the evidence, it appears that $\lim_{t\to 0} \frac{\tan 3t}{5t} = 0.6 = \frac{3}{5}$.

b. The graph of $\frac{\tan 3t}{5t}$ is shown.



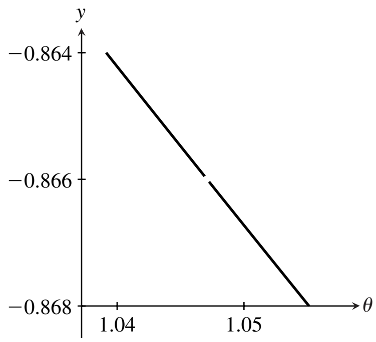
It appears that $\lim_{t\to 0} \frac{\tan 3t}{5t} = 0.6 = \frac{3}{5}$.

27. a. The table shows values of $\frac{\cos\theta - \frac{1}{2}}{\theta = \frac{\pi}{3}}$ for values of θ close to $\frac{\pi}{3}$.

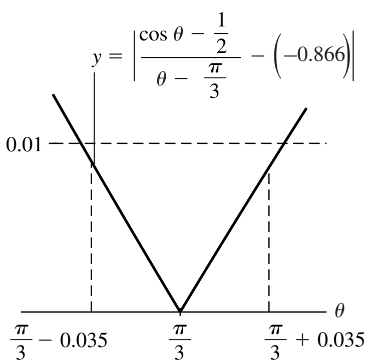
θ	$\frac{\cos\theta - \frac{1}{2}}{\theta - \frac{\pi}{3}}$
$\frac{\pi}{3} + 0.1$	–0.889562
$\frac{\pi}{3} - 0.1$	–0.839604
$\frac{\pi}{3} + 0.01$	–0.868511
$\frac{\pi}{3} - 0.01$	–0.863511
$\frac{\pi}{3} + 0.001$	–0.866275
$\frac{\pi}{3} - 0.001$	–0.865775

Based on the evidence, it appears that $\lim_{\theta\to\pi/3} \frac{\cos\theta - \frac{1}{2}}{\theta - \frac{\pi}{3}} \approx 0.866$. Note that $-\frac{\sqrt{3}}{2} \approx -0.866$.

b. The graph of $\dfrac{\cos\theta - \frac{1}{2}}{\theta - \frac{\pi}{3}}$ is shown.



It appears that $\lim\limits_{\theta \to \pi/3} \dfrac{\cos\theta - \frac{1}{2}}{\theta - \frac{\pi}{3}} \approx -0.866.$



δ can be any positive number < 0.035 (sample answer; other answers are possible).

29. $f(x) = 2x - 5$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{2(x+h) - 5 - (2x-5)}{h} \\ &= \lim_{h\to 0} \frac{2h}{h} \\ &= \lim_{h\to 0} 2 \\ &= 2 \end{aligned}$$

31. $f(x) = x + \dfrac{1}{x}$

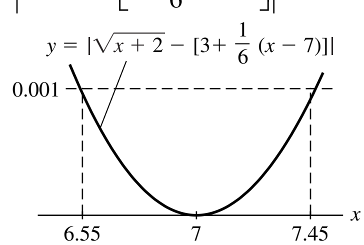
$$\begin{aligned} f'(x) &= \lim_{t\to x} \frac{f(t) - f(x)}{t - x} \\ &= \lim_{t\to x} \frac{t + \frac{1}{t} - \left(x + \frac{1}{x}\right)}{t - x} \\ &= \lim_{t\to x} \frac{t - x + \frac{1}{t} - \frac{1}{x}}{t - x} \\ &= \lim_{t\to x} \left[1 + \frac{1}{t-x}\left(\frac{x-t}{xt}\right)\right] \\ &= \lim_{t\to x} \left(1 - \frac{1}{xt}\right) \\ &= 1 - \frac{1}{x^2} \end{aligned}$$

33. We seek the equation of the tangent line to the graph of $y = f(x) = \sqrt{x+2}$ through the point (7, 3). The equation for the tangent line is $y - 3 = f'(7)(x - 7)$. We compute $f'(7)$:

$$\begin{aligned} f'(7) &= \lim_{t\to 7} \frac{f(t) - f(7)}{t - 7} \\ &= \lim_{t\to 7} \frac{\sqrt{t+2} - \sqrt{7+2}}{t - 7} \\ &= \lim_{t\to 7} \frac{\sqrt{t+2} - 3}{t - 7} \cdot \frac{\sqrt{t+2} + 3}{\sqrt{t+2} + 3} \\ &= \lim_{t\to 7} \frac{t + 2 - 9}{(t-7)\left(\sqrt{t+2} + 3\right)} \\ &= \lim_{t\to 7} \frac{1}{\sqrt{t+2} + 3} \\ &= \frac{1}{6} \end{aligned}$$

The tangent line is given by $y - 3 = \dfrac{1}{6}(x - 7)$ or $y = 3 + \dfrac{1}{6}(x - 7)$. We wish to find $r > 0$ so that for a value of x within r units of 7, we have that $\left|\sqrt{x+2} - \left[3 + \dfrac{1}{6}(x-7)\right]\right| < 0.001.$

The figure shows the graph of $\left|\sqrt{x+2} - \left[3 + \dfrac{1}{6}(x-7)\right]\right|$ near x = 7.



The graph suggests that any positive $r \le 0.45$ will work.

35. Sample answer (other answers are possible): $\lim_{x \to a} f(x) = L$ means that given any number $\varepsilon > 0$, all values of $f(x)$ will lie between $L - \varepsilon$ and $L + \varepsilon$ when x is close enough to a. If the limit does not exist, then no matter what L is, you can find an ε so that no matter how close you are to a, there are x-values for which $f(x)$ does not lie between $L - \varepsilon$ and $L + \varepsilon$. In other words, the values of $f(x)$ do not "settle down" to one number as x gets close to a. For illustrations of functions with points at which no limit exists, see Figs. 1.50, 1.51, 1.66 and 1.68.

37. Sample answer (other answers are possible): If f is not continuous at $x = a$, then there is some sort of "break" in the graph at $(a, f(a))$. Thus, we will never see a "straight line" when zooming in on the graph of $y = f(x)$ at $(a, f(a))$.