Java Applet
Tangent Lines
Explores the connection between the slope of a line tangent to the graph of a function and the function's derivative. Students can graph a function $f(x)$ and the line tangent to the graph of $y = f(x)$ for a given x value.

WEB Expression (8) says that when x is close to a, the function value $f(x)$ is approximated by the corresponding y-value for the tangent line.

EXAMPLE 6 Find the equation of the line tangent to

$$f(x) = 1/(x+1)$$

at $(2, 1/3)$ and use this equation to approximate $f(1.93) = 1/(1.93 + 1)$.

Solution

The tangent line passes through the point of tangency, $(2, 1/3)$. The slope of the tangent line is the derivative of f at $x = 2$,

$$f'(2) = \lim_{x\to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2} \frac{\dfrac{1}{x+1} - \dfrac{1}{3}}{x - 2}.$$

Add the fractions in the numerator and simplify the resulting complex fraction. We then have

$$f'(2) = \lim_{x\to 2} \frac{2 - x}{3(x+1)(x-2)} = \lim_{x\to 2} \frac{-1}{3(x+1)} = -\frac{1}{9}.$$

The equation of the line tangent to $y = 1/(x+1)$ at $(2, 1/3)$ is

$$y = f(2) + f'(2)(x - 2) = \frac{1}{3} - \frac{1}{9}(x - 2).$$

When x is close to 2, the y coordinates of the graph of $y = 1/(x+1)$ and the tangent line should be close. See Fig. 1.78. Hence for x near 2,

$$\frac{1}{x+1} \approx \frac{1}{3} - \frac{1}{9}(x - 2).$$

Substitute $x = 1.93$ into this expression to get

$$f(1.93) = \frac{1}{1.93 + 1} \approx \frac{1}{3} - \frac{1}{9}(1.93 - 2) \approx 0.341111.$$

How does this compare with the value of $(1.93 + 1)^{-1}$ given by your calculator?

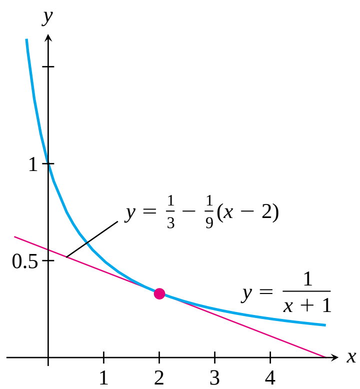

FIGURE 1.78 The line tangent to $y = 1/(x+1)$ at $(2, 1/3)$.

Error in the Tangent Line Approximation We can use the techniques developed in Section 1.6 to estimate the error in the tangent line approximation. Let

$$\frac{f(x) - f(a)}{x - a} - f'(a) = E(x). \tag{9}$$

Because

$$\lim_{x\to a} E(x) = \lim_{x\to a}\left(\frac{f(x) - f(a)}{x - a} - f'(a)\right) = f'(a) - f'(a) = 0,$$