

TABLE 1.1 Iditarod winners.

Year	Name	Time	Year	Name	Time
1973	Dick Wilmarth	20:00:49:41	1987	Susan Butcher	11:02:05:13
1974	Carl Huntington	20:15:02:07	1988	Susan Butcher	11:11:41:40
1975	Emmitt Peters	14:14:43:45	1989	Joe Runyan	11:05:24:34
1976	Jerry Riley	18:22:58:17	1990	Susan Butcher	11:01:53:23
1977	Rick Swenson	16:27:13:00	1991	Rick Swenson	12:16:35:39
1978	Dick Mackey	14:18:52:24	1992	Martin Buser	10:19:17:15
1979	Rick Swenson	12:08:45:02	1993	Jeff King	10:15:38:15
1980	Joe May	14:07:11:51	1994	Martin Buser	10:13:02:39
1981	Rick Swenson	12:08:45:02	1995	Doug Swingley	9:02:42:19
1982	Rick Swenson	16:04:40:10	1996	Jeff King	9:05:43:13
1983	Rick Mackey	12:14:10:44	1997	Martin Buser	9:08:30:45
1984	Dean Osmar	12:15:07:33	1998	Jeff King	9:05:52:26
1985	Libby Riddles	18:00:20:17	1999	Doug Swingley	9:14:31:07
1986	Susan Butcher	11:15:06:00	2000	Doug Swingley	9:00:58:06

**Java Applet***Graphing Functions*

Allows students to graph up to five functions simultaneously in Cartesian coordinates.

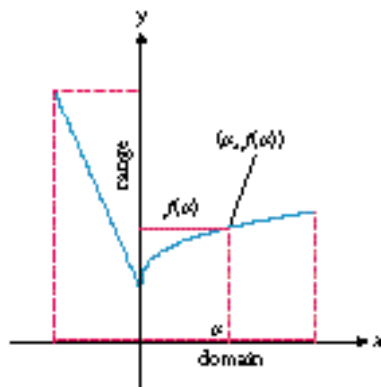


FIGURE 1.5 Graphical representation of a function.



of points with coordinates of the form $(t, I(t))$, for t in the domain of I . Because the domain of I has 28 elements, there are only 28 such points. These points make up the graph of I . What are some other functions suggested by the table?

When a function f is represented graphically, we assume, unless told otherwise, that the domain is represented by some part of the horizontal axis and the range by some part of the vertical axis. The graph of f consists of all points with coordinates $(x, f(x))$ where x is in the domain of f . The domain of the function contains the set of points on the horizontal axis that lie above or below the graph. The range of the function contains the set of points on the vertical axis that lie to the left or right of the graph. Given an a in the domain (on the horizontal axis), we find the corresponding element in the range by extending a dotted line from a , either up or down, until it meets the graph at the point $(a, f(a))$. From this point, extend the dotted line horizontally until it meets the vertical axis at $f(a)$. See Fig. 1.5. Graphs are a very useful way to study and represent functions. However, as the next example shows, graphs may not give complete information about a function.

EXAMPLE 4 Discuss the domain and range of the function f represented by the graph in Fig. 1.6.

Solution

The graph appears to lie above or below the interval $[-2, 3] = \{x: -2 \leq x \leq 3\}$ on the horizontal axis. Thus the domain of f contains the interval $[-2, 3]$. We cannot be certain that this is the entire domain of f because the portion of the graph shown does not tell us whether

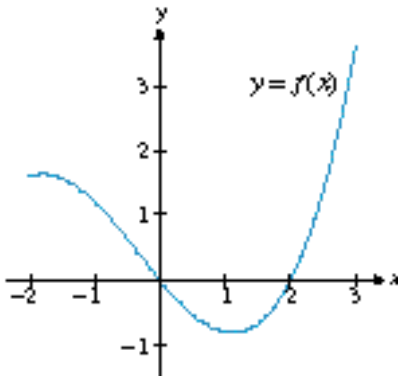


FIGURE 1.6 Graphical representation of f .



Java Applet

Tracing Graphs of Functions

Students can graph and trace two functions simultaneously, enabling them to explore function values and points of intersection.

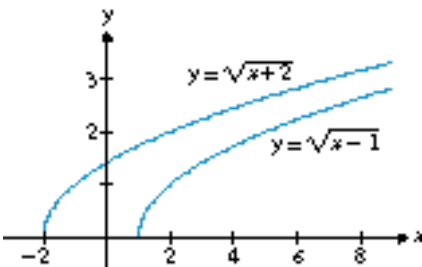


FIGURE 1.7 The graphs of $y = f(x) = \sqrt{x - 1}$ and $y = g(x) = \sqrt{x + 2}$.

f is defined for real numbers outside of $[-2, 3]$; that is, the picture may show the graph for only part of the domain of f . The graph appears to lie to the left and right of (approximately) the interval $[-0.7, 3.8] = \{y : -0.7 \leq y \leq 3.8\}$; that is, all y for which $-0.7 \leq y \leq 3.8$ on the vertical axis. Hence, we conclude that the range of f contains this interval.

Interval Notation In the previous example, we used the notation $[-2, 3]$ to denote the set of x values between -2 and 3 , with -2 and 3 included. Square brackets $[]$ are used to indicate that an endpoint is to be included in the interval described. Parentheses $()$ are used when an endpoint is not to be included. Thus, $(-2, 3]$ is the set of x values between -2 and 3 , not including -2 but including 3 . We will also use this notation to describe intervals of infinite length. For example,

$(-\infty, 6)$ describes the set of real numbers x such that $x < 6$,

and

$[\sqrt{2}, \infty)$ describes the set of real numbers x such that $x \geq \sqrt{2}$.

Combining Functions Usually we work with functions that are defined by equations, as in Examples 1 and 2. Although such functions may be complicated, they are usually constructed from simple pieces with which we are already familiar. These pieces are often referred to as the **elementary functions**. They include



- The constant functions, given by $f(x) = c$, where c is a real number.
- the absolute value function, $|x|$;
- the power functions: x, x^2, x^3, \dots ;
- the reciprocal function defined by $f(x) = 1/x$;
- the root functions: $\sqrt{x}, \sqrt[3]{x}, \sqrt[4]{x}, \dots$;
- the trigonometric functions: $\sin x, \cos x, \tan x, \sec x, \dots$;
- the inverse trigonometric functions: $\arcsin x, \arctan x, \dots$;
- the logarithmic functions: $\ln x, \log_{10} x, \log_2 x, \dots$;
- the exponential functions: $e^x, 10^x, 2^x, \dots$.

Though you are probably familiar with these functions, we will review many of them in later sections. In the remainder of this section, we review a few methods for combining functions to build new functions.

EXAMPLE 5 Discuss the domains of the sum and product of the functions f and g defined by $f(x) = \sqrt{x - 1}$ and $g(x) = \sqrt{x + 2}$. The graphs of these functions are shown in Fig. 1.7.

Solution

The sum of f and g is denoted $f + g$ and is defined by

$$(f + g)(x) = f(x) + g(x) \tag{2}$$

for all x in the domain of $f + g$. The product function is denoted $f \cdot g$ and is defined by

$$(f \cdot g)(x) = f(x) \cdot g(x) \tag{3}$$