given, we can try to determine the "likely" domain by using knowledge of algebra and trigonometry.

EXAMPLE 2 Discuss the domain and range of the function g described by the equation $g(x) = \sqrt{x}$.

Solution

Because we've all heard that "you can't take the square root of a negative number," it seems natural that the domain of g is the set $[0, \infty) = \{x : x \geq 0\}$ of nonnegative real numbers. We may take the range of the square root function to be the set of nonnegative real numbers. Thus if x is a nonnegative real number, then $\sqrt{x}$ denotes the nonnegative square root of x. Calculators and computer algebra systems follow this convention.

Sometimes a CAS or graphing calculator can help in determining the domain of a function represented by an equation. The graph in Fig. 1.3 was produced by having a CAS graph $y = \sqrt{x}$ for $-5 \leq x \leq 5$. The CAS responded that $\sqrt{x}$ is not a real number for $x < 0$, and then produced the graph for those x values for which $\sqrt{x}$ is real. The graph produced is consistent with our assumption that the domain of g is $x \geq 0$.

However, be careful when you use graphics packages to help determine the domain of a function. Sometimes the results are deceptive. Always think about the calculator or CAS result and interpret it in light of the problem that you are trying to solve. (See Exercises 37 and 38.)

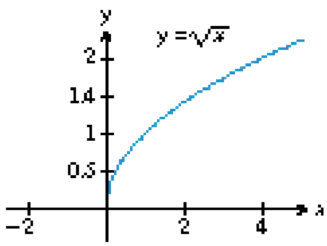
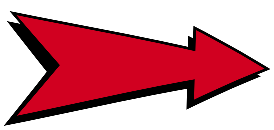


FIGURE 1.3 A CAS-produced graph of $y = \sqrt{x}$ for $-5 \leq x \leq 5$.



EXAMPLE 3 The Iditarod Trail sled dog race is an annual event to commemorate the courage of Leonhard Seppala and other dog mushers who delivered badly needed diphtheria serum to Nome, Alaska, in 1925. The 1100-mile course of the race runs from Anchorage to Nome. The first race was run in 1973. Table 1.1 gives the times of the winners for the races through 2000. Time is given as days:hours:minutes:seconds. This table represents a function whose domain is the set of years in which the race has been run and whose range is the set of winning times. Discuss this function.

Solution

Let I (for Iditarod) be the function that takes as input the year of the race and, as output, gives the winning time for that year. For example,

$$I(1980) = 14{:}07{:}11{:}51 \quad \text{and} \quad I(1994) = 10{:}13{:}02{:}39.$$

The domain of I is the set of years that the race has been run:

$$\{1973, 1974, 1975, \ldots, 2000\}.$$

The range of I is the set of winning times. A graphical representation of this function can be constructed by first forming the ordered pairs $(t, I(t))$, for t in the domain of I,

$$(1973, 20{:}00{:}49{:}41), (1974, 20{:}15{:}02{:}07), \ldots, (2000, 9{:}00{:}58{:}06).$$

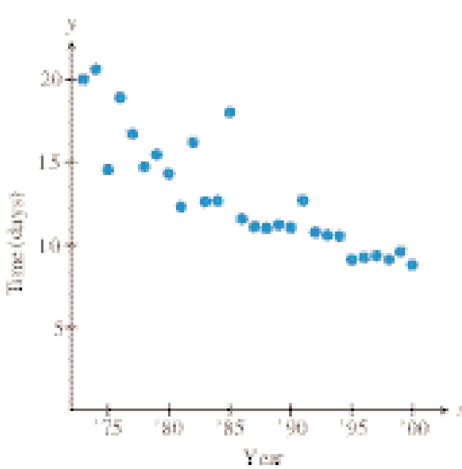


FIGURE 1.4 Winning times for the Iditarod Trail sled dog race.

A graph representing I is shown in Fig. 1.4. Note that the graph is not a curve, but consists of several points. The graph of I is simply the collection

TABLE 1.1 Iditarod winners.

Year	Name	Time	Year	Name	Time
1973	Dick Wilmarth	20:00:49:41	1987	Susan Butcher	11:02:05:13
1974	Carl Huntington	20:15:02:07	1988	Susan Butcher	11:11:41:40
1975	Emmitt Peters	14:14:43:45	1989	Joe Runyan	11:05:24:34
1976	Jerry Riley	18:22:58:17	1990	Susan Butcher	11:01:53:23
1977	Rick Swenson	16:27:13:00	1991	Rick Swenson	12:16:35:39
1978	Dick Mackey	14:18:52:24	1992	Martin Buser	10:19:17:15
1979	Rick Swenson	12:08:45:02	1993	Jeff King	10:15:38:15
1980	Joe May	14:07:11:51	1994	Martin Buser	10:13:02:39
1981	Rick Swenson	12:08:45:02	1995	Doug Swingley	9:02:42:19
1982	Rick Swenson	16:04:40:10	1996	Jeff King	9:05:43:13
1983	Rick Mackey	12:14:10:44	1997	Martin Buser	9:08:30:45
1984	Dean Osmar	12:15:07:33	1998	Jeff King	9:05:52:26
1985	Libby Riddles	18:00:20:17	1999	Doug Swingley	9:14:31:07
1986	Susan Butcher	11:15:06:00	2000	Doug Swingley	9:00:58:06

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of points with coordinates of the form $(t, I(t))$, for t in the domain of I. Because the domain of I has 28 elements, there are only 28 such points. These points make up the graph of I. What are some other functions suggested by the table?

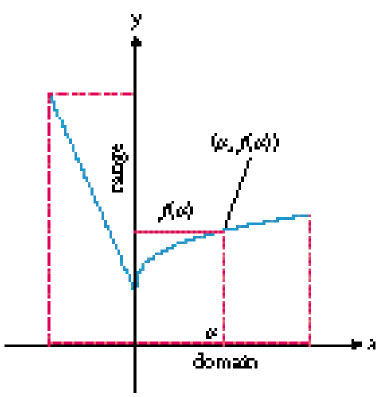

FIGURE 1.5 Graphical representation of a function.

When a function f is represented graphically, we assume, unless told otherwise, that the domain is represented by some part of the horizontal axis and the range by some part of the vertical axis. The graph of f consists of all points with coordinates $(x, f(x))$ where x is in the domain of f. The domain of the function contains the set of points on the horizontal axis that lie above or below the graph. The range of the function contains the set of points on the vertical axis that lie to the left or right of the graph. Given an a in the domain (on the horizontal axis), we find the corresponding element in the range by extending a dotted line from a, either up or down, until it meets the graph at the point $(a, f(a))$. From this point, extend the dotted line horizontally until it meets the vertical axis at $f(a)$. See Fig. 1.5. Graphs are a very useful way to study and represent functions. However, as the next example shows, graphs may not give complete information about a function.

EXAMPLE 4 Discuss the domain and range of the function f represented by the graph in Fig. 1.6.

Solution

The graph appears to lie above or below the interval $[-2, 3] = \{x : -2 \leq x \leq 3\}$ on the horizontal axis. Thus the domain of f contains the interval $[-2, 3]$. We cannot be certain that this is the entire domain of f because the portion of the graph shown does not tell us whether