

29. A geostationary satellite is one moving so that it stays above a fixed location on the Earth. Assuming that such a satellite has a circular orbit, find its altitude.
30. Kepler discovered three laws of planetary motion. The first is that the planets orbit the sun in ellipses, with the sun at one focus. His third law is that the square of a planet's orbital period T is proportional to the cube of the semimajor axis a of its orbit. Excepting only Mercury and Pluto, the orbits of the planets are very nearly circular. First, verify empirically that Kepler's third law holds. Use the periods and semimajor axes of the orbits of Earth, Venus, Jupiter, and Uranus. Data for the last three are given in the table. The periods are measured in tropical years, the time required for the Earth to complete one orbit about the sun. The semimajor axes are measured in astronomical units (AU), the length of the semimajor axis of Earth's orbit.

	T	a
Venus	0.61521	0.7233316
Jupiter	11.86224	5.202561
Uranus	84.01247	19.21814

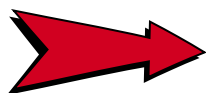
Table for Exercise 30.

Does Kepler's third law follow from an equation similar to (26), but with M_E replaced by the mass M_S of the sun? Why or why not?

Review of Key Concepts

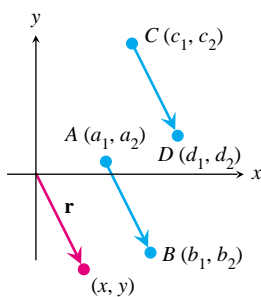
This chapter extends the work of Chapter 1 on rates of change into two dimensions. A major goal was to discuss the velocity and acceleration of objects in motion, two important rates of change. To describe the velocity and acceleration of an object moving in two dimensions, we defined vectors and several vector operations. The trajectories or orbits followed by objects in motion were described

using parametric equations. We defined the dot product of two vectors and used it to calculate the angle between two vectors, to project a vector on a unit vector, and to calculate the work done by a force acting on an object. We introduced antidifferentiation and initial value problems in connection with Newton's three laws and his universal gravitation law.



Chapter Summary

Equivalent Vectors



Vectors \overrightarrow{AB} , \overrightarrow{CD} , and \mathbf{r} , where $\mathbf{r} = \langle x, y \rangle$ is a position vector with initial point at the origin, are equivalent if they have the same magnitude and direction. Specifically, vectors \overrightarrow{AB} , \overrightarrow{CD} , and \mathbf{r} are equivalent if

$$b_1 - a_1 = d_1 - c_1 = x$$

and

$$b_2 - a_2 = d_2 - c_2 = y.$$

If A , B , C , and D are the points

$$(6, 1), (10, -7), (9, 12), \text{ and } (13, 4)$$

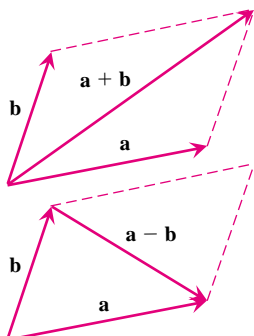
and $\mathbf{r} = \langle 4, -8 \rangle$, then \overrightarrow{AB} , \overrightarrow{CD} , and \mathbf{r} are equivalent because

$$10 - 6 = 13 - 9 = 4$$

and

$$-7 - 1 = 4 - 12 = -8.$$

Sum and Difference of Vectors



The sum or difference of vectors $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$ is

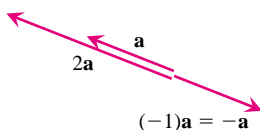
$$\mathbf{a} \pm \mathbf{b} = \langle a_1 \pm b_1, a_2 \pm b_2 \rangle.$$

For $\mathbf{a} = \langle -5, 2 \rangle$ and $\mathbf{b} = \langle 4, 7 \rangle$,

$$\mathbf{a} + \mathbf{b} = \langle -1, 9 \rangle$$

$$\mathbf{a} - \mathbf{b} = \langle -9, -5 \rangle.$$

Product of Scalar and Vector



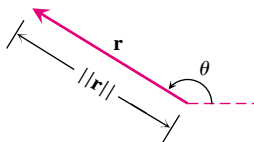
The product of a vector $\mathbf{a} = \langle a_1, a_2 \rangle$ and a scalar (real number) s is

$$s\mathbf{a} = s\langle a_1, a_2 \rangle = \langle sa_1, sa_2 \rangle.$$

For $\mathbf{a} = \langle -5, 2 \rangle$ and $s = 2$,

$$s\mathbf{a} = 2\langle -5, 2 \rangle = \langle -10, 4 \rangle.$$

Length and Direction of a Vector



The length of $\mathbf{r} = \langle x, y \rangle$, and of any vector equivalent to \mathbf{r} , is

$$\|\mathbf{r}\| = \sqrt{x^2 + y^2}.$$

The direction of $\mathbf{r} = \langle x, y \rangle$, and of any vector equivalent to \mathbf{r} , is the angle θ through which the positive x -axis must be rotated counterclockwise to align with \mathbf{r} .

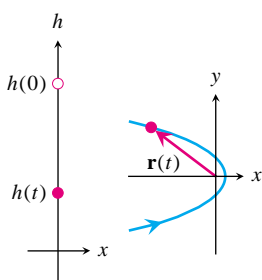
For $\mathbf{r} = \langle -9, 5 \rangle$,

$$\|\mathbf{r}\| = \sqrt{(-9)^2 + 5^2} = \sqrt{106}.$$

The direction of \mathbf{r} is

$$\theta = \pi - \arctan(5/9) \approx 2.63.$$

Average Velocity and Velocity



For one-dimensional motion, average velocity on $[t_1, t_2]$ and velocity at t are

$$v(t_1, t_2) = \frac{h(t_2) - h(t_1)}{t_2 - t_1}$$

$$v(t) = \lim_{t_2 \rightarrow t} v(t, t_2) = h'(t).$$

For two-dimensional motion, average velocity on $[t_1, t_2]$ and velocity at t are

$$\mathbf{v}(t_1, t_2) = \frac{1}{t_2 - t_1} (\mathbf{r}(t_2) - \mathbf{r}(t_1))$$

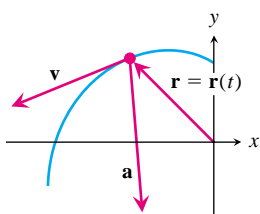
$$\mathbf{v}(t) = \lim_{t_2 \rightarrow t} \mathbf{v}(t, t_2) = \mathbf{r}'(t).$$

If the position of an object at time t is $\mathbf{r} = (1 - t^2)\mathbf{i} + t\mathbf{j}$, its average velocity on the interval $[0, t_2]$ is

$$\mathbf{v}(0, t_2) = \frac{1}{t_2 - 0} ((1 - t_2^2)\mathbf{i} + t_2\mathbf{j} - \mathbf{i})$$

$$= -t_2\mathbf{i} + \mathbf{j}.$$

Velocity and Acceleration



If the position of an object at time t is $\mathbf{r} = \mathbf{r}(t)$, its velocity and acceleration vectors are

$$\mathbf{v} = \frac{d}{dt} \mathbf{r}$$

$$\mathbf{a} = \frac{d}{dt} \mathbf{v}.$$

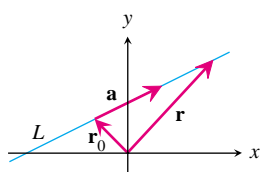
If the position of an object at time t is $\mathbf{r} = \langle t \cos t, t \sin t \rangle$, its velocity and acceleration vectors at t are

$$\mathbf{v} = \langle \cos t - t \sin t, \sin t + t \cos t \rangle$$

$$\mathbf{a} = \langle -2 \sin t - t \cos t, 2 \cos t - t \sin t \rangle.$$

Evaluated at $t = 3\pi/4$, $\mathbf{v} \approx \langle -2.4, -1.0 \rangle$, and $\mathbf{a} \approx \langle 0.3, -3.1 \rangle$.

Parametric Equation of a Line



A parametric equation describing the line through a point with position vector \mathbf{r}_0 and parallel to the vector \mathbf{a} is

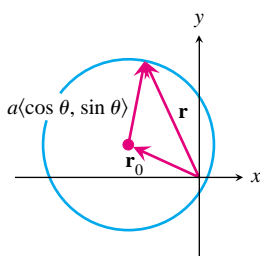
$$\mathbf{r} = \mathbf{r}_0 + s\mathbf{a}, \quad -\infty < s < \infty.$$

A parametric equation describing the line through the point with position vector $\langle -1, 1 \rangle$ and parallel to the vector $\langle \cos 30^\circ, \sin 30^\circ \rangle$ is

$$\mathbf{r} = \langle -1, 1 \rangle + s \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle,$$

$$-\infty < s < \infty.$$

Parametric Equation of a Circle



A circle with center $\mathbf{r}_0 = \langle h, k \rangle$ and radius a is described by

$$\mathbf{r} = \mathbf{r}_0 + a \langle \cos \theta, \sin \theta \rangle,$$

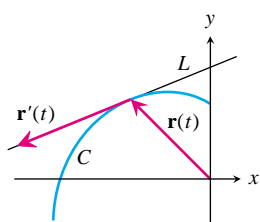
$$0 \leq \theta \leq 2\pi.$$

A parametric equation describing the circle with center $\mathbf{r}_0 = \langle -2, 1 \rangle$ and radius 3 is

$$\mathbf{r} = \langle -2, 1 \rangle + 3 \langle \cos \theta, \sin \theta \rangle,$$

$$0 \leq \theta \leq 2\pi.$$

Curve, Tangent Vector, Slope of Tangent Line



A curve C in an (x, y) -plane is the range of a parametric function

$$\mathbf{r} = \mathbf{r}(t) = \langle x(t), y(t) \rangle, \quad t \in I,$$

where I is an interval and the functions $x = x(t)$ and $y = y(t)$ are continuous on I . In case the functions x and y are differentiable, a tangent vector to C at $\mathbf{r}(t)$ is

$$\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle.$$

For the curve C described by

$$\mathbf{r}(t) = \langle t \cos t, t \sin t \rangle,$$

the tangent vector and slope at $\mathbf{r}(3\pi/4)$ are

$$\mathbf{r}'(3\pi/4) = \left\langle -\frac{\sqrt{2}}{2} - \frac{3\pi\sqrt{2}}{8}, \frac{\sqrt{2}}{2} - \frac{3\pi\sqrt{2}}{8} \right\rangle$$

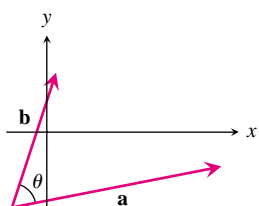
Curve, Tangent Vector, Slope of Tangent Line (continued)

From the Tangent Vector and Slope Theorem, the slope of the tangent line L to C at $\mathbf{r}(t)$ is

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}.$$

$$\begin{aligned} \frac{dy}{dx} &= \left. \frac{dy/dt}{dx/dt} \right|_{t=3\pi/4} \\ &= \frac{3\pi - 4}{3\pi + 4} \approx 0.404. \end{aligned}$$

Dot Product and Angle between Vectors



The dot product of vectors $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$ is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2.$$

The angle θ between \mathbf{a} and \mathbf{b} can be calculated from

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}.$$

Nonzero vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

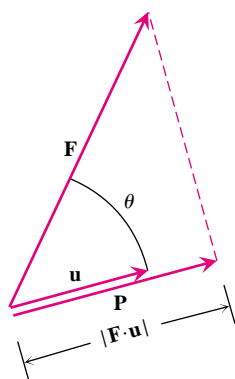
The dot product of the vectors

$$\mathbf{a} = \langle 28.1, 5.4 \rangle \quad \text{and} \quad \mathbf{b} = \langle 6.0, 18.2 \rangle,$$

is $\mathbf{a} \cdot \mathbf{b} = 266.88$. The angle between them is

$$\begin{aligned} \theta &= \arccos\left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}\right) \\ &\approx 61^\circ. \end{aligned}$$

Projection of a Vector on a Unit Vector



The projection \mathbf{P} of a vector \mathbf{F} onto a unit vector \mathbf{u} is

$$\mathbf{P} = (\mathbf{F} \cdot \mathbf{u})\mathbf{u}.$$

The length of this projection is

$$\|\mathbf{P}\| = |\mathbf{F} \cdot \mathbf{u}| = \|\mathbf{F}\| |\cos \theta|,$$

where θ is the angle between \mathbf{F} and \mathbf{u} .

For $\mathbf{F} = \langle 1, 2 \rangle$ and $\mathbf{u} = \langle \cos 15^\circ, \sin 15^\circ \rangle$,

$$\mathbf{P} = (\mathbf{F} \cdot \mathbf{u})\mathbf{u} \approx 1.48\mathbf{u} \approx \langle 1.43, 0.38 \rangle.$$

The length of this projection is

$$\|\mathbf{P}\| \approx 1.48.$$

Chapter Review Exercises

- Give the definitions of average velocity for objects moving along a line and for objects moving in the plane.
- An object is moving on the x -axis and its coordinate positions at $t = 36$ s and $t = 39$ s are $x = 35.8$ m and $x = 24.7$ m. What is its average velocity during this 3-second interval?
- An object is moving in the (x, y) -plane and its position vectors at $t = 5.5$ s and $t = 7.2$ s are $\langle -10, 3 \rangle$ and $\langle 1, 4 \rangle$. What is its average velocity during this 1.7-second interval?
- The position x of the point of contact between a freight car and a snubber (see Section 3.1, Example 3) is

$$x = x(t) = -0.5e^{-0.25t} \sin(1.5t),$$