$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

The chain rule.

$$(f \circ g)'(x) = (f(g(x)))' = f'(g(x))g'(x)$$

or

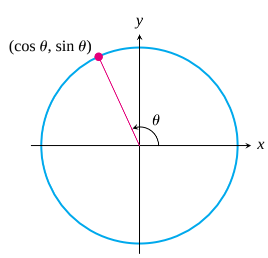
$$\text{If } y = f(u) \text{ and } u = g(x), \text{ then } \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

The Derivatives of the Elementary Functions

Function	Derivative	If u is a function of x:	Function	Derivative	If u is a function of x:
c (a constant)	0	—	$\ln x$	$\frac{1}{x}$	$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$
x^n (n constant)	nx^{n-1}	$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$	a^x $(a > 0, a \neq 1)$	$(\ln a)a^x$	$\frac{d}{dx}a^u = (\ln a)a^u\frac{du}{dx}$
$\sin x$	$\cos x$	$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$	$\log_a x$ $(a > 0, a \neq 1)$	$\frac{1}{x \ln a}$	$\frac{d}{dx}\log_a u = \frac{1}{u \ln a}\frac{du}{dx}$
$\cos x$	$-\sin x$	$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$	$\arcsin x$	$\frac{1}{\sqrt{1 - x^2}}$	$\frac{d}{dx}\arcsin u = \frac{1}{\sqrt{1 - u^2}}\frac{du}{dx}$
$\tan x$	$\sec^2 x$	$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$	$\arctan x$	$\frac{1}{1 + x^2}$	$\frac{d}{dx}\arctan u = \frac{1}{1 + u^2}\frac{du}{dx}$
$\sec x$	$\sec x \tan x$	$\frac{d}{dx}\sec u = \sec u \tan u\frac{du}{dx}$	$\arccos x$	$-\frac{1}{\sqrt{1 - x^2}}$	$\frac{d}{dx}\arccos u = -\frac{1}{\sqrt{1 - u^2}}\frac{du}{dx}$
e^x	e^x	$\frac{d}{dx}e^u = e^u\frac{du}{dx}$	$\operatorname{arcsec} x$	$\frac{1}{\lvert x\rvert\sqrt{1 + x^2}}$	$\frac{d}{dx}\operatorname{arcsec} u = \frac{1}{\lvert u\rvert\sqrt{1 + u^2}}\frac{du}{dx}$

Chapter Summary

Sine and Cosine Functions



Measure angle θ counterclockwise from the positive x-axis. The terminal side of the angle intersects the circle $x^2 + y^2 = 1$ in the point $(\cos\theta, \sin\theta)$.

Because $(\cos\theta, \sin\theta)$ is a point on the circle $x^2 + y^2 = 1$, many identities can be obtained by using properties of the circle. For example

$$\cos^2\theta + \sin^2\theta = x^2 + y^2 = 1.$$

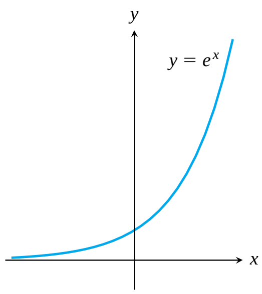
By reflection about $y = x$,

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

and

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta.$$

The Exponential Function



There is a number, denoted by e, such that

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

By exploring with a calculator, we find

$$e \approx 2.718281828459045\ldots.$$

The function exp defined by

$$\exp(x) = e^x$$

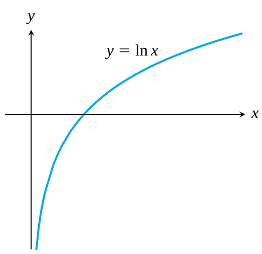
is called the exponential function.

Let $f(x) = ce^{kx}$. Then

$$f'(x) = kce^{kx} = kf(x).$$

Hence the rate of change of f is proportional to f. Many quantities change at a rate proportional to the quantity itself: population, amount of radioactive material, drug level in a body, and so on. Exponentials are essential in modeling such phenomena.

The Natural Logarithm Function



The logarithm of x to the base b is the number s for which $b^s = x$. We use the notation

$$s = \log_b x$$

to describe s. Hence

$$b^{\log_b x} = x.$$

The logarithm to the base e is called the natural logarithm and is denoted ln. Hence for $x > 0$,

$$\ln x = \log_e x.$$

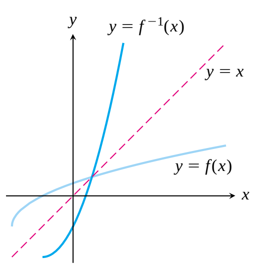
Logarithms satisfy many useful identities. We present some of these for the natural logarithm:

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\ln(x^r) = r \ln x.$$

Inverse Function



Let f be one-to-one on its domain $\mathcal{D}$. A function g is the inverse of f if the domain of g is equal to the range of f and

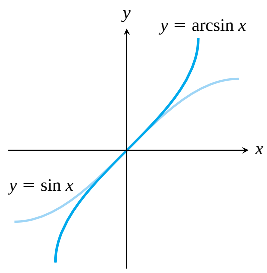
$$(g \circ f)(x) = g(f(x)) = x$$

for all x in $\mathcal{D}$. We write $g = f^{-1}$. If g is the inverse of f, then f is also the inverse of g.

If g is the inverse of f, then the graph of $y = g(x)$ is the reflection about the line $y = x$ of the graph of $y = f(x)$. From this it follows that

$$g'(x) = (f^{-1})'(x) = \frac{1}{f'(g(x))}.$$

The Inverse Sine Function



Let $-1 \leq x \leq 1$. The inverse sine of x is the number θ satisfying

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

and

$$\sin \theta = x.$$

We denote the inverse sine function by arcsin and write

$$\theta = \arcsin x.$$

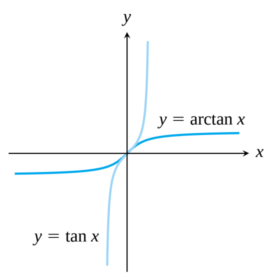
If $-1 \leq x \leq 1$, then

$$\sin(\arcsin x) = x.$$

If $-\pi/2 \leq \theta \leq \pi/2$, then

$$\arcsin(\sin \theta) = \theta.$$

The Inverse Tangent Function



Let x be a real number. The inverse tangent of x is the number θ satisfying

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

and

$$\tan \theta = x.$$

We denote the inverse tangent function by arctan and write

$$\theta = \arctan x.$$

For any real number x,

$$\tan(\arctan x) = x.$$

If $-\pi/2 < \theta < \pi/2$, then

$$\arctan(\tan \theta) = \theta.$$

Chapter Review Exercises

Exercises 1–20: Find the derivative.

1. $y = -4x^5 + 7x^4 - \sqrt{2}x^2 + 3x - \pi$

2. $f(x) = (x^4 + x^2 + 1)^4$

3. $r = \left(\theta^2 - \dfrac{3}{\theta}\right) \sin \theta$

4. $Q(t) = \dfrac{3t^2 - 4t + 7}{-t^2 + 4}$

5. $y = \ln(4x^3 - 2x + 3)$

6. $b(s) = se^{s^2}$

7. $k(p) = \dfrac{p}{\sqrt{p^2 + 3}}$

8. $f(x) = \sqrt{\dfrac{2x - 3}{4x + 5}}$

9. $g(x) = \ln\left(\dfrac{2x - 3}{4x + 5}\right)$

10. $r(s) = 2^{2s}3^{3s}$

11. $y = \sqrt{t} \arcsin t^2$

12. $F(x) = (\sqrt{x} + 1)(\sqrt{x} + 2)(\sqrt{x} + 3)(\sqrt{x} + 4)$

13. $s(t) = \ln(\tan 2t)$

14. $z = \sqrt[3]{\dfrac{e^{2w} + 2w}{e^{-2w} - 2w}}$