large population for one species has an adverse effect on the other species. Suppose that:

a. The number of offspring produced by species A is proportional to $A(t)$, and a similar statement holds for species B.

b. Competition for scarce resources tends to limit the growth of each population. For each species, this effect is proportional to the number of encounters between members of different species.

 i. Based on these assumptions, write equations for dA/dt and dB/dt. Describe how each term in your equations is reflected in the assumptions.

 ii. Are there any circumstances under which both dA/dt and dB/dt are 0? What do you think happens to the populations in this case?

14. (Continuation of Exercise 13) Every member of a species is in competition for resources with every other member of the species. Thus, too many members of species A will have a detrimental effect on the growth of species A.

a. Alter the model in Exercise 13 to obtain a model that accounts for this intraspecies competition.

b. With this new model, are there any circumstances under which both dA/dt and dB/dt are 0? What do you suppose happens to the populations in this case?

15. Consider an environment with two animal populations, rabbits and foxes. Assume that the rabbits are the only food source for the foxes, but that food for the rabbits is plentiful. Let $F(t)$ and $R(t)$ be, respectively, the number of foxes and rabbits at time t. Assume the following:

a. New rabbits are born at a rate proportional to the rabbit population.

b. Rabbits are killed and eaten by foxes at a rate proportional to the number of encounters between rabbits and foxes.

c. Foxes can reproduce only if they have plenty of food. Thus, new foxes are born at a rate proportional to the number of encounters between rabbits and foxes.

d. The fox population is very vulnerable to population pressures. This effect on the growth rate is proportional to the number of foxes.

 i. Based on these assumptions, write expressions for dR/dt and dF/dt. Tell how each term in your equations is reflected in the assumptions.

 ii. Under what conditions are dR/dt and dF/dt both 0? What happens to the populations in this case?

Review of Key Concepts

In this chapter we derived techniques and formulas for finding derivatives quickly and efficiently. We first developed a collection of "general rules" for finding the derivative of functions built from one or more simple functions. Among these were the product rule, the quotient rule, and the chain rule.

Next we reviewed the elementary functions out of which many other functions are built. These include the power functions, the trigonometric functions, the exponential function, the natural logarithm, and the inverse trigonometric functions. We used the definition of the derivative to find the derivatives of many of these functions and then applied the "general rules" developed earlier for others.

If we remember the derivatives of the elementary functions and know how to apply rules such as the product rule and the chain rule, then we can, with practice, quickly write down the derivatives of many functions. These rules and formulas are valuable because with them we can avoid using the definition of derivative every time we need to perform a differentiation. However, it is important to always keep the definition of derivative in mind. The definition suggests many interpretations of the derivative: as a rate of change, as a slope, and so on. These interpretations are important in helping us understand what the derivative means in the context of a problem.

We summarize the derivatives of elementary functions and the general rules in table form. We also provide a review grid for the elementary functions introduced and reviewed in this chapter.

General Rules for Differentiation

Let f and g be differentiable functions.

The derivative of a constant times a function.

Let c be a constant.

$$(cf(x))' = cf'(x)$$

The derivative of a sum or difference of functions.

$$(f(x) + g(x))' = f'(x) + g'(x)$$
$$(f(x) - g(x))' = f'(x) - g'(x)$$

The product rule.

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

The quotient rule.

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

The chain rule.

$$(f \circ g)'(x) = (f(g(x)))' = f'(g(x))g'(x)$$

or

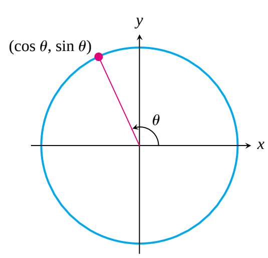
If $y = f(u)$ and $u = g(x)$, then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

The Derivatives of the Elementary Functions

Function	Derivative	If u is a function of x:	Function	Derivative	If u is a function of x:
c (a constant)	0	—	$\ln x$	$\frac{1}{x}$	$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$
x^n (n constant)	nx^{n-1}	$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$	a^x $(a > 0, a \neq 1)$	$(\ln a)a^x$	$\frac{d}{dx}a^u = (\ln a)a^u\frac{du}{dx}$
$\sin x$	$\cos x$	$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$	$\log_a x$ $(a > 0, a \neq 1)$	$\frac{1}{x\ln a}$	$\frac{d}{dx}\log_a u = \frac{1}{u\ln a}\frac{du}{dx}$
$\cos x$	$-\sin x$	$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\arcsin u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$
$\tan x$	$\sec^2 x$	$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$	$\arctan x$	$\frac{1}{1+x^2}$	$\frac{d}{dx}\arctan u = \frac{1}{1+u^2}\frac{du}{dx}$
$\sec x$	$\sec x\tan x$	$\frac{d}{dx}\sec u = \sec u\tan u\frac{du}{dx}$	$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\arccos u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$
e^x	e^x	$\frac{d}{dx}e^u = e^u\frac{du}{dx}$	$\operatorname{arcsec} x$	$\frac{1}{\lvert x\rvert\sqrt{1+x^2}}$	$\frac{d}{dx}\operatorname{arcsec} u = \frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}$

Chapter Summary

Sine and Cosine Functions



Measure angle θ counterclockwise from the positive x-axis. The terminal side of the angle intersects the circle $x^2 + y^2 = 1$ in the point $(\cos\theta, \sin\theta)$.

Because $(\cos\theta, \sin\theta)$ is a point on the circle $x^2 + y^2 = 1$, many identities can be obtained by using properties of the circle. For example

$$\cos^2\theta + \sin^2\theta = x^2 + y^2 = 1.$$

By reflection about $y = x$,

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

and

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta.$$