Exercises 26–29: Use the result of Exercise 25 to find the derivative of the given function.

26. $f(x) = \dfrac{2}{x^3}$

27. $r(\theta) = \theta + 6 + \dfrac{1}{\theta}$

28. $s(t) = 3t^3 - 4t + 7 - 6t^{-1} + \sqrt{2}t^{-5}$

29. $h(r) = \left(1 + \dfrac{2}{r^2}\right)^2$

30. Two balloonists, Sir Bass and Madam Alto, leave the ground in their balloons at the same instant. The height of Sir Bass's balloon at time t minutes is $h(t)$ meters, while the height of Madam Alto's balloon at time t is $2h(t)$ meters. At any given time, what can be said about the speeds at which the two balloons are ascending or descending? Give reasons for your answer.

31. Two gymnasts, Matt and Bart, climb a rope. At time $t = 0$, Matt starts climbing from ground level while Bart starts from 10 feet above ground. It is observed that at all times during the climb, Matt is ascending twice as fast as Bart. At time $t = 20$ minutes, Bart is 200 feet above the ground. How high is Matt at this time? Give reasons for your answer. Interpret the problem in terms of derivatives.

32. In this problem we outline another proof for (9), the rule for the derivative of a sum. If $f'(a)$ exists, then, as seen in (10) of Section 1.7,

$$f(t) = f(a) + f'(a)(t - a) + E_f(t)(t - a)$$

where $\lim_{t\to a} E_f(t) = 0$. Similarly,

$$g(t) = g(a) + g'(a)(t - a) + E_g(t)(t - a)$$

where $\lim_{t\to a} E_g(t) = 0$. Use these expressions to show that

$$\begin{aligned}(f + g)'(a) &= \lim_{t\to a} \frac{(f + g)(t) - (f + g)(a)}{t - a} \\ &= f'(a) + g'(a).\end{aligned}$$

2.2 Derivatives of Products and Quotients

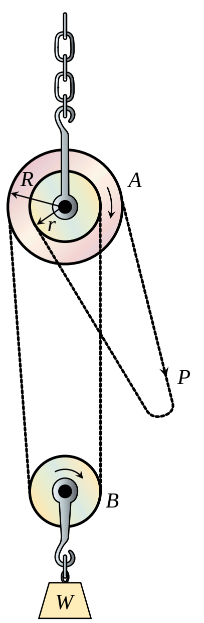

FIGURE 2.4 The chain block. Pull at P to raise the object below wheel B.

The chain block hoist, shown in Fig. 2.4, is an ingenious mechanism that allows people to hoist loads of three or four tons by hand. This device is often found in garages, where it is used to move engines into and out of cars. When the chain is pulled at point P to rotate wheel A through one revolution, wheel B is raised $\pi(R - r)$ units. To lift an object of weight W, one has to pull at P with a force of

$$F = W\frac{R - r}{2Re} \tag{1}$$

where e is the mechanical efficiency of the mechanism (usually about 0.30 because of friction). Note that when R and r are close, it takes very little force to raise the weight, though in this case, the weight is not lifted very far. To understand the fascinating physics behind mechanisms like the chain block, we need to be able to study the derivative of expressions such as (1). See Exercises 29 and 30. Such expressions often involve products and quotients of functions with known derivatives. In this section we see how to differentiate product or quotient functions of the form

$$u(x)v(x) \quad \text{or} \quad \frac{u(x)}{v(x)},$$

provided that we already know the derivatives of u and v.

The Product Rule

Suppose that we know the rates of change of the functions u and v. What can we say about the rates of change of the product uv? The product rule tells us how the derivative of the product is related to the derivatives of the factors.