CHAPTER 2 SUMMARY

KEY TERMS & SYMBOLS

2.1 Graphs of Basic Functions and Relations; Symmetry

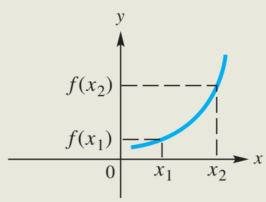
continuity
increasing function
decreasing function
constant function
identity function
degree
squaring function
parabola
vertex
symmetry
cubing function
inflection point
square root function
cube root function
absolute value function
even function
odd function

KEY CONCEPTS

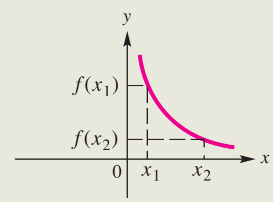
CONTINUITY

A function is **continuous** over an interval of its domain if its hand-drawn graph over that interval can be sketched without lifting the pencil from the paper.

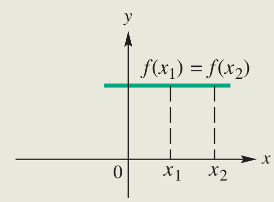
INCREASING, DECREASING, AND CONSTANT FUNCTIONS



When $x_1 < x_2$, $f(x_1) < f(x_2)$.
f is **increasing**.

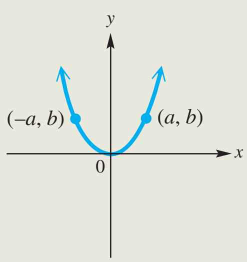
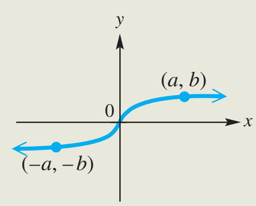
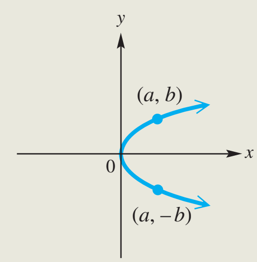


When $x_1 < x_2$, $f(x_1) > f(x_2)$.
f is **decreasing**.



For x_1 and x_2, $f(x_1) = f(x_2)$.
f is **constant**.

TYPES OF SYMMETRY

***y*-axis Symmetry** $f(-x) = f(x)$	**Origin Symmetry** $f(-x) = -f(x)$	***x*-axis Symmetry (not possible for a function)**
		

EVEN AND ODD FUNCTIONS

A function f is called an **even function** if $f(-x) = f(x)$ for all x in the domain of f. (Its graph is symmetric with respect to the y-axis.)

A function f is called an **odd function** if $f(-x) = -f(x)$ for all x in the domain of f. (Its graph is symmetric with respect to the origin.)

BASIC FUNCTIONS AND RELATIONS

- The **identity function** defined by $f(x) = x$ is increasing and continuous on its entire domain, $(-\infty, \infty)$.
- The **squaring function** defined by $f(x) = x^2$ decreases on the interval $(-\infty, 0]$, increases on the interval $[0, \infty)$, and is continuous on its entire domain, $(-\infty, \infty)$.
- The **cubing function** defined by $f(x) = x^3$ increases and is continuous on its entire domain, $(-\infty, \infty)$.
- The **square root function** defined by $f(x) = \sqrt{x}$ increases and is continuous on its entire domain, $[0, \infty)$.
- The **cube root function** defined by $f(x) = \sqrt[3]{x}$ increases and is continuous on its entire domain, $(-\infty, \infty)$.
- The **absolute value function** defined by $f(x) = |x|$ decreases on the interval $(-\infty, 0]$, increases on the interval $[0, \infty)$, and is continuous on its entire domain, $(-\infty, \infty)$.

KEY TERMS & SYMBOLS	KEY CONCEPTS
2.2 Vertical and Horizontal Shifts of Graphs translation	**VERTICAL SHIFTING OF THE GRAPH OF A FUNCTION** If $c > 0$, the graph of $\mathbf{y = f(x) + c}$ is obtained by shifting the graph of $y = f(x)$ *upward* a distance of c units. The graph of $\mathbf{y = f(x) - c}$ is obtained by shifting the graph of $y = f(x)$ *downward* a distance of c units. **HORIZONTAL SHIFTING OF THE GRAPH OF A FUNCTION** If $c > 0$, the graph of $\mathbf{y = f(x - c)}$ is obtained by shifting the graph of $y = f(x)$ to the *right* a distance of c units. The graph of $\mathbf{y = f(x + c)}$ is obtained by shifting the graph of $y = f(x)$ to the *left* a distance of c units.
2.3 Stretching, Shrinking, and Reflecting Graphs	**VERTICAL STRETCHING OF THE GRAPH OF A FUNCTION** If $c > 1$, the graph of $\mathbf{y = c \cdot f(x)}$ is obtained by vertically *stretching* the graph of $y = f(x)$ by a factor of c. In general, the larger the value of c, the greater the stretch. **VERTICAL SHRINKING OF THE GRAPH OF A FUNCTION** If $0 < c < 1$, the graph of $\mathbf{y = c \cdot f(x)}$ is obtained by vertically *shrinking* the graph of $y = f(x)$ by a factor of c. In general, the smaller the value of c, the greater the shrink. **REFLECTING THE GRAPH OF A FUNCTION ACROSS AN AXIS** For a function defined by $\mathbf{y = f(x)}$, **(a)** the graph of $\mathbf{y = -f(x)}$ is a reflection of the graph of f across the x-axis. **(b)** the graph of $\mathbf{y = f(-x)}$ is a reflection of the graph of f across the y-axis.
2.4 Absolute Value Functions: Graphs, Equations, Inequalities, and Applications	**PROPERTIES OF ABSOLUTE VALUE** For all real numbers a and b, **1.** $\|ab\| = \|a\| \cdot \|b\|$. **2.** $\left\|\frac{a}{b}\right\| = \frac{\|a\|}{\|b\|}$ $(b \neq 0)$. **3.** $\|a\| = \|-a\|$. **4.** $\|a\| + \|b\| \geq \|a + b\|$ (the triangle inequality). **GRAPH OF $\mathbf{y = \|f(x)\|}$** The graph of $y = \|f(x)\|$ is obtained from the graph of $y = f(x)$ by reflecting the portion of the graph below the x-axis across the x-axis, and leaving the graph unchanged for the portion on or above the x-axis. **SOLVING ABSOLUTE VALUE EQUATIONS AND INEQUALITIES** To solve $\|ax + b\| = c$, $c > 0$, solve the compound statement $\mathbf{ax + b = c}$ **or** $\mathbf{ax + b = -c}$. To solve $\|ax + b\| < c$, $c > 0$, solve the compound statement $\mathbf{-c < ax + b < c}$. To solve $\|ax + b\| > c$, $c > 0$, solve the compound statement $\mathbf{ax + b > c}$ **or** $\mathbf{ax + b < -c}$.

KEY TERMS & SYMBOLS	KEY CONCEPTS
2.5 Piecewise-Defined Functions piecewise-defined function greatest integer function step function	**PIECEWISE-DEFINED FUNCTION** A piecewise-defined function is defined by different rules over different subsets of its domain. **GREATEST INTEGER FUNCTION** $f(x) = [\![x]\!] = \begin{cases} x \text{ if } x \text{ is an integer} \\ \text{the greatest integer less than } x \text{ if } x \text{ is not an integer} \end{cases}$
2.6 Operations and Composition difference quotient composite function $g \circ f$	**OPERATIONS ON FUNCTIONS** Given two functions f and g, then for all values for which both $f(x)$ and $g(x)$ are defined, the functions $f + g$, $f - g$, fg, and $\frac{f}{g}$ are defined as follows. **Sum** $(f + g)(x) = f(x) + g(x)$ **Difference** $(f - g)(x) = f(x) - g(x)$ **Product** $(fg)(x) = f(x) \cdot g(x)$ **Quotient** $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$ The domains of $f + g$, $f - g$, and fg include all real numbers in the intersection of the domains of f and g, while the domain of $\frac{f}{g}$ includes those real numbers in the intersection of the domains of f and g for which $g(x) \neq 0$. **DIFFERENCE QUOTIENT** $\frac{f(x+h) - f(x)}{h}, \quad h \neq 0$ **COMPOSITION OF FUNCTIONS** If f and g are functions, then the composite function, or composition, of g and f is $(g \circ f)(x) = g[f(x)]$ for all x in the domain of f such that $f(x)$ is in the domain of g.

CHAPTER 2 REVIEW EXERCISES

Concept Check *Draw sketches of the graphs of the basic functions introduced in Section 2.1, avoiding the temptation to look back at them in the text. Also, do not use your calculator. They are:*

$$f(x) = x, \quad f(x) = x^2, \quad f(x) = x^3,$$
$$f(x) = \sqrt{x}, \quad f(x) = \sqrt[3]{x}, \quad f(x) = |x|.$$

Use your sketches to determine whether each statement is true *or* false*. If* false*, tell why.*

1. The range of $f(x) = x^2$ is the same as the range of $f(x) = |x|$.
2. $f(x) = x^2$ and $f(x) = |x|$ increase on the same interval.
3. $f(x) = \sqrt{x}$ and $f(x) = \sqrt[3]{x}$ have the same domain.
4. $f(x) = \sqrt[3]{x}$ decreases on its entire domain.
5. $f(x) = x$ has its domain equal to its range.
6. $f(x) = \sqrt{x}$ is continuous on the interval $(-\infty, 0)$.
7. None of the functions shown decreases on the interval $[0, \infty)$.
8. Both $f(x) = x$ and $f(x) = x^3$ have graphs that are symmetric with respect to the origin.

9. Both $f(x) = x^2$ and $f(x) = |x|$ have graphs that are symmetric with respect to the y-axis.

10. None of the graphs shown is symmetric with respect to the x-axis.

In Exercises 11–18, give the interval that describes the following.

11. domain of $f(x) = \sqrt{x}$

12. range of $f(x) = |x|$

13. range of $f(x) = \sqrt[3]{x}$

14. domain of $f(x) = x^2$

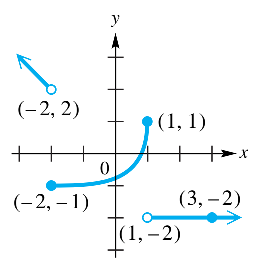
15. the largest interval over which $f(x) = \sqrt[3]{x}$ is increasing

16. the largest interval over which $f(x) = |x|$ is increasing

17. domain of $x = y^2$

18. range of $x = y^2$

19. Consider the function whose graph is shown here.



Give the interval(s) over which the function

(a) is continuous.

(b) increases.

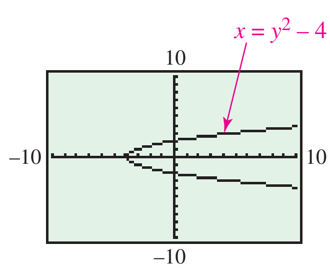
(c) decreases.

(d) is constant.

(e) What is the domain of the function?

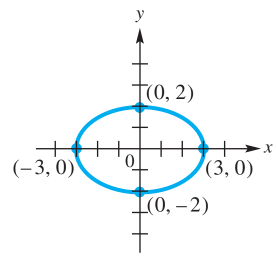
(f) What is the range of the function?

20. The screen shows the graph of $x = y^2 - 4$. Give the two functions that must be used to graph this relation if the calculator is in function mode.



In Exercises 21–26, determine whether the given relation has x-axis symmetry, y-axis symmetry, origin symmetry, *or* none of these *symmetries. (More than one choice is possible.) Also, if the relation is a function, determine whether it is an* even function, *an* odd function, *or* neither.

21.



22. $F(x) = x^3 - 6$

23. $y = |x| + 4$

24. $f(x) = \sqrt{x - 5}$

25. $y^2 = x - 5$

26. $f(x) = 3x^4 + 2x^2 + 1$

Concept Check *Decide whether each statement is* true *or* false. *If* false, *tell why.*

27. The graph of a function (except for the constant function defined by $f(x) = 0$) cannot be symmetric with respect to the x-axis.

28. The graph of an even function is symmetric with respect to the y-axis.

29. The graph of an odd function is symmetric with respect to the origin.

30. If (a, b) is on the graph of an even function, so is $(a, -b)$.

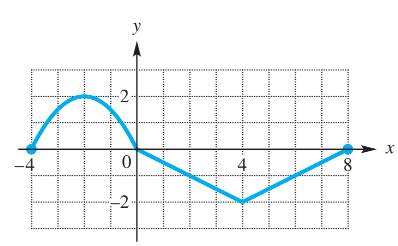
31. If (a, b) is on the graph of an odd function, so is $(-a, b)$.

32. The constant function defined by $f(x) = 0$ is both even and odd.

33. Use the terminology of Sections 2.2 and 2.3 to describe how the graph of $y = -3(x + 4)^2 - 8$ can be obtained from the graph $y = x^2$.

34. Given the equation for the function whose graph is obtained by reflecting the graph of $y = \sqrt{x}$ across the y-axis, then reflecting across the x-axis, shrinking vertically by a factor of $\frac{2}{3}$, and, finally, translating 4 units upward.

The graph of a function f is shown in the figure. Sketch the graph of each function defined as follows.



35. $y = f(x) + 3$

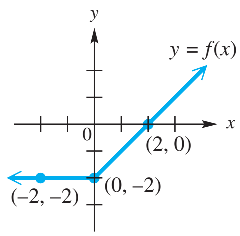
36. $y = f(x - 2)$

37. $y = f(x + 3) - 2$

38. $y = |f(x)|$

The graph of a function defined by $y = f(x)$ is given. Sketch the graph of $y = |f(x)|$.

39.



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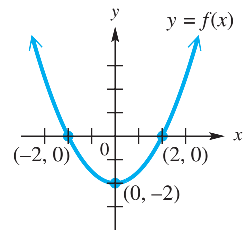

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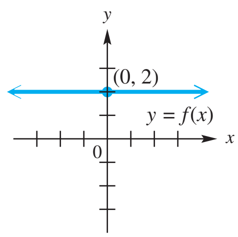

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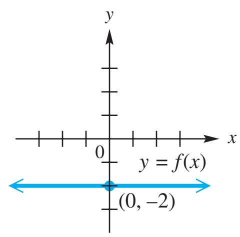


Solve each equation or inequality analytically.

43. $|4x + 3| = 12$

44. $|-2x - 6| + 4 = 1$

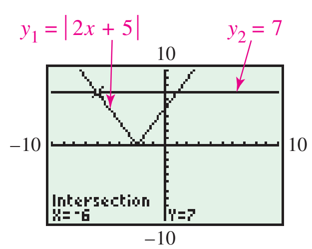
45. $|5x + 3| = |x + 11|$

46. $|2x + 5| = 7$

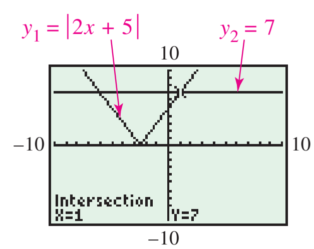
47. $|2x + 5| \le 7$

48. $|2x + 5| \ge 7$

49. The graphs of $y_1 = |2x + 5|$ and $y_2 = 7$ are shown, along with the two points of intersection of the graphs. Explain how these screens support the answers in Exercises 46–48.



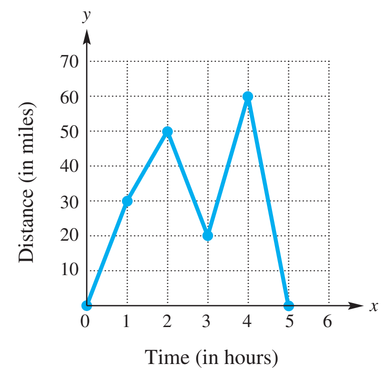
First View



Second View

50. Solve the equation $|x + 1| + |x - 3| = 8$ graphically. Then, give an analytic check by substituting the values in the solution set directly into the left-hand side of the equation.

51. ***(Modeling) Distance from Home*** The graph depicts the distance y that a person driving a car on a straight road is from home after x hours. Interpret the graph. What speeds did the car travel?



52. ***(Modeling) Water in a Tank*** A 500-gallon water tank is initially full, then emptied at a constant rate of 50 gallons per minute. Then the tank is filled by a pump that outputs 25 gallons of water per minute. Sketch a graph that depicts the amount of water in the tank after x minutes.

Sketch the graph of each function by hand.

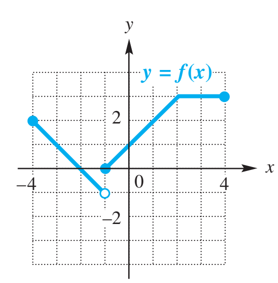
53. $f(x) = \begin{cases} 3x + 1 & \text{if } x < 2 \\ -x + 4 & \text{if } x \ge 2 \end{cases}$

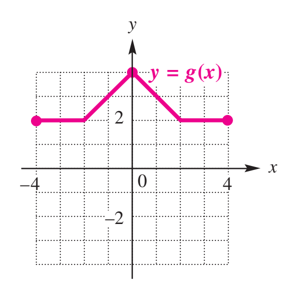
54. $f(x) = \begin{cases} |x| & \text{if } x < 3 \\ 6 - x & \text{if } x \ge 3 \end{cases}$

55. Graph the function in Exercise 53, using a graphing calculator with the window $[-10, 10]$ by $[-10, 10]$.

56. Use a graphing calculator to graph $f(x) = [\![x - 3]\!]$ in the window $[-5, 5]$ by $[-5, 5]$.

The graphs of functions f and g are shown. Use these graphs to find each value.





57. $(f + g)(1)$

58. $(f - g)(0)$

59. $(fg)(-1)$

60. $\left(\dfrac{f}{g}\right)(2)$

61. $(f \circ g)(2)$

62. $(g \circ f)(2)$

63. $(g \circ f)(-4)$

64. $(f \circ g)(-2)$

Use the table to evaluate each expression, if possible.

x	$f(x)$	$g(x)$
-1	3	-2
0	5	0
1	7	1
3	9	9

65. $(f + g)(1)$

66. $(f - g)(3)$

67. $(fg)(-1)$

68. $\left(\dfrac{f}{g}\right)(0)$

Tables for f and g are given. Evaluate each expression.

x	$f(x)$
-2	1
0	4
2	3
4	2

x	$g(x)$
1	2
2	4
3	-2
4	0

69. $(g \circ f)(-2)$

70. $(f \circ g)(3)$

For the given function, find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

71. $f(x) = 2x + 9$

72. $f(x) = x^2 - 5x + 3$

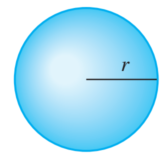
Find functions f and g such that $(f \circ g)(x) = h(x)$.

73. $h(x) = (x^3 - 3x)^2$

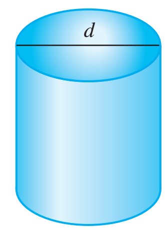
74. $h(x) = \dfrac{1}{x - 5}$

(Modeling) *Solve each problem.*

75. ***Volume of a Sphere*** The formula for the volume of a sphere is $V(r) = \frac{4}{3}\pi r^3$, where r represents the radius of the sphere. Construct a model function D representing the amount of volume gained when a sphere of radius r inches is increased by 3 inches.



76. ***Dimensions of a Cylinder*** A cylindrical can with a top and bottom makes the most efficient use of materials when its height is the same as the diameter of its top.



(a) Express the volume V of such a can as a function of the diameter d of its top.

(b) Express the surface area S of such a can as a function of the diameter d of its top. (*Hint:* The curved side is made from a rectangle whose length is the circumference of the top of the can.)

77. ***Relationship of Measurement Units*** There are 36 inches in 1 yard, and there are 1760 yards in 1 mile. Express the number of inches in x miles by forming two functions and then considering their composition.

78. ***Perimeter of a Rectangle*** Suppose the length of a rectangle is twice its width. Let x represent the width of the rectangle. Write a formula for the perimeter P of the rectangle in terms of x alone. Then use $P(x)$ notation to describe it as a function. What type of function is this?

CHAPTER 2 TEST

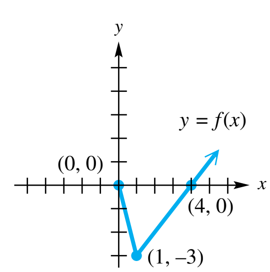
1. Match the set described in Column I with the correct interval notation from Column II. Choices in Column II may be used once, more than once, or not at all.

I	II
(a) domain of $f(x) = \sqrt{x} + 3$	**A.** $[-3, \infty)$
(b) range of $f(x) = \sqrt{x - 3}$	**B.** $[3, \infty)$
(c) domain of $f(x) = x^2 - 3$	**C.** $(-\infty, \infty)$
(d) range of $f(x) = x^2 + 3$	**D.** $[0, \infty)$
(e) domain of $f(x) = \sqrt[3]{x - 3}$	**E.** $(-\infty, 3)$
(f) range of $f(x) = \sqrt[3]{x} + 3$	**F.** $(-\infty, 3]$
(g) domain of $f(x) = \lvert x \rvert - 3$	**G.** $(3, \infty)$
(h) range of $f(x) = \lvert x + 3 \rvert$	**H.** $(-\infty, 0]$

(i) domain of $x = y^2$

(j) range of $x = y^2$

2. The graph of $y = f(x)$ is shown here.



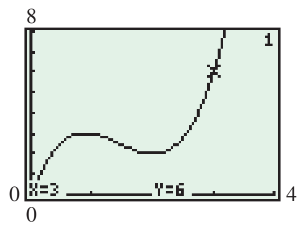
Sketch the graph of each of the following. Use ordered pairs to indicate three points on the graph.

(a) $y = f(x) + 2$ **(b)** $y = f(x + 2)$

(c) $y = -f(x)$ **(d)** $y = f(-x)$

(e) $y = 2f(x)$ **(f)** $y = |f(x)|$

3. Observe the coordinates displayed at the bottom of the screen showing only the right half of the graph of $y = f(x)$. Answer each of the following based on your observation.



(a) If the graph is symmetric with respect to the y-axis, what are the coordinates of another point on the graph?

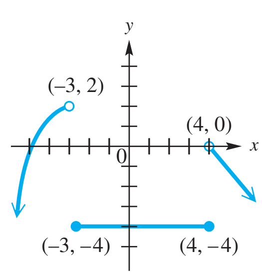
(b) If the graph is symmetric with respect to the origin, what are the coordinates of another point on the graph?

(c) Suppose the graph is symmetric with respect to the y-axis. Sketch a typical viewing window with dimensions $[-4, 4]$ by $[0, 8]$. Then, draw the graph you would expect to see in this window.

4. (a) Write a description of how the graph of $y = 4\sqrt[3]{x + 2} - 5$ can be obtained by translation of the graph of $y = \sqrt[3]{x}$.

(b) Sketch by hand the graph of $y = -\frac{1}{2}|x - 3| + 2$. Give the domain and range.

5. Consider the graph of the function shown here.



(a) Give the interval over which the function is increasing.

(b) Give the interval over which the function is decreasing.

(c) Give the interval over which the function is constant.

(d) Give the intervals over which the function is continuous.

(e) What is the domain of this function?

(f) What is the range of this function?

6. Solve each equation or inequality analytically. Then graph $Y_1 = |4x + 8|$ and $Y_2 = 4$ in the standard viewing window of a graphing calculator, and state how the graphs support your solution in each case.

(a) $|4x + 8| = 4$ **(b)** $|4x + 8| < 4$ **(c)** $|4x + 8| > 4$

7. Given $f(x) = 2x^2 - 3x + 2$ and $g(x) = -2x + 1$, find each of the following. Simplify the expressions when possible.

(a) $(f - g)(x)$ **(b)** $\left(\frac{f}{g}\right)(x)$

(c) the domain of $\frac{f}{g}$ **(d)** $(f \circ g)(x)$

(e) $\dfrac{f(x + h) - f(x)}{h}$ $(h \neq 0)$

8. Consider the piecewise-defined function defined by

$$f(x) = \begin{cases} -x^2 + 3 & \text{if } x \leq 1 \\ \sqrt[3]{x} + 2 & \text{if } x > 1 \end{cases}.$$

(a) Graph it by hand.

(b) Use a graphing calculator to obtain an accurate graph in the window $[-4.7, 4.7]$ by $[-5.1, 5.1]$.

9. *(Modeling) Long-Distance Call Charges* A certain long-distance carrier provides service between Podunk and Nowheresville. If x represents the number of minutes for the call, where $x > 0$, then the function f defined by

$$f(x) = .40[\![x]\!] + .75$$

gives the total cost of the call in dollars.

(a) Using dot mode and window $[0, 10]$ by $[0, 6]$, graph this function on a graphing calculator.

(b) Use the graph to find the cost of a call that is 5.5 minutes long.

10. *(Modeling) Cost/Revenue/Profit Analysis* Tyler McGinnis starts up a small business manufacturing bobble-head figures of famous baseball players. His initial cost is \$3300. Each figure costs \$4.50 to manufacture.

(a) Write a cost function C, where x represents the number of figures manufactured.

(b) Find the revenue function R, if each figure in part (a) sells for \$10.50.

(c) Give the profit function P.

(d) How many figures must be produced and sold before Tyler earns a profit?

(e) Support the result of part (d) graphically.

Chapter 2 Project

Modeling the Movement of a Cold Front

A weather map of the United States on April 22, 1996, is shown in Figure A. A cold front was traveling in a southeast direction, roughly in the shape of a circular arc passing north of Dallas and west of Detroit. The center of the arc was located near Pierre, South Dakota, with a radius of about 750 miles. Rectangular coordinate axes have been superimposed on the map, with Pierre at the origin. Thus Pierre has coordinates $(0, 0)$, and the equation of the front can be modeled by

$$f(x) = -\sqrt{750^2 - x^2},$$

where $0 \leq x \leq 750$. (*Source:* AccuWeather, Inc.)

St. Louis is located at $(535, -400)$, and Nashville is at $(730, -570)$. Figure B shows the graph of f along with these two cities plotted as data points. Notice that the cold front had passed through St. Louis but had not yet reached Nashville.

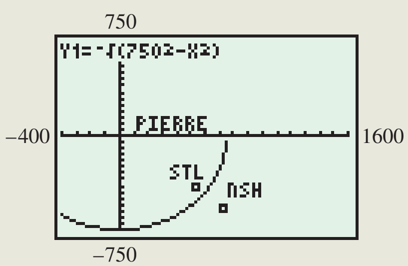


FIGURE B

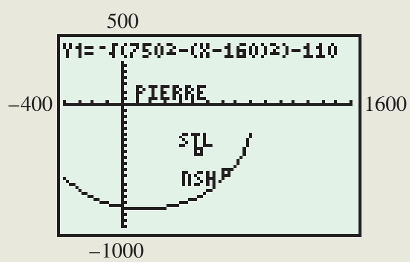


FIGURE C

During the next 12 hours, the center of the front moved approximately 110 miles south and 160 miles east. If we assume that the front did not change shape, its new position can be modeled by

$$g(x) = -\sqrt{750^2 - (x - 160)^2} - 110.$$

We can see from Figure C that the cold front did indeed reach Nashville after 12 hours.

Activity

Suppose that a cold front is passing through the United States at noon with a shape described by the graph of

$$f(x) = \frac{1}{20}x^2.$$

Des Moines, Iowa, is located at $(0, 0)$. See Figure D.

1. If the cold front is moving south at 40 miles per hour for 4 hours and retains its present shape, what would be the equation of its graph at that time?

2. Suppose that by midnight the vertex of the front has moved 250 miles south and 210 miles east of Des Moines, maintaining the same shape. With the class divided into three groups (a), (b), and (c), each group should determine whether the front has reached the designated city.

- **(a)** Columbus, Ohio, which is 550 miles east and 80 miles south of Des Moines
- **(b)** Memphis, Tennessee, which is 190 miles east and 430 miles south of Des Moines
- **(c)** Louisville, Kentucky, which is 420 miles east and 230 miles south of Des Moines