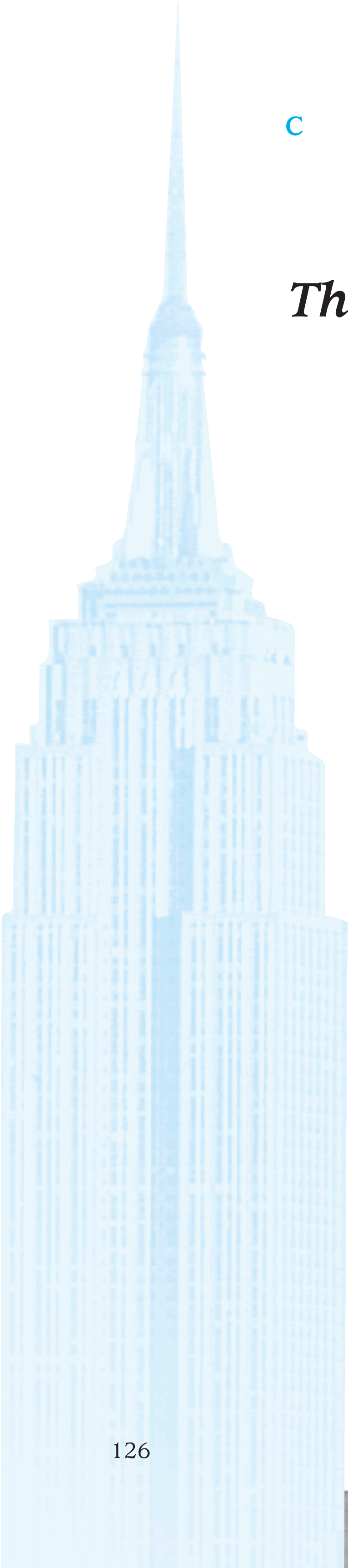


CHAPTER 6

The Time Value of Money

Chapter Objectives

By the end of this chapter you should be able to:

1. Compute how much a sum deposited today will grow to in the future
2. Compute the value today of a sum you will not receive until some future date
3. Compute how much a series of equal payments will grow to over time
4. Compute the value today of a series of equal payments
5. Apply the time value of money methods to answer a variety of questions

When plumbers or carpenters tackle a job, they begin by opening their toolboxes, which hold a variety of specialized tools to help them perform their jobs. The financial manager also needs a toolbox in order to perform the job. In this chapter we begin to fill the toolbox with the skills required to solve many of the financial problems faced by businesses and investors. The particular tools learned here will be applied extensively throughout the text.

The title of this chapter, "The Time Value of Money," implies that money has different values at different times. This point can be demonstrated by asking yourself whether you would rather have $100 now or a year from now. You probably answered that you would rather have it now. But suppose you know that you really do not need it until next year. You would still rather have it now because you could invest the $100 and have more than $100 in 1 year. The point is that your preference depends on when the $100 is received. You probably do not care too much whether the money is paid today or tomorrow, but you probably have a strong preference for receiving it now rather than in 10 years. This means that there

will be a small difference in value if the delay in payment is short, but a large difference if the delay is long.

The time value of money (TVM) depends on the investment opportunities that could be taken if the money were currently available. However, another factor also influences the TVM. Chapter 1 noted that not all cash flows are known with certainty. Future cash flows are subject to fluctuations that can only be estimated in advance. Consider your preference for receiving funds now rather than later. In addition to the fact that you could invest them, it is also possible that the future cash flow may not actually materialize. The fluctuations inherent in business enterprises mean that the money may not be available as promised. With this in mind, it makes sense that the riskier the cash flows, the greater your preference for receiving them now.

Recall that one of the assumptions introduced in Chapter 1 is that the value of money depends on when it is received. In this chapter you will learn to compute precisely how the value of money changes over time. You will use these methods repeatedly throughout the balance of the text. In fact, *of all the tools you will acquire in this course, an understanding of the TVM is the most important.* There are many self-test questions in this chapter. It is important that you have your calculator out to work these problems as you read through each topic. If you are unable to work the self-test problems, go back and review the topic again before proceeding to new material.

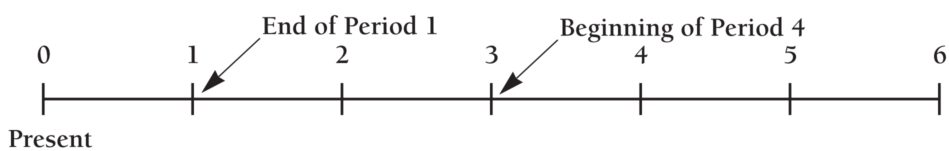
BASICS OF THE TIME VALUE OF MONEY

Graphing Time

Sometimes the English language is less precise, or at least not as efficient, as we might like. For example, what does it mean to say that you received $100 per year for 5 years? Did the money arrive at the beginning, the end, or in the middle of each year? A simple tool to solve this problem is the **time line,** a graphic representation of cash flows. Review the following diagram:

Study Tip

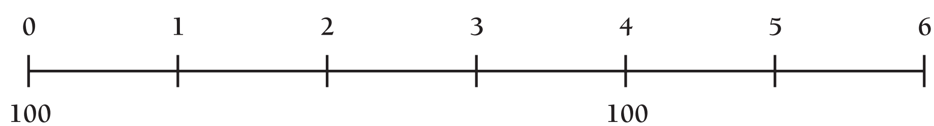
Be particularly careful when putting the cash flows given in a word problem onto a time line. If the problem says the cash flow occurs at the end of the time period, do not put it at the beginning of that time period.



Time 0 is the present. It is also the beginning of the first period. Time 3 is the end of the third and the beginning of the fourth period.

The interval between ticks on the time line can be years, months, weeks, or even days. The cash flow that occurs at each interval is written directly on the time line.

Suppose you will receive $100 at the beginning of period 1 and at the end of period 4. These cash flows are placed on the following time line:



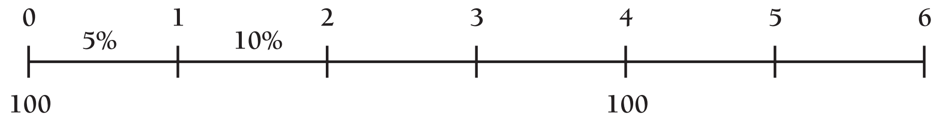
Study Tip

Pay attention to the difference between points in time and periods of time. The numbers on our time line represent points in time, but growth in the value of money occurs during the intervals between the points.

Study Tip

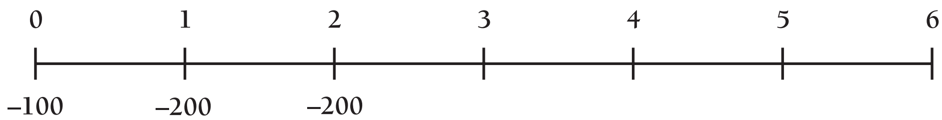
Many students are reluctant to use time lines when solving TVM problems. Although they do take a few extra moments to draw, you will find this time is well spent. Skipping this step often results in having to redo problems because you did the computations using the wrong number of time periods.

If an interest rate is specified, it is placed between the tick marks as in the following diagram:



In this example, the interest rate is 5% during period 1 and 10% during periods 2 through 4. By convention, you do not need to repeat the interest rate between each interval.

The sign of the cash flow is used to indicate whether the cash flow is a cash inflow or a cash outflow. Inflows, cash you receive, have a positive sign (typically the + sign is dropped), as implied in the above examples. A cash outflow, cash you spend, has a negative sign, as shown in the following time line:



In this example, a $100 investment is made at the beginning of the first period and $200 investments are made at the beginning of the second and third periods.

Time lines are useful as an aid to solving TVM problems. They can help keep you organized, especially when the cash flows become complex. Even experienced finance professors rely on them to help visualize problems. To practice using them, we will show how to use time lines as we solve several of the examples in this chapter.

Compound Interest

Simple returns do not earn interest on reinvested interest. A balance is deposited and at the end of the investment period, interest is computed and paid. In the following example, we compute the simple interest earned on a deposit.

Example 6.1 Simple Interest

Compute the simple interest earned on a 1-year $100 deposit that earns 5% per year.

Solution

The interest is computed as the principal amount multiplied by the interest rate:

$$\text{Interest} = \$100 \times 0.05 = \$5$$

Simple interest is easy to compute and understand, but most of the time, we want our interest payments to be reinvested. For example, if you put money into a savings account, the bank automatically deposits the monthly or quarterly interest payments back into the account so that during subsequent periods interest is earned on a higher balance. This is an example of **compound interest,** which is interest earned on interest. Virtually all of the calculations performed in finance require compound interest calculations. We will begin our study of compound interest by finding future balances.

FUTURE VALUE OF A SUM

Annual Compounding

If you make a \$100 deposit into a bank that pays 5% interest once per year, you will have \$105 at the end of one year. We call the amount we have today the **present value** and the future balance the **future value.** In this case, we found the future value by multiplying the \$100 by one plus the interest rate $(1 + i)$. Where did the "1" come from? The "1" adds the original balance back to the interest that has been earned to give the total balance in the account. The formula for computing the balance after one period is

$$FV_1 = PV_0(1 + i)$$
$$\$105 = \$100(1 + 0.05)$$

The process of computing a future balance is called **compounding** because the investor is earning compound interest. Notice the use of the subscripts in the above formula. The subscript denotes the point on the time line when the cash flow occurs.

Now suppose that you leave the above deposit in the bank to compound for another year without withdrawing any money. Using the above formula, the balance grows to \$110.25:

$$FV_2 = FV_1(1 + i)$$
$$\$110.25 = \$105(1 + 0.05)$$

During the first year the account earned \$5, but during the second year it earned \$5.25. The extra \$0.25 was from the interest earned on the first period's interest. We can simplify these calculations by noting that FV_1 is equal to $PV_0(1 + i)$. By substituting $PV_0(1 + i)$ for FV_1 into the above equation, we get

$$FV_2 = PV_0(1 + i)(1 + i)$$
$$FV_2 = PV_0(1 + i)^2$$
$$\$110.25 = \$100(1 + 0.05)^2$$

If the funds are left untouched for a third year, the balance continues to grow:

$$FV_3 = FV_2(1 + i)$$
$$\$115.76 = \$110.25(1.05)$$

Again, by substituting $PV_0(1 + i)^2$ for FV_2, we get

$$FV_3 = PV_0(1 + i)(1 + i)^2$$
$$FV_3 = PV_0(1 + i)^3$$
$$\$115.76 = \$100(1.05)^3$$

Similarly, each subsequent period of compounding increases the exponent by one. The generalized equation for finding the future value of a deposit is given as

$$FV_n = PV_0(1 + i)^n \qquad (6.1)$$

Study Tip

Note the subscripts on the FV and PV variables. These subscripts refer to the time periods. PV_0 means the PV at time 0. FV_1 refers to the FV at time 1.

where

FV_n = the future value of a deposit at the end of the *n*th period
PV_0 = the initial deposit made at time 0
i = the interest rate earned during each period
n = the number of periods the deposit is allowed to compound

Let us illustrate with an example.

EXAMPLE 6.2 Future Value, Beginning-of-Period Deposit

What will the balance be in an account if $1,500 is deposited and allowed to compound for 20 years at 8% with interest being paid annually? How much interest will be earned on the original deposit?

Solution

Substituting these figures into Equation 6.1,

$$FV_n = PV_0(1 + i)^n$$
$$FV_{20} = \$1{,}500(1.08)^{20}$$
$$FV_{20} = \$1{,}500(4.66) = \$6{,}991.44$$

The balance at the end of 20 years will be $6,991.44.
To find the interest earned, we subtract the principal (the original amount) from the ending balance:[1]

$$\text{Interest earned} = FV - PV$$
$$\text{Interest earned} = \$6{,}991.44 - \$1{,}500 = \$5{,}491.44$$

If simple interest, rather than compound interest, had been earned in Example 6.2, $120 per year would have been earned ($1,500 × 0.08 = $120). Twenty times $120 is only $2,400. So $3,091.44 was earned because of compounding ($5,491.44 – $2,400 = $3,091.44). Put another way, $3,091.44 in interest was earned on interest. In this example, more interest was earned on the interest than was earned on the original principal balance! *This is the power of compounding.*

Not all deposits are made at the present time. We sometimes need to compute future balances on deposits that will be made in the future. The method does not change. Simply count the intervals between when the deposit is made and the ending point on the time line to determine the number of periods.

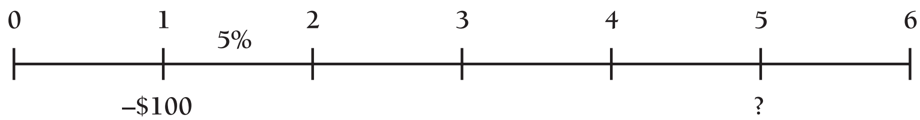
[1]It is easy to compute the value of a sum raised to any exponent by using the **Y^x** key on your financial calculator. In the example given, first enter 1.08 and then press the **Y^x** key. Next enter the number of periods for which compounding will occur, in this case 20. Now press the **=** key. You should get 4.6610. If your calculator only shows two digits to the right of the decimal, you may want to format it. On the TI BA-II Plus you do this by pressing the **2nd** button and then **Format**. Now enter the number of decimal places you want to see and press **Enter**.

EXAMPLE 6.3 Future Value, End-of-Period Deposit

If a deposit of \$100 is made at the end of the current period, what will be the balance in the account at the end of the fifth period if interest is paid annually at 5%?

Solution

To solve this problem, begin by drawing a time line. Note that the initial \$100 investment is given a negative sign because it is a cash outflow (money is going from your pocket into an investment):



To find the number of compounding periods (n), count the intervals during which the deposit can grow. There is one interval between 1 and 2, another between 2 and 3, a third between 3 and 4, and a fourth between periods 4 and 5. Because there are four intervals, $n = 4$. Using Equation 6.1 we find

$$FV_n = PV_1(1 + i)^n$$

$$FV_4 = \$100(1.05)^4$$

$$FV_4 = \$100(1.21) = \$121.55$$

Self-Test Review Questions*

1. You are about to graduate from college at the age of 22. You just learned that your grandfather invested \$10,000 in your name when you were born and it is available for you to withdraw today. If the stock market earned an average 12% over the last 22 years, how much should be in the account today?
2. Suppose that instead of investing the \$10,000 in the stock market, your grandfather invested it in Treasury bonds, which averaged 5.5%. How much would the investment be worth today?

Study Tip

Pay attention to the terminology used in Example 6.3. The initial deposit was made at the end of the first period, yet we still called it the *present value*. The cash flow farthest to the left on the time line is always the PV. The cash flow to the right is always the FV.

Nonannual Compounding Periods

In the above examples, interest was paid once per year. This is called **annual compounding.** Often, interest is paid more often than once per year. Most bank accounts, for example, pay interest every month. We must adjust our formula to allow for any interest payment schedule that may arise.

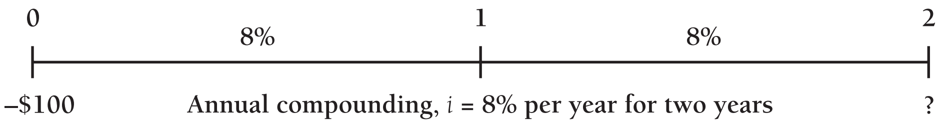
Semiannual compounding occurs when interest is paid every 6 months. To compute the future value under semiannual compounding, recognize that the exponent in the formula for future value is the number of *periods*, not the number of years. Similarly, the interest rate is the interest paid during the *period*, not the annual interest rate. In the last

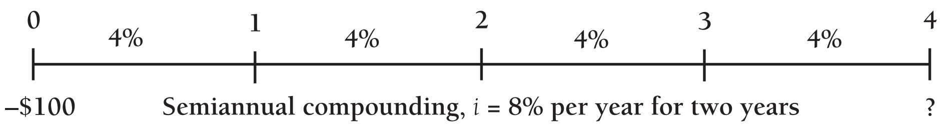
Study Tip

If you skipped the Self-Test Review Questions, *stop* and go back! It is critical that you read this chapter with calculator in hand and work each example and self-test question as they occur.

*1. $FV = \$10,000(1.12)^{22} = \$121,003.10$.
2. $\$10,000(1.055)^{22} = \$32,475.37$. Notice how important the investment choice is to the future value.

section, when we were discussing annual compounding, each period was 1 year so n equaled the number of years and i was the annual interest rate. If we want to compute the future value when each period is 6 months long, we must adjust n and i to reflect the number of 6-month periods and the interest paid during each 6-month period. This is demonstrated by the following time lines:





Suppose that you deposit \$100 for 2 years, and the bank pays interest semiannually at an annual rate of 8%. There are four 6-month periods in 2 years, so n must be multiplied by 2. If the interest rate is 8% for 1 year, then 4% is being paid every 6 months. Substituting these figures into Equation 6.1, we have

$$FV_n = PV_0(1 + i)^n$$

$$FV_2 = \$100(1.04)^4 = \$116.99$$

Notice that the balance at the end of two years is \$0.35 higher under semiannual compounding than it would be with annual compounding [$\$100(1.08)^2 = \$116.64$].

To adjust to semiannual compounding, we multiplied the number of years (n) by the number of periods per year (m) and we divided the annual interest rate by the number of periods per year. Equation 6.2 shows how to adjust for any number of compounding periods per year:

$$FV_n = PV\left(1 + \frac{i}{m}\right)^{mn} \tag{6.2}$$

where

FV_n = the future value of a deposit at the end of the nth year
PV = the initial deposit
i = the annual interest rate
n = the number of years the deposit is allowed to compound
m = the number of times compounding occurs during the year

EXAMPLE 6.4 Multiple Compounding Periods

What is the future value of a \$1,500 deposit after 20 years, with an annual interest rate of 8%, compounded quarterly?

Solution

Substituting these numbers into Equation 6.2,

$$FV_n = PV\left(1 + \frac{i}{m}\right)^{mn}$$

$$FV_{20} = \$1{,}500\left(1 + \frac{0.08}{4}\right)^{4\times 20} = \$1{,}500(1+0.02)^{80} = \$7{,}313.16$$

In Example 6.2 we found that $1,500 deposited for 20 years at 8% compounded annually grew to $6,991.44. It turns out that the more frequently interest is paid, the greater is the future value. Now review Figure 6.1. This figure shows the future value of $100 at 12% compounded at different frequencies. The increase in future value from additional compounding periods increases at a decreasing rate because the length of the compound periods is getting smaller.

The last column shows the future value, assuming continuous compounding. In this case we assume that the number of compounding periods increases into infinity.[2]

Effective Interest Rates

If the bank increases the number of compounding periods, the amount earned on a deposit increases. This implies that a higher **effective interest rate** (EIR) is being earned. The effective interest rate is the amount you would need to earn with annual compounding to be as well off as you are with multiple compounding periods per year. To determine the exact effective interest rate, given multiple compounding periods per year, we find the FV of $1 after 1 year and then subtract the initial dollar. What is left is the earnings for the year, which equals the effective interest rate. The FV of $1 is found by applying Equation 6.2 with PV = 1 and $n = 1$. The effective rate is computed by subtracting 1. Equation 6.3 computes the effective interest rate:

$$\text{Effective rate} = \left(1 + \frac{i}{m}\right)^{m} - 1 \qquad (6.3)$$

[2]The formula for computing the future value, assuming continuous compounding, is $FV_n = PV \times e^{in}$, where $e = 2.71828$, i = annual interest rate, and n = number of years.

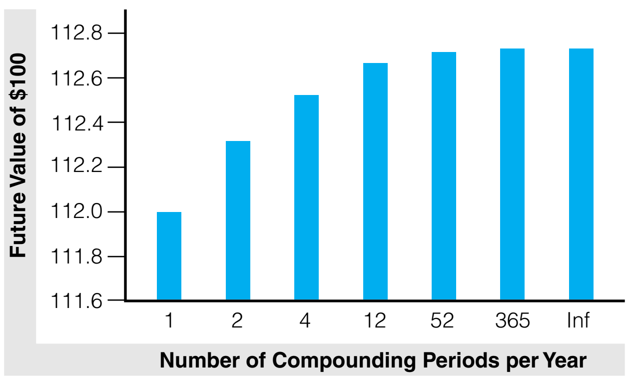


FIGURE 6.1
Effective of Different Compounding Frequencies on Future Value (i = 12%)

EXAMPLE 6.5 Effective Interest Rate Calculation

What is the effective rate of interest if 12% is compounded monthly?

Solution

Applying Equation 6.3 and substituting the numbers, we get

$$\text{Effective rate} = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 0.1268 = 12.68\%$$

Study Tip

Be sure to use the decimal equivalent when computing effective rates. For example, enter .08 into your calculator—for an 8% interest rate.

This result is interpreted to mean that an annually compounded interest rate of 12.68% is equivalent to earning a 12% annual rate that is compounded 12 times per year. Table 6.1 shows the effective rate at different compounding intervals when the annual rate is 12%. As in Figure 6.1, we see that the effective rate increases at a decreasing rate.

Investors are more interested in the effective rate of interest than they are in the annual rate. Suppose that you were attempting to choose between two bank savings accounts. The first pays 5% compounded annually and the second pays 4.9% compounded monthly. Which would you prefer? To answer this, you must compute the effective rate of each alternative and pick the largest one. The effective rate of the first is unchanged, 5%. The effective rate of the second is 5.01%, so you would choose the bank offering 4.9% compounded monthly.

Self-Test Review Questions*

1. What is the future value of $2,000 invested for 20 years with quarterly compounding at 8%?
2. What is the effective rate of 8% compounded quarterly?

Using Financial Tables

In working the examples above, we needed to compute exponents. Many calculators have this function available. An alternative way to solve TVM problems is to use tables

TABLE 6.1 Effective Interest Rates with 12% Annual Rate

Compounding Interval	Equation 6.3	Effective Rate
Annual	$FV = (1.12)^1 - 1$	12.00%
Semiannual	$FV = (1.06)^2 - 1$	12.36
Quarterly	$FV = (1.03)^4 - 1$	12.55
Monthly	$FVC = (1.01)^{12} - 1$	12.68
Weekly	$FV = (1.0023)^{52} - 1$	12.73
Daily	$FV = (1.0003288)^{365} - 1$	12.7475
Continuously	$FV = e^{0.12} - 1$	12.7496

*1. $FV = \$2,000(1.02)^{80} = \$9,750.88$.
2. Effective rate $= (1 + 0.02)^4 - 1 = 8.24\%$.

TABLE 6.2 Future Value Interest Factors (FVIF)

n	4%	5%	6%	7%	8%	9%	10%
3	1.125	1.158	1.191	1.225	1.260	1.295	**1.331**
4	1.170	1.216	1.162	1.311	1.260	1.412	1.464
5	1.217	1.276	1.338	1.403	1.469	1.539	1.611

that have worked a portion of the problem for you. Review Equation 6.1, $FV_n = PV_0(1 + i)^n$. This equation is sometimes written as

$$FV_n = PV_0(FVIF_{n,i}) \tag{6.4}$$

$FVIF_{n,i}$ is the *future value interest factor,* and it is reported in tables at the end of the text and on the inside of the front cover. *$FVIF_{n,i}$ is equal to $(1 + i)^n$*. The tables contain factors for commonly used values of *i* and *n*. A portion of Table A.1, found in the Appendix, is reproduced in Table 6.2. To find the FVIF for $n = 3$ and $i = 10\%$, look down the left column until you find 3. Then look across the top row until you find 10%. The row and column intersect at **1.331**.

EXAMPLE 6.6 Future Value Using FVIF Table

Use the FVIF table to find the future value of \$100 deposited at 10% for five periods with annual compounding.

Solution

The $FVIF_{5,10\%} = 1.611$. Substituting this into Equation 6.4, we get

$$FV_5 = \$100(FVIF_{5,10\%})$$
$$FV_5 = \$100(1.611) = \$161.10$$

This result can be verified by using Equation 6.1:

$$FV_5 = \$100(1.10)^5 = \$161.05$$

The results of using the table and the formula are close, but not exactly the same. The table rounds to the nearest three decimal places, whereas your calculator will probably use at least eight decimal places when the formula is applied.

Study Tip

Many students experience frustration trying to use financial calculators. Additionally, they often make errors when using them on exams. The only way to get past this problem is to work many problems using both financial calculators and the equations. If your answers do not match, it is almost always due to the issues discussed in this section.

Using a Financial Calculator

Financial calculators are available today at very reasonable prices and can ease solving many types of time value problems. It is best to begin by learning to solve the problems using the equations, then to progress to using the calculator. By using the formulas, you learn to adjust for nonannual compounding and other issues that will surface later in the chapter. These issues do not disappear when you begin using a calculator.

A financial calculator is distinguished by a row of buttons that correspond to the inputs to TVM problems. The five most common buttons are

N **I** **PV** **PMT** **FV**

where

N = the number of periods
I = the interest rate per period
PV = the present value or the initial deposit
PMT = the payment (this button is used only when there is more than one equal payment)
FV = the future value

These buttons appear either on the face of the calculator or on the calculator's screen.

There are a few common errors students make when using financial calculators. First, be sure that the number of compounding periods is set correctly. The default is usually 12 periods per year. If the number of periods is set to 12 periods per year, the calculator divides the interest rate by 12 before performing calculations. Most new calculators are preprogrammed to divide the interest rate by 12. Additionally, whenever the batteries are replaced the calculator will revert to dividing by 12. (Check your calculator now and set it to one period per year. Refer to your owner's manual.)[3]

A second common error pertains to the assumption the calculator makes regarding the timing of cash flows. A cash flow can arrive at the beginning or at the end of a period. Most calculators have a way of setting the mode to match the timing. Be sure this mode is set correctly.

There is no consensus regarding whether it is best for students of finance to use calculators, equations, or tables to learn to solve TVM problems. With this in mind, this chapter will show some examples solved all three ways.

Accumulating a Future Balance

Another application of future value is computing the initial deposit required to accumulate a future balance. Suppose you want to have $5,000 in your bank account at the end of 10 years. If you can earn 5% annually, how much must you deposit today? To solve a problem like this, begin by writing down the formula for future value, then plug in the values that you know. For example,

$$FV_n = PV_0(1 + i)^n$$

The future value amount must be $5,000, so

$$\$5{,}000 = PV(1 + 0.05)^{10}$$

Now solve for PV:

$$\$5{,}000/(1.05)^{10} = PV$$

$$\$3{,}069.57 = PV$$

If you deposit $3,069.57 today in an account that pays 5% annually, you will have a balance of $5,000 at the end of 10 years. (Of course, if you invested in the stock market and earned 12% rather than 5%, your future balance would be $9,533.62 [$\$3{,}069.57(1.12)^{10} = \$9{,}533.62$] instead of $5,000.)[4]

[3]To check the number of periods on the TI BA-II plus press the **2nd** key and then the **I/Y** button. Your calculator should display P/Y = 1.00. If not, enter 1 on the keypad and press **Enter**.

[4]The stock market has earned an average of 12% per year over the last 65 years. In some years it suffered losses and in others it earned substantially more than 12%.

A similar application of future value is computing the change in purchasing power due to inflation.

EXAMPLE 6.7 Inflation Adjustment

How much will you need in 20 years to have the same purchasing power that $100 has today, if inflation averages 3% per year?

Solution

This example is a little different from the others in this section because there is no deposit and no future balance to calculate. On the other hand, the concept of future value applies. The $100 will be compounded at 3% for 20 years. To find the equivalent future amount, you can use any of the following methods.

Equation solution:

$$FV_n = PV_1(1 + i)^n$$

$$FV_{20} = \$100(1.03)^{20}$$

$$FV_{20} = \$180.61$$

Table solution:

$$FV_n = PV_1(FVIF_{i,n})$$

$$FV_{20} = \$100(1.806) = \$180.60$$

Calculator solution:

20	3	–$100	0	
N	I	PV	PMT	FV
				= $180.60

If an item costs $100 today and inflation averages 3%, the item will cost approximately $180.60 in 20 years.

Study Tip

if you are entering both a PV and an FV, most calculators will require one to have a negative sign. In this example, the –$100 means that you are spending $100.

Self-Test Review Question*

Suppose that you are trying to decide when you will be able to retire. You decide that if you have the equivalent annual spending power given by $50,000 today, you will be happy. You realize that with inflation you will actually need more than $50,000 every year during your retirement. If you plan to retire in 35 years and if you estimate that inflation will average 4% per year, how much will you need during your first year of retirement?

Solving for Number of Periods and Interest Rates

In all of the examples shown so far we have used the basic future value equation to solve for how much a deposit today can grow to in the future. There are occasions, however, when we already know both the future and present values and want to solve for one of the other variables in the equation. For example, we may want to know how long it will take to accumulate a future balance given a known interest rate and initial deposit. We

*$\$50,000(1.04)^{35} = \$50,000(3.9461) = \$197,304.45$ (You will need nearly $200,000 per year to live as well as you can with only $50,000 in today's dollars.)

may want to solve for what average compounded rate of interest has been earned if we know how long a deposit has been invested and its current balance.

Solving for these different variables is not difficult. Simply identify the future and present value amounts and at least one other variable and plug them into Equation 6.1 or 6.4. Use algebra to solve the equation for the unknown variable. In the next example we solve for the interest rate.

Example 6.8 Future Value, Solving for *i*

Just as you were about to enter college you learned that a great aunt had established a college trust fund for you when you were born 20 years ago. She deposited $5,000 initially. If the balance is now $19,348.42, what average compounded rate of return has been earned?

Solution

This problem can be solved most easily using either financial factor tables or a financial table.

Table solution:

$$\begin{aligned} FV_n &= PV_0(FVIF_{i,n}) \\ FV_{20} &= PV_0(FVIF_{i,20}) \\ \$19{,}348.42 &= \$5{,}000(FVIF_{i,20}) \\ 3.8697 &= (FVIF_{i,20}) \end{aligned}$$

Now go to the FVIF table and look for the FVIF equal to 3.8697 on the row corresponding to 20 periods. When you find the factor, look at the top of the column to determine the interest rate. In this example the interest rate is 7%. The trouble with solving these types of problems using the factor tables is that if the investment rate is not a round number, you will not be able to determine the exact answer.

Calculator solution:

20		–$5000	0	$19,348.42
N	**I**	**PV**	**PMT**	**FV**
	= 7%			

Notice that the sign on the $5,000 initial deposit is negative and the sign on the future value is positive. Some calculators require that the signs be different. Other calculators require that the signs be the same. You will have to experiment with yours to see which method works.

Equation solution:

$$\begin{aligned} FV &= PV(1+i)^n \\ \$19{,}348.42 &= \$5{,}000(1+i)^{20} \\ 3.86968 &= (1+i)^{20} \\ \sqrt[20]{3.86968} &= (1+i) \\ i &= \sqrt[20]{3.86968} - 1 \\ i &= 1.0700 - 1 \\ i &= 7\% \end{aligned}$$

You will have an opportunity to solve TVM problems for the unknown interest rate in Example 6.22.

Intuition Behind Compounding

It is important that the student of finance develop an understanding of how the compounding process works beyond simply applying the equations. Two things should be noted. First, as the *number of compounding periods increases*, the future balance increases. Second, as the *interest rate increases*, the future balance increases. Figure 6.2 shows how the future balance changes given different interest rates and number of compounding periods.

One of the more important features of TVM calculations is that the methods can be applied to anything that grows. In Example 6.7 we showed how the effects of inflation were computed. We can also use the same equations to find future sales, if sales grow at a constant rate. We can estimate future stock dividends as well, if dividends are assumed to grow at a constant rate. The method can be applied to any constant growth situation, whether it be money, sales, profits, populations, or trees.

Self-Test Review Questions*

1. If sales are currently $1,000 per year and are expected to grow at 5%, what are the projected sales 5 years in the future?
2. If the U.S. population is 273 million today and is projected to grow at 3% per year, what will the population be in 35 years?[5]

*1. $\$1{,}000(1.05)^5 = \$1{,}276.28$.
2. $273{,}000{,}000(1.03)^{35} = 768{,}184{,}450$.

[5]We can compute the average compounded growth rate between 1990 and 1999 as well. www.ameristat.org/estproj/basics.htm shows the 1990 U.S. population as 249 million. This data gives us a 1.03% annual growth rate.

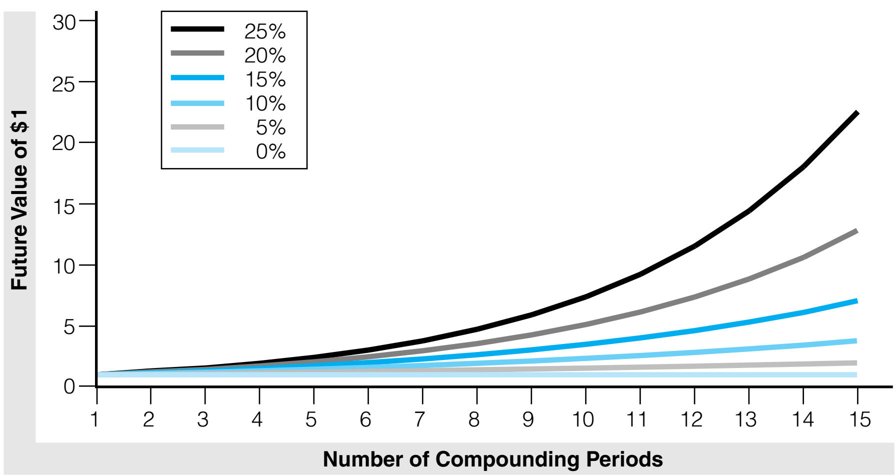


FIGURE 6.2
Future Value of $1 at Different Compounding Periods and Interest Rates

FUTURE VALUE OF AN ANNUITY

In the last section, we computed the future balance of a single deposit or sum. Often, many equal deposits or payments are made. For example, you may make equal monthly deposits into a retirement account. You could find the future value of these deposits by computing the future value of each one separately and then adding them together, but this becomes tedious if there are many deposits. An easier way involves using annuities.

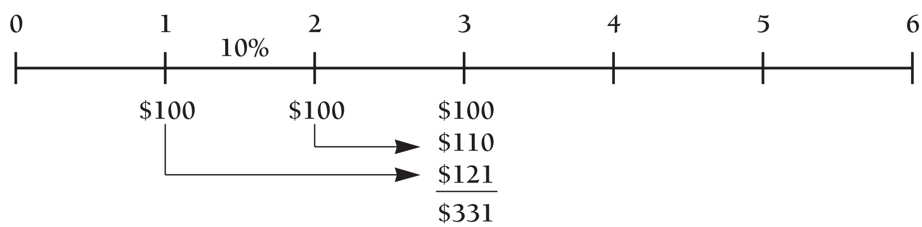
What Is an Annuity?

An **annuity** is a series of equal payments made at equal intervals. Despite being called annuities, annuity payments do not have to be made annually. They can be made monthly, weekly, or even daily. The critical factors are that the payments equal each other and that the interval between each one is the same.

An annuity in which payments are made at the *end* of each period is an **ordinary annuity.** For example, deposits to a retirement account typically are made at the end of each month when you get paid. An annuity in which payments are made at the *beginning* of each period is an **annuity due.** Apartment rental payments usually are due at the beginning of the month, so the annuity is called an annuity due. ***Ordinary annuities are more common than annuities due (as the term implies), so in this book, the annuity may be assumed to be ordinary unless otherwise specified***.

Computing the Future Value of an Ordinary Annuity

Suppose that you want to know the future balance in your account after 3 years if you make three equal annual deposits that each earn 10%. To solve this problem using Equation 6.1 you would find the future value of each deposit, then add them together:



Notice that this is an ordinary annuity because the cash flows are received at the end of the period. Also note that the last payment does not earn any interest. In other words, the last payment is deposited and the balance in the account is immediately checked.

If the total future value is the sum of the future values of the individual payments, the following equation applies:

$$FV_{\text{ordinary annuity}} = PMT_1 + PMT_2(1+i) + PMT_3(1+i)^2 + \cdots + PMT_n(1+i)^{n-1}$$

$$FV_{\text{annuity}} = PMT\sum_{t=1}^{n}(1+i)^{n-t} \qquad (6.5)$$

Unfortunately, Equation 6.5 is not much help in solving annuity problems. Students of algebra will recognize that Equation 6.5 is a geometric progression for which standardized solutions have been found. Annuity problems are best solved using either the tables or a financial calculator because the formulas are fairly complex.[6] (The solutions are listed in Appendix A, Table A.2.)

Financial tables compute the value of

$$\sum_{t=1}^{n}(1+i)^{n-t}$$

These values are called the future value interest factor of an annuity (FVIFA).

If the financial tables are to be used to solve annuities, the formula becomes

$$FV_{\text{annuity}} = PMT(FVIFA_{i,n}) \qquad (6.6)$$

where

PMT = the equal payment made at regular intervals

$FVIFA_{i,n}$ = the future value interest factor of an annuity from Table A.2

EXAMPLE 6.9 FV of Ordinary Annuity, Accumulating a Nest Egg

Suppose that you win the lottery. The winnings consist of 20 equal annual payments of $50,000. You decide to save all of this money for your retirement, and deposit it into an account that earns 8% per year. What is the amount of your retirement nest egg?

Solution

Using the financial tables and applying Equation 6.6, we get

$$FV_{\text{annuity}} = PMT(FVIFA_{i,n})$$
$$FV_{20} = \$50{,}000(FVIFA_{8\%,20})$$
$$FV_{20} = \$50{,}000(45.762) = \$2{,}288{,}100$$

Calculator solution:

20	8	0	–$50,000	
N	I	PV	PMT	FV
				= $2,288,098.21

[6]The factors in Table A.2 are the sum of the geometric progression. The formula is

$$FVIFA_{i,n} = \frac{(1+i)^n - 1}{i}$$

Fortunately, just as compounding makes the effects of inflation seem severe, compounding makes it easier to reach a future goal. Let us look at an example that shows how small regular deposits can accumulate large ending balances.

Study Tip

Without exception, the number of periods in an annuity is equal to the number of payments received. This is true even if payments do not start for several periods. The factors in the tables and the algorithms in the calculators assume that *n* payments are actually made.

EXAMPLE 6.10 FV of Ordinary Annuity, Computing Future Balances

Because we cannot depend on winning the lottery, we must prepare for our retirement with our own funds. What will be in your retirement account after 35 years if you make $2,000 annual deposits that earn (a) 10%, (b) 5%, and (c) 12%?

Solution

Again, we can use the financial tables and apply Equation 6.6, with (a) $i = 10\%$, (b) $i = 5\%$, and (c) $i = 12\%$.

(a) If $i = 10\%$,

$$FV_{annuity} = PMT(FVIFA_{i,n})$$
$$FV_{35} = \$2{,}000(FVIFA_{10\%,35})$$
$$FV_{35} = \$2{,}000(271.024) = \$542{,}048$$

Calculator solution:

35	10	0	–2,000	
N	I	PV	PMT	FV
				= $542,048.74

(b) If $i = 5\%$,

$$FV_{35} = \$2{,}000(FVIFA_{5\%,35})$$
$$FV_{35} = \$2{,}000(90.3203)$$
$$FV_{35} = \$180{,}640.61$$

(c) If $i = 12\%$,

$$FV_{35} = \$2{,}000(431.6635)$$
$$FV_{35} = \$863{,}327.00$$

To solve with $i = 5\%$ and 12% using the calculator, input the variables as you did for 10%, then input $i = 5\%$ and compute FV. Then input $i = 12\%$ and compute FV.

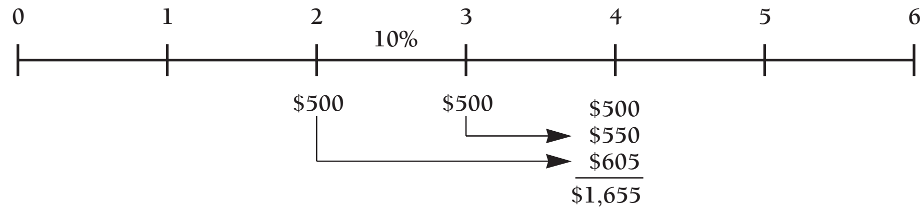
Not all annuities begin with the current period. We sometimes have a delay before the annuity begins paying. For example, if you start a business, it may not begin making a profit for several years. The next example demonstrates how to compute the future value of a **deferred annuity**.

EXAMPLE 6.11 Deferred Annuity

In 2 years you will begin receiving an annual payment of $500 that will be made for 3 years. If the annual interest rate is 10%, what will be the balance in your account at the end of the fourth year?

Solution

Begin by drawing a time line:



The first \$500 deposit compounds for two periods. The second deposit compounds for one period, and the final deposit does not earn any interest. To solve this problem using tables, find the $FVIFA_{10\%,3}$:

$$FV_5 = \$500(FVIFA_{10\%,3}) = \$1{,}655.00$$

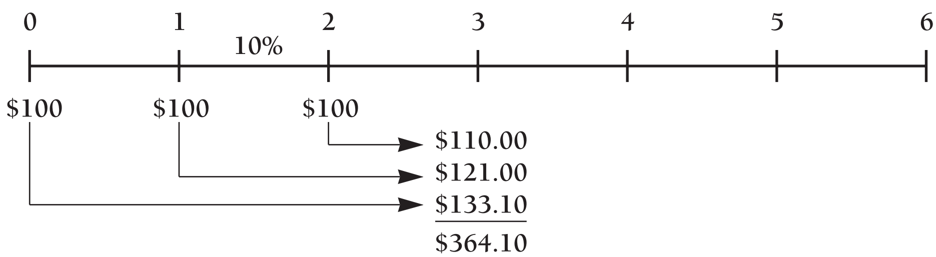
Notice that even though we are computing the balance at the end of the fourth period, we looked up the *three-period* annuity. This is because *three payments* were made.

Computing the Future Value of an Annuity Due

As we noted earlier, sometimes cash flows occur at the *beginning* of a time period rather than at the end. When an annuity's payments occur at the beginning of each period, we call the cash flows an annuity due. The following time line shows a three-period annuity due:

Study Tip

An ordinary annuity is just an annuity due that has been deferred for one period.



Compare this time line with the one shown for a three-period ordinary annuity. Each cash flow earns interest one period longer in this example. For instance, the final cash flow is compounded one period whereas the ordinary annuity's final cash flow is not compounded at all. Similarly, the first and second cash flows are compounded for one more period.

The tables at the end of the book for computing future values and the default settings on your calculators all assume ordinary annuities with end-of-period cash flows. Most of the time you will want to use either tables or a calculator to solve annuity problems, even if it is an annuity due. ***To compute the future value of an annuity due, multiply the future value of an ordinary annuity by (1+i). This compounds the cash flows for one more period***.

EXAMPLE 6.12 Annuity Due

Compute the future value of the annuity due shown on the above time line, assuming i = 10%.

Solution

The future value of a three-period ordinary annuity of $100 is found by multiplying $100 times the $FVIFA_{10\%,3}$:

$$FV_{\text{ordinary annuity}} = \$100 \times (3.31) = \$331$$

Now, multiply the result by 1.10:

$$FV_{\text{annuity due}} = FV_{\text{ordinary annuity}} \times (1 + i)$$

$$FV_{\text{annuity due}} = \$331 \times (1.10) = \$364.10$$

Study Tip

If you choose to change the mode of your calculator to compute annuities due, remember to change it back or all future annuity problems will be computed in that mode.

To solve this problem using a financial calculator, compute the future value as if the cash flows were for an ordinary annuity, then multiply by 1.10. An alternative calculator solution is to change your financial calculator from *end* to *begin* mode. The default mode on most calculators is *end,* which means that the calculator will treat the cash flows as if they occur at the end of each period. When the mode is switched to *begin,* the calculator treats the cash flows as if they occur at the beginning of each period. (Refer to your owner's manual for directions on how to change the mode of your particular calculator.)

Let us look at a slightly more complex example.

Study Tip

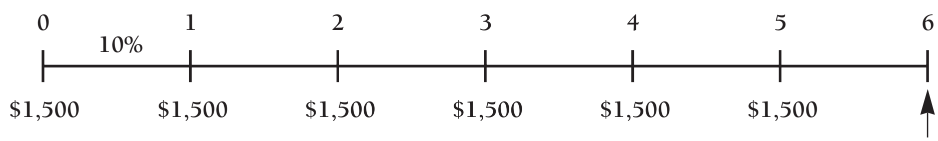
The tables and your calculator will compute the value of an annuity at the point in time when the *final payment of the annuity occurs.* In Example 6.13, this is at the end of period 5. Ordinary annuities always provide a value at this point. The annuity due adjustment simply computes the FV one period later.

EXAMPLE 6.13 Annuity Due

Cash flows of $1,500 occur at the beginning of periods 1, 2, 3, 4, 5, and 6. What is the future value of this cash flow at the end of period 6, assuming i = 10%?

Solution

First draw a time line as follows:



We want to compute the future value of the cash flows where the arrow points. We can do this by computing the future value of an ordinary annuity, which will give us the value at the end of period 5, and then multiplying by 1.10:

$$FV_{\text{annuity due}} = \$1{,}500(FVIFA_{10\%,6\text{ yr}})(1.10)$$

$$FV_{\text{annuity due}} = \$1{,}500(7.716)(1.10) = \$12{,}731.40$$

PRESENT VALUE OF A SUM

Often we need to determine what the value is today of sums that will be received in the future. For example, if a firm is evaluating an investment that will generate future income, it must compare today's expenditures with expected future revenues. To compare sums across time, future values must be adjusted to what they are worth today.

There are many applications of present value methods. Assets are valued as the present value of the cash flows they generate. For example, stock is valued as the present value of the future cash flows the investor expects to receive, and bonds are valued as the present value of the interest payments plus the maturity payment. Capital investments are evaluated by subtracting the present value of cash expenditures from the present value of cash revenues generated by the investment. Loan payments are found by applying present value techniques. These applications make this section particularly important to grasp.

Luckily, if the concepts discussed under future value make sense, it will not be difficult to understand present value.

Study Tip

Go back and review future value concepts in this chapter if you have doubts about your understanding so far. TVM concepts build on one another. You must master each level before moving onto the next.

Present Value Equation

We already derived Equation 6.1, which establishes the future value of a deposit. Take another look at this equation:

$$FV_n = PV_0(1 + i)^n$$

Future value has been defined in terms of present value. To find the equation for computing present value, we only need to rearrange the terms in the above equation. If both sides are divided by $(1 + i)^n$, we get the equation for present value:

$$PV_0 = \frac{FV_n}{(1+i)^n} \quad \text{or} \quad PV_0 = FV_n\left[\frac{1}{(1+i)^n}\right] \tag{6.7}$$

The process of computing the present value of a future sum is called **discounting**. Remember that a future sum is not worth as much as a sum you have today. To convert future amounts to their present values, the future amount must be reduced, or *discounted*.

Study Tip

It is important to keep the terminology straight with regard to TVM. *Compounding* refers to the process of computing *future* values. *Discounting* refers to the process of computing *present* values. Further, the **discount rate** is the interest rate used when discounting.

EXAMPLE 6.14 Present Value

In 1 year John expects to receive $100 in repayment of a loan. He really needs the money now, however. How much would you give him today if he were to transfer the future payment to you? Assume that you have investment opportunities available at 10% that have similar risk.

Solution

You will pay John the present value of the future amount. Applying Equation 6.7,

$$PV_0 = \frac{FV_1}{(1 + i)^1}$$

$$PV_0 = \frac{\$100}{1.10} = \$90.91$$

You would pay $90.91 for the right to receive $100 in 1 year. You would pay this much because if you invested the $90.91 it would be worth $100 in one year:

$$\$90.91(1.10) = \$100$$

We are devoting a great deal of time to learning present value techniques. One reason for this emphasis is that many investment decisions are made by converting all relevant cash flows into present value terms. In the next example an investment opportunity is evaluated.

EXAMPLE 6.15 Present Value

Your great-grandmother has left you $100,000 in a trust fund that you cannot have for another 5 years. You have decided that you really need this money now to pay for your college expenses. You discuss your problem with your attorney, who offers you $75,000 for an assignment of the proceeds of the trust. If you can get a student loan at 8%, should you accept your attorney's offer?

Solution

This problem can be solved two ways. You can find the future value of $75,000 and see whether it is more or less than $100,000, or you can find the present value of the $100,000 and see whether it is more or less than the $75,000. Using the present value method,

$$PV_0 = \frac{FV_5}{(1+i)^5}$$

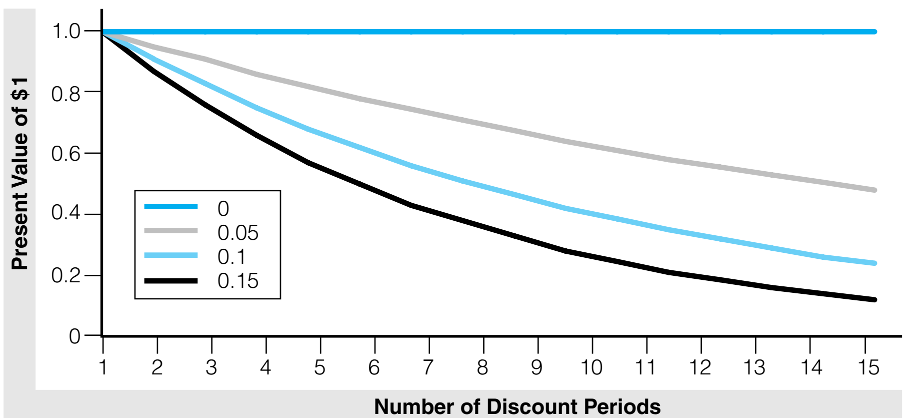
$$PV_0 = \frac{\$100{,}000}{(1.08)^5} = \frac{\$100{,}000}{1.4693} = \$68{,}059.62$$

Because the $100,000 trust payment is worth only $68,059.62 to you today, you would be happy to receive $75,000 for it.

You can think of discounting as moving dollars to the left on the time line. Whenever dollars are moved to the left, their value gets smaller. The further to the left it moves and the higher the interest rate, the less the dollars are worth. Figure 6.3 shows the relationship between the interest rate, the number of periods, and the present value of $1.

Figure 6.3 shows that as you discount money by moving it backward in time (to the left on the time line) the value of $1 decreases. As always, if you compound money by investing it, the value of $1 increases.

FIGURE 6.3
Present Value of $1 at Different Discount Rates and Compounding Periods



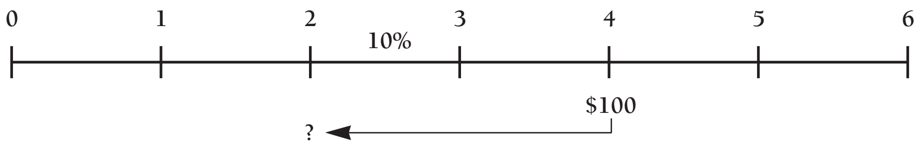
You do not always want to move cash flows all the way to time period 0. Present value calculations include moving future sums to any earlier period.

EXAMPLE 6.16 Present Value, Deferred Cash Flows

Compute the value of \$100 at the end of the second period. Assume that it will be received at the end of the fourth period and there is a 10% interest rate.

Solution

Begin by drawing a time line:



There are two periods for the \$100 to discount. PV_2 is the value at the end of the second period, which is computed using Equation 6.7:

$$PV_2 = \frac{\$100}{(1.10)^2} = \$82.64$$

Using PVIF Tables

Financial tables have been computed to make present value calculations easier. Review the present value equation. The right-hand side can be separated into two terms:

$$PV = \frac{FV_n}{(1+i)^n}$$

$$PV = FV_n \times \left[\frac{1}{(1+i)^n}\right]$$

Table A.3 in the Appendix contains the expression in parentheses for various commonly used interest rates and periods. When the PV interest factors from the tables are used, the present value equation becomes

$$PV = FV_n(PVIF_{i,n}) \quad (6.8)$$

Study Tip

Students often use the wrong table when under the stress and time pressure of a test. There are two things you can do to combat this problem. When doing homework and studying examples, get used to the normal values of the factors. For example, PVIFs are always less than 1 and FVIFs are always greater than 1. Second, take a highlighter and label each table in broad 4-inch letters.

EXAMPLE 6.17 Present Value, Using Factors

What is the present value of a \$500 sum that will be received in 6 years if the interest rate is 8%?

Solution

Using Equation 6.8 and Table A.3, we get

$$PV = \$500(PVIF_{8\%,6})$$

$$PV = \$500(0.630) = \$315.00$$

Intuition Behind Present Values

We will use the present value concept often throughout the balance of this course. It is important for your future success in finance that you understand the intuition behind the concept.

What does it mean that the present value of $100 to be received in 1 year at 10% is $90.91? ***It means that you are indifferent which you get: the $90.91 today or the $100 in 1 year***. What if you do not really need the money today but expect to need it in 1 year? Then you can invest the $90.91 today and it will grow to be exactly $100 by the time you need it. The interest rate used to discount the $100 back to the present is selected so that you will be properly compensated for the risk that you may not be paid and for the delay in receiving the cash flow.

Still another way of defining present value is that if you have the present value of a future sum, ***you can exactly match that future sum by investing what you have today at the discount rate***.

Present Value of Mixed Streams

Thus far, we have focused on computing the present value of a single, lump sum future cash flow. However, there are many occasions when you must find the present value of a series of *unequal cash flows*. For example, if you are evaluating an investment in a business, it is unlikely that each year's cash flows will be the same. To find the present value of mixed streams, simply find the present value of each cash flow individually, then add them together.[7]

Study Tip

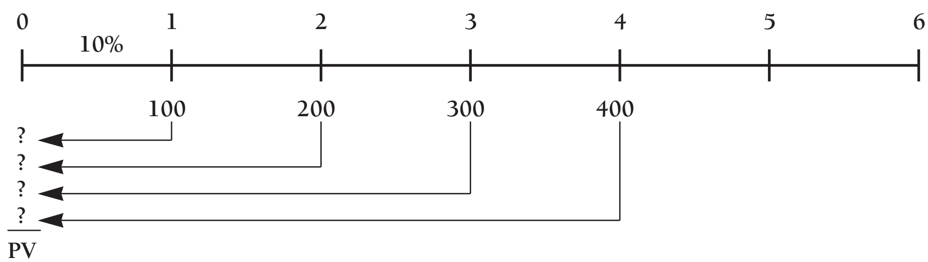
If you solve this problem by calculator, your answer may display as –$754.80. Your calculator will always give the opposite sign on the PV as you have on the FV. Simply ignore the negative sign in this case.

EXAMPLE 6.18 Present Value of Mixed Streams

You are contemplating introducing a new product line that is expected to generate cash flows of $100 at the end of the first year, $200 at the end of the second year, $300 at the end of the third year, and $400 at the end of the fourth year. What is the present value of the cash flows if the appropriate interest rate is 10%?

Solution

Begin by drawing a time line:



[7]If you have an advanced financial calculator such as the TI BA-II plus or the HP-10, you can enter the cash flows and the interest rate to compute the present value. Refer to your calculator's owners manual under NPV calculations.

$$PV = \$100(PVIF_{10\%,1}) + \$200(PVIF_{10\%,2}) + \$300(PVIF_{10\%,3}) + \$400(PVIF_{10\%,4})$$

$$PV = \$100(0.909) + \$200(0.826) + \$300(0.751) + \$400(0.683)$$

$$PV = \$754.60$$

The calculator solution is $754.80.

Self-Test Review Question*

Suppose that you have estimated the cost of your room, board, and tuition for the next 4 years of your education (you have decided to get your MBA) to be as follows: $5,000 due at the end of the first year, $5,500 at the end of the next year, $6,000 at the end of the third year, and $6,500 at the end of the fourth year. Assume a discount rate of 7%. How much must you invest today to pay for your future education?

Increasing the Compounding Periods

Computing present values with multiple compounding periods per year is very similar to computing future values with multiple compounding periods per year. The annual interest rate is divided by the number of periods per year to find the interest rate per period. The number of years is multiplied by the number of periods per year to get the number of periods the future value will be discounted. Present value with multiple compounding periods is computed as

$$PV = \frac{FV_n}{\left(1 + \frac{i}{m}\right)^{mn}} \qquad (6.9)$$

where

m = number of compounding periods per year
n = number of years
i = annual interest rate

On occasion you may need to find the present value of a sum that will be received in a few days or months. The concept remains the same. Convert the interest rate to the interest rate in effect for that period and raise the denominator by the number of periods.

EXAMPLE 6.19 Present Value with Monthly Compounding

H&R Block offers taxpayers instant tax return refunds for a fee. Suppose you had $1,000 coming from the IRS and you were offered an instant refund for a $35 fee. Would you take the

*Find the present value of each cash flow separately and add them together: ($5,000/1.07) + ($5,500/1.07^2) + ($6,000/$1.07^3$) + ($6,500/1.07^4) = $19,333.42. If you have $19,333.42 now and invest at 7%, you will be able to pay for the balance of your education.

instant refund if you could borrow $1,000 at 12% and if you expected the IRS to pay you in 2 months?

Solution

The first step is to convert the annual interest rate into a monthly interest rate. This is done by dividing 12% by 12. The monthly interest rate is 1%. Next plug the monthly interest rate and the number of periods into Equation 6.9:

$$PV = \frac{\$1{,}000}{\left(1 + \frac{0.12}{12}\right)^{12 \times \frac{2}{12}}} = \frac{\$1{,}000}{(1.01)^2} = \$980.30$$

This problem can also be solved using the factors where the interest rate is the interest rate per month and the number of periods is the number of months:

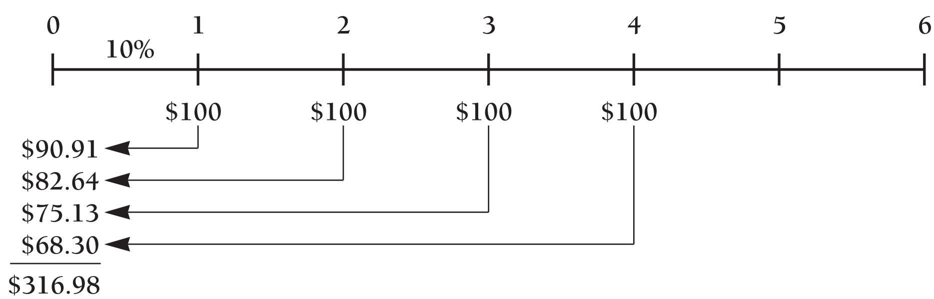
$$PV = \$1{,}000(PVIF_{1\%,2})$$
$$PV = \$1{,}000(0.98) = \$980.00$$

Your $1,000 future tax refund is worth $980 today. If you take the instant refund, you are essentially selling the refund today for $965 ($1,000 − $35 = $965). Because you are a rational investor, you would not sell something worth $980 for $965, so you would reject H&R Block's offer.

PRESENT VALUE OF AN ANNUITY

Just as we sometimes need to find the future value of a stream of equal cash flows, we also may need to find the *present value* of a stream of equal cash flows. If the cash flows are different from each other, there is no shortcut other than using a financial calculator. If the cash flows are equal to each other and occur at regular intervals, we can find the present value using the present value of annuity tables.

Review the following time line:



One way to find the present value of the annuity shown on the time line is to find the present value of each cash flow separately and then add them together, as shown. Obviously, if there are many cash flows, this can become tedious and time consuming. The solution to the above problem could be written as

$$PV = \$100(PVIF_{10\%,1}) + \$100(PVIF_{10\%,2}) + \$100(PVIF_{10\%,3}) + \$100(PVIF_{10\%,4})$$

The \$100 can be factored out of each term in the equation. The solution is then rewritten as

$$PV = \$100 \times (PVIF_{10\%,1} + PVIF_{10\%,2} + PVIF_{10\%,3} + PVIF_{10\%,4})$$

This is another example of a geometric progression that has a standardized solution. The solution to the equation for various interest rates and number of periods has been computed and appears in Appendix A, Table A.4, at the end of the book.[8] Using these factors, the equation for the present value of an annuity can be written as

$$PV_{annuity} = PMT(PVIFA_{i,n}) \tag{6.10}$$

where PMT = the amount of the equal annual payments.

EXAMPLE 6.20 Present Value of Annuities

Find the present value of the cash flow stream on the above time line using Equation 6.10.

Solution

The payment is \$100, n is 4, and i is 10%. Plugging these figures into Equation 6.10 gives us

$$PVIFA = \$100(PVIFA_{10\%,4})$$
$$PVIFA = \$100(3.170) = \$317.00$$

Financial calculators are most valuable when used to solve annuity problems. The problem with using tables is that, although they are easy to use, they limit you to whole interest rates and certain numbers of periods. By using the financial calculator, you can solve problems with present values of fractional interest rates, as well as any number of periods. To solve the above problem with a financial calculator, input the data as follows:

4	10		–\$100	0
N	I	PV	PMT	FV
		= \$316.99		

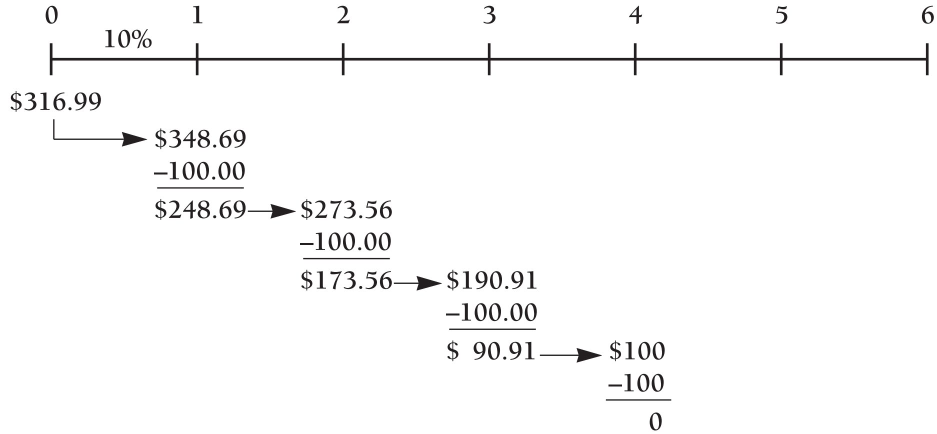
In the last section we noted that the present value of a future sum can be used to exactly match the future sum by investing what you have today at the discount rate. We can reemphasize this important point using the present value of an annuity.

We have already computed that the present value of a four-period stream of equal \$100 cash flows is \$316.99. This means that if we have \$316.99 to invest today we can exactly reproduce the future stream of cash flows. You would invest the \$316.99 at 10% and have \$348.69 1 year later. The first \$100 cash flow is then subtracted from the balance, which is reinvested. One year later the balance is \$273.56 and an additional \$100

[8] The PVIFAs (present value interest factors for annuities) in Table A.4 are the solutions for the following equation:

$$PVIFA = \frac{1 - \left[1/(1+i)^n\right]}{i}$$

is subtracted. This process continues as shown below until the final \$100 is subtracted and the remaining balance is zero:



This example provides an important clue as to when you will want to use present value equations. Anytime you want to know how much you need today to create a future cash flow stream, find the present value of the cash flow. For example, in Box 6.1, present value and future value methods are used to compute how much must be saved to achieve a particular retirement goal.

Box 6.1 Will You Retire a Millionaire?

Have you ever asked yourself whether it is likely that you will end up a millionaire by the time you retire? If you are pursuing a degree that offers career opportunities with modest salaries and if you are not planning to go into business for yourself, you may feel that the chances are slim. Let us review this question in a little more detail.

Suppose you determine that you will need the spending power provided today by \$50,000 every year during your retirement. However, we expect inflation to erode the spending power of the sum, so we must adjust for inflation. If inflation averages 3% over the next 35 years, you will need \$140,693 during the first year after you retire ($\$50{,}000 \times 1.03^{35} = \$140{,}693$). If inflation continues during your retirement years, you will need \$286,000 during your 25th year of retirement to have the same spending power provided by \$50,000 today.

To determine how much you will need in your retirement nest egg to finance a series of annual withdrawals, we need to find the present value of the series of cash flows. Remember that the present value of a cash flow stream is sufficient to exactly reproduce that income stream if the present value amount is invested at the discount rate. In this case, using an Excel spreadsheet that first computes the annual inflation-adjusted cash flow each year and then computes the present value of that amount, we find we need a nest egg of \$1,621,493, assuming we can invest at 10%.

Small changes in our assumptions make a big difference in how much money we will need to retire. Suppose that inflation averages 4% instead of 3% and that we invest the nest egg at 7% instead of 10%, because most retirees are conservative. Under this set of assumptions we will need \$2,386,258 in our nest egg.

Clearly, you will need to be a millionaire by the time you retire. In fact, to be safe you will probably want to be a multimillionaire. Because of the effects of compounding, it turns out that this is not as farfetched as it may seem. If you invest \$371.06 per month at an average rate of return of 12%, you will have \$2,386,258 in your nest egg 35 years later. Because most companies match their employees' contributions to their retirement plans, you would only need to invest half of \$371.01, or \$185.53 per month to meet your goal.

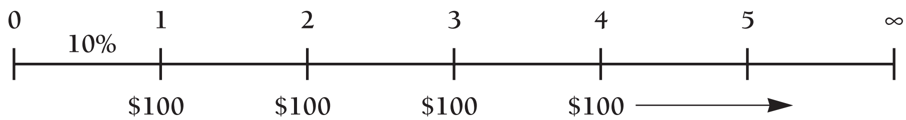
Self-Test Review Question*

You have just won the lottery and will receive equal payments of $50,000 at the end of each year for the next 20 years. Your attorney offers to pay you $500,000 today if you will agree to give him the stream of payments. If you think that you can invest at 10%, should you accept this offer?

PERPETUITIES

Annuities are equal payments that go on for a fixed number of periods. If the annuity continues forever, it is a **perpetuity.** The term *perpetuity* comes from the word *perpetual*, which means continuous or indefinitely. The cash flows received from preferred stock are an example of a perpetuity. Preferred dividends are fixed at a known level at the time the stock is issued. The dividends do not rise or fall with the fortunes of the firm. Because stock never matures, the cash flows are perpetual.[9]

The cash flows from a perpetuity are illustrated by the following time line:



To find the present value of this perpetuity algebraically, we would begin with the expression

$$PV_0 = \frac{\$100}{(1.10)^1} + \frac{\$100}{(1.10)^2} + \frac{\$100}{(1.10)^3} + \frac{\$100}{(1.10)^4} + \cdots$$

Because the $100 appears in every term, it can be factored out:

$$PV_0 = \$100\left[\frac{1}{(1.10)^1} + \frac{1}{(1.10)^2} + \frac{1}{(1.10)^3} + \frac{1}{(1.10)^4} + \cdots\right]$$

The term in parentheses is a geometric series. The solution to the sum of this endless geometric series is

$$PV_0 = \frac{\$100}{0.10} = \$1{,}000$$

[9]Most preferred stock has fixed dividends and does not mature, but firms are being more creative with the terms of securities they issue. It is now possible to buy floating rate preferred stock that matures.

*To evaluate this question, compute the present value of the lottery payments and compare this amount to $500,000: $PV_{lottery} = \$50{,}000(PVIFA_{10\%,20}) = \$50{,}000(8.5136) = \$425{,}680$. Because $PV_{lottery} < \$500{,}000$, you should accept the offer.

Study Tip

The idea behind the perpetuity equation is really very simple. For the cash flow to continue indefinitely, only the earned interest can be withdrawn. The amount of interest earned each year is the principal times the interest rate ($PV \times i$) = PMT. By rearranging this equation we get Equation 6.11.

In general terms, the formula for a perpetuity is

$$PV_0 = \frac{PMT}{i} \tag{6.11}$$

Let us illustrate Equation 6.11 using an example. Suppose we wanted to determine the present value of a share of preferred stock. We know that it promises to pay a $2 dividend forever and we assume an 8% interest rate.

The dividend of $2 is the constant payment. The interest rate is 8%. Plugging these figures into Equation 6.11 yields

$$PV_0 = \frac{\$2}{0.08} = \$25.00$$

As an investor, we would be willing to pay $25.00 for a share of this stock.

Self-Test Review Question*

An endowed faculty chair is created when a benefactor makes a donation of sufficient size that the earnings from the donation pay the salary and benefits of a professor. How much would need to be donated to endow a chair in your name if the salary and benefits were $80,000 and the after-inflation interest rate was 7%?

UNEVEN STREAMS

Study Tip

The PVIFA equation (Equation 6.10) finds the present value *one period before* the first payment is received. Go back and review the time line just before Example 6.19 to see this.

It is not unusual that cash flows have some payments that differ as well as some that are the same. When an annuity is mixed with other irregular payments we have an *imbedded* annuity. One reason we will see this cash flow pattern in this course is that we can often estimate the next couple of cash flows with some degree of confidence. However, as we make predictions about cash flows that are far in the future, it is often a rational simplification to assume that they will remain constant.

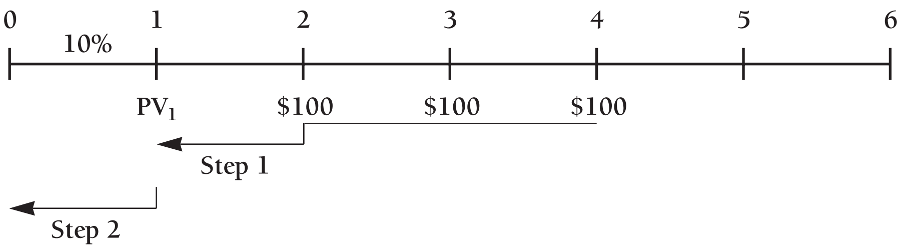
To solve for the present value of mixed cash flow streams, we find the present value of the annuity, then add the present value of any other cash flows. The simplest mixed cash flow is one in which there are several periods before the annuity begins. Thus, the stream consists of several periods of zero cash flows plus an annuity.

Consider the following example. Suppose an annuity does not begin at the end of the first period, as in the earlier examples. Instead, it begins one period later. As we noted in our earlier discussion of future value (see "Computing the Future Value of an Ordinary Annuity"), an annuity that begins *any time after* the end of the first period is called a *deferred annuity*.

*Because the chair position is expected to go on forever, the cash flow is a perpetuity. The given figures should be substituted as follows:

$$PV = \frac{\$80,000}{0.07} = \$1,142,857.14$$

A sum of $1,142,857.14 would need to be donated to create a chair in your name.



To solve this type of problem, we first apply the present value method to the annuity. When we do this using the Equation 6.10, we find the value of the annuity one period *before* the first payment (we label this point as PV_1 on the time line):

Study Tip
Pay close attention to when the PVIFA and the PVIF factors are used.

Step 1 $$PV_1 = \$100(PVIFA_{3,10\%}) = \$248.70$$

We are not finished yet because we need the value at time zero, not at time 1 (the end of the first period). To move the value of the annuity back in time one more period, we treat the solution to Step 1 as a new problem, and find its present value:

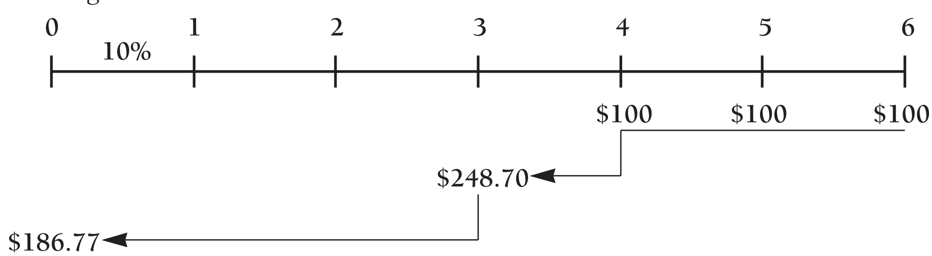
Step 2 $$PV_0 = \$248.70(PVIF_{1,10\%}) = (\$248.70)(0.909) = \$226.07$$

Steps 1 and 2 can be combined as follows to save time:

$$PV_0 = \$100(PVIFA_{3,10\%})(PVIF_{1,10\%}) = (\$100)(2.487)(0.909) = \$226.07$$

Let us review carefully what we have done. $\$100(PVIFA_{3,10\%})$ discounts the value of the three-period annuity back one period to the end of period 1. Multiplying by $PVIF_{1,10\%}$ discounts the annuity back to time 0. Our goal was to obtain the present value at time 0.

What happens if the annuity is deferred more than one period? For example, suppose that an annuity does not begin until three periods later. We find that the method is unchanged:



$$PV = \$100(PVIFA_{3,10\%})(PVIF_{3,10\%}) = (\$100)(2.487)(0.751) = \$186.77$$

Study Tip
The number of periods in the annuity is *always* equal to the number of payments, even when the annuity is deferred. One way to view annuities is to recognize that the annuity begins one period before the first payment is made and ends when the last payment is made.

You use the three-period PVIFA because three payments of $100 are received. This discounts the annuity back to the end of period 3. Next, discount back three more periods to compute the PV at time zero.

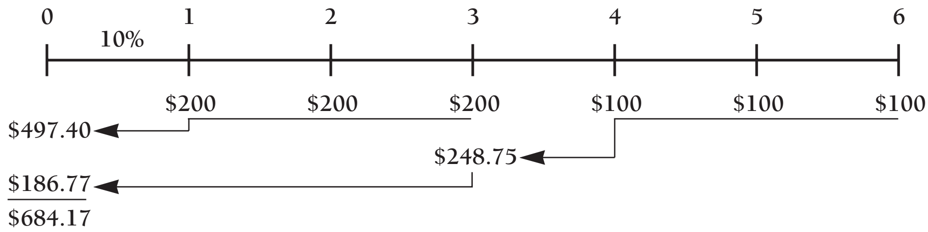
A slightly more complex example has two annuities. In the following example, find the present value of each annuity separately, and then add them together.

Study Tip

Be sure you thoroughly understand and can do each of these examples. These models will appear over and over in different sections of the text.

EXAMPLE 6.21 Present Value of an Annuity

Suppose that you have two annuities, one beginning one period later and another beginning four periods later. What are the present values of each of the annuities and what is their combined PV?



Solution

$$PV = \$100(PVIFA_{3,10\%})(PVIF_{3,10\%}) + 200(PVIFA_{3,10\%})$$

$$PV = \$100(2.487)(0.751) + \$200(2.487) = \$684.17$$

The present values of the separate annuities are $186.77 and $497.40. Their combined present value is $684.17.

Note that you find the PV of each annuity separately, then add them together. Instead of multiplying by $PVIF_{n,i}$, you could multiply by $1/(1 + i)^n$. Pay close attention to when PVIFA and PVIF are used.

Self-Test Review Question*

You are considering buying a business. You have determined that it will generate $1,000 per year for 5 years, then $2,000 per year for an additional 5 years. How much would you be willing to pay for this business if you felt the discount rate should be 15%? (Hint: The value of a business asset is simply the present value of the cash flows.)

EXTENSION 6.1

Loans

Loan Amortization

Another useful application of TVM methods is the calculation of loan payments and loan amortization schedules. Loan payment calculations are performed using the present value of an annuity formula. This is because the bank making a loan must be indifferent between the money it is lending at the present time and the future cash flows it will receive in return. In other words, the loan amount must be the present value of the loan payments.

Loan Payments

In the section above, we learned how to compute the present value of an annuity. Suppose that we already know the present value, but we want to know what annuity payment equals that present value. To find this, we need to know the length of the annuity and the interest rate. Given these figures, we can compute the payment amount.

*$PV = \$1,000(PVIFA_{15\%,5}) + \$2,000(PVIFA_{15\%,5})(PVIF_{15\%,5}) = \$1,000(3.3522) + \$2,000(3.3522)(0.4972) = \$6,685.63$. You would be willing to pay $6,685.63.

Let's illustrate using an example. Suppose you have decided to buy a new car as a reward for acing your finance exam. You will need a loan of \$15,000 and would like to make payments for 2 years. The dealership has offered a 12% interest rate on the loan. What will your monthly payment be?

You begin by writing down the formula for the present value of an annuity:

$$PV = PMT(PVIFA_{i,n})$$

Note that there are 24 monthly periods and that the interest rate is 1% per period. Because you will receive the loan proceeds today, the loan amount is the present value. Substituting these figures into the above equation yields

$$\$15{,}000 = PMT\left(PVIFA_{1\%,24}\right)$$

$$\$15{,}000 = PMT\left(21.2434\right)$$

$$\frac{\$15{,}000}{21.2434} = PMT = \$706.10$$

Monthly payments of \$706.10 will be required to pay off this loan. To solve this equation using a financial calculator, you would enter the numbers as follows:

24	1	15,000		0
N	I	PV	PMT	FV
			= –\$706.08	

As the length of the loan increases, the amount of the monthly payment falls, but the amount of interest you pay over the course of the loan increases. Box 6.2 demonstrates the importance of selecting the shortest possible repayment period for a mortgage loan.

Loan Amortization In the above example, monthly loan payments of \$706.10 are required. These payments pay the interest that has accrued from the last payment as well as a portion of the principal owed. Each period, the interest due falls because the principal balance is declining. As a result, the portion of the payment that goes to reducing the principal increases. An amortization schedule details the distribution of funds between principal and interest. To prepare an amortization schedule, compute the amount of interest that has accrued and subtract this from the payment to find the principal reduction.

The sample amortization schedule that follows is for a 3-year loan of \$10,000 at 10% with three annual payments:

A Beginning Balance	B Payment	C Accrued Interest (A × 0.10)	D Principal Payment (B − C)	E Ending Balance (A − D)
\$10,000	\$4,021.15	\$1,000	\$3,021.15	\$6,978.85
6,978.85	4,021.15	697.988	3,323.27	3,655.58
3,655.58	4,021.15	365.56	3,655.58	0

Box 6.2 How to Save $150,000 Dollars

Suppose that you have graduated, gotten married, secured that good job, and now want to buy a house. After a diligent search, you and your spouse decide to buy a $170,000 house on a quiet cul-de-sac in a nice neighborhood to raise kids. You go into your local bank to apply for financing and are asked whether you would prefer a 30-year loan or a 15-year loan. Knowing that the 30-year loan will have lower payments, you select that. What is the long-term implication of this decision? To find out, let us compute the payment and total interest paid in each scenario.

Assume that the interest rate on the 30-year loan is 8% and on the 15-year loan it is 7.5% (remember that the usual shape of the yield curve causes short-term rates to be less than long-term rates). You pay a 5% down payment, so you will borrow $161,500. Using a financial calculator, the payments are as follows:

30-year loan: 360 payments of $1,185.03

15-year loan: 180 payments of $1,497.12

The total interest paid in each case is found by multiplying the number of payments by the monthly payment amount and subtracting the loan amount.

30-year loan interest paid

$$= 360 \times \$1{,}185.03 - \$161{,}500 = \$265{,}110.80$$

15-year loan interest paid

$$= 180 \times \$1{,}497.12 - \$161{,}500 = \underline{\$107{,}982.49}$$

$$\$157{,}128.31$$

By choosing the 30-year mortgage, you will pay an additional $157,128.31 in interest over the life of the loan. Most loans allow you to pay more than is required each month. You could take the 30-year loan, but make payments as if it were a 15-year loan. However, you will have to pay a higher interest rate on the 30-year loan than if you had just obtained a 15-year loan to start with. Some people prefer having the option of being able to reduce their payments should they get into a financial pinch, and this option is worth the extra interest cost to them. Others feel they need the discipline a 15-year loan will impose on them. Otherwise, they may not make the larger payments.

Notice that as the beginning balance falls the accrued interest also falls. This leaves a greater portion of each payment to go to reducing the principal.

Self-Test Review Question*

You plan to finance a $100,000 plant expansion over 5 years at a 12% annual interest rate. What will the monthly payment be?

EXTENSION 6.2

Compounded Growth

Growth Rates

The following table shows that K-Mart sales increased over the period 1990–1994:

*PV = PMT(PVIFA), $100,000 = PMT($PVIFA_{1\%,60}$), $100,000/44.955 = $2,224.45.

K-Mart Sales 1990–1994 (thousands)				
1990	**1991**	**1992**	**1993**	**1994**
\$27,670	\$28,133	\$29,419	\$31,416	\$34,557

You can use the TVM equations to compute the average compounded growth in sales. You might be tempted to find the average growth rate by computing the percentage increase in sales for each year and then taking an average. This might seem logical at first glance, but on second thought, it does not work. The problem is that taking an average of each year's increase ignores the effects of compounding. The proper method is to recognize that a growth rate is the same thing as the interest rate used with compound interest problems. You can use either the future or present value formula to solve for the interest rate.

EXAMPLE 6.22 Compound Growth Rate

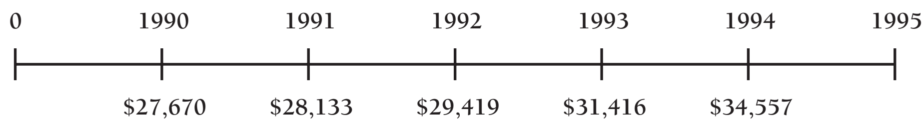
What is the average compound growth rate in sales for K-Mart between 1990 and 1994?

Solution

Let us use Equation 6.1:

$$FV_n = PV_0(1 + i)^n$$

The PV is the earlier occurring sales figure (this is the figure farthest to the left if the data were plotted on a time line, in this case \$27,670). The future value is the most recent sales amount, or \$34,557.



We can determine the number of periods by counting the intervals during which growth can occur. In this example, there are four periods of growth. Substituting these numbers into the equation yields

$$\$34{,}557 = \$27{,}670(1 + g)^4$$

A little algebra results in the following:

$$g = \sqrt[4]{\frac{\$34{,}557}{\$27{,}670}} - 1$$

$$g = \sqrt[4]{1.2489} - 1$$

$$g = 0.0571 = 5.71\%$$

The average compound growth rate for K-Mart's sales between 1990 and 1994 was therefore 5.71%.

Note that you can find fractional roots on most calculators by entering the exponent as a fraction.[10] This problem could also be worked by solving for the FVIF and looking

Study Tip

Note that on most calculators the sign of the PV and FV terms must be different or an error message results.

[10]You can find any root if your calculator has a Y^X button (or otherwise lets you compute exponents).

up the interest rate that corresponds to four periods. The problem is that only whole interest rates are reported in the tables, so only an approximation can be found.

Growth rates are easily computed using financial calculators. Input the PV, FV, and number of periods and solve for i:

4		27,670	0	–34,557
N	I	PV	PMT	FV
	= 5.71%			

Study Tip

In setting up problems, it may help to draw or visualize a time line. Amounts on the left of the time line are present values. Amounts on the right are future values. Begin each problem by putting everything on the time line that is in the text of the problem. Solve for what is missing.

IS IT PRESENT VALUE OR FUTURE VALUE?

Mastering the TVM techniques presented in this chapter is a significant step toward solving TVM problems. However, an added layer of complexity is the need to determine whether a particular problem is a present value or a future value problem. Consider the following example. You want to determine the amount that must be accumulated to pay a fixed $25,000 per year during your expected 20-year retirement, assuming a 10% interest rate. Is this a present value, future value, present value of an annuity, or future value of an annuity problem?

For most students, the hard part of learning TVM concepts is not mastering the equations, but rather determining which equation to use to solve a particular word problem. Table 6.3 offers some guidelines to help. Remember, there were only four different equations presented in this chapter:

- Future value of a lump sum: $FV = PV(1 + i)^n$
- Future value of an annuity: $FV = PMT(FVIFA_{i,n})$
- Present value of a lump sum: $PV = FV/(1 + i)^n$
- Present value of an annuity: $PV = PMT(PVIFA_{i,n})$

To determine how to solve new, unfamiliar types of TVM problems and to pick which of the above equations to use, follow these steps:

1. Review the cash flows in the question to determine whether they are annuities. Once you have made this determination, only two equations remain. In the above example, because regular equal payments will be made, it is an annuity problem.
2. Review the information given in the problem and jot down what is known. If you are confused, draw a time line. PVs are to the left and FVs are to the right. For

TABLE 6.3 Guidelines for TVM Problems

- Use *future value* to find the balance resulting from an interest-earning deposit.
- Use *future value of an annuity* to find the payment needed to achieve a known future balance.
- Use either *present* or *future value* (without an annuity) to compute growth rates.
- Use *present value* on all loan calculations.
- Use *present value* to value assets.
- Use *present value* to evaluate investments.

instance, in the above example, we know that the regular payment amount is \$25,000, so write down PMT = \$25,000. We also know n = 20 and i = 10%. We do *not* know PV or FV.

3. Determine what you want to solve for. In the above example, we want to know how much is needed now to generate a series of payments (PV = ?). This lets us choose an equation. We will use the PV of an annuity equation because it is the only one that includes all of the variables that are given plus the variable for which we need to solve.

Here is one last suggestion. If you are attempting to solve a problem and find that you cannot determine what to plug into the equation for each of the variables, it may be because you are using the wrong equation. Try another.

Careers in Finance

Financial Planners

Financial planners help individuals plan their financial futures: How are you going to cover your retirement needs? What do you have to do today to put your children through college?

This work can be personally and financially rewarding and requires excellent interpersonal skills. A good financial planner understands investments, taxes, and estate planning issues, and he or she knows how to listen. This work can be done within a company such as IDS Financial Services or by yourself, as a sole proprietorship.

Most planners go solo or work within smaller practices. It is essential that you have a certain amount of entrepreneurship, given that you will be running your own business. The work pays well and is rewarding if you like to help people. Many financial planners obtain the Certified Financial Planner (CFP) designation. This certification adds a great deal of credibility to the planner, and many customers look for it. The CFP designation requires work experience and the passage of a multipart exam.

CHAPTER SUMMARY

In this chapter we learned that dollars received in the future are not worth as much as dollars received today. As a result, we must adjust for differences in the time value of money. Four basic equations are used either independently or in combination to move money around in time.

We can compute the value of money invested over time using the future value equation. A lump sum invested today will grow to a larger amount over time. The longer the investment period and the greater the interest rate, the more will be the future balance.

At times, we may want to compute how much a series of equal payments will accumulate. For example, if you make regular contributions to a retirement fund, what will be available for retirement? Problems of this nature are solved using the future value of an annuity equation. If we know what the future value should be, we can also use the future value of an annuity equation and solve for the payment required to reach this balance.

More often in the remainder of this text, we are going to need to compute the present value of a future sum to be received. We use present value often because it is best to evaluate investments by evaluating all cash flows in the present so that cash inflows can be compared with cash outflows. There are two equations for determining present value. We can find the present value of a lump sum or the present value of an annuity.

TVM techniques are useful in many applications. For instance, we find the present value of the cash flows from an investment to determine how much that asset is worth. We can compute loan payments and amortization schedules using present value of an annuity methods. In later chapters we will use these methods to evaluate projects and acquisitions.

KEY WORDS

annual compounding 131
annuity 140
annuity due 140
compound interest 128
compounding 129
deferred annuity 142
discount rate 145
discounting 145
effective interest rate (EIR) 133
future value 129
ordinary annuity 140
perpetuity 153
present value 129
time line 127

DISCUSSION QUESTIONS

1. Why are you indifferent between having the present value of a cash flow and the cash flow itself?
2. Define the term *annuity.* What is an embedded annuity? What is a deferred annuity?
3. Would you rather make a deposit into an account that pays 6% compounded annually, quarterly, monthly, or daily? Does it make any difference which account you choose?
4. What is the relationship between factors found on the FVIF table and those on the PVIF table? (Hint: How could you mathematically convert one into the other?)
5. What is meant by the term *discounting* and what is meant by the term *compounding*?
6. Demonstrate how you can derive the equation for the present value of a future amount from the equation for the future value of an investment.
7. What is the distinction between an annuity and a perpetuity?
8. What is meant by the statement, "You can exactly reproduce a future cash flow if you have the present value of that cash flow"?
9. What is a loan amortization schedule and why might you want to construct one?
10. Describe how you would determine the compound growth rate in sales over a period of time.

PROBLEMS

1. The financial manager at Zimmer Industries is considering an investment that requires an initial outlay of \$30,000 and is expected to result in cash inflows of \$5,000 at the end of year 1, \$8,000 at the end of years 2 and 3, \$15,000 at the end of year 4, \$12,000 at the end of year 5, and \$10,000 at the end of year 6. Without doing any calculations,
   a. Draw and label a time line depicting the cash flows associated with Zimmer Industries' proposed investment.
   b. Use arrows on the time line to demonstrate how compounding to find future values can be used to measure all cash flows at the end of year 6.
   c. Use arrows on the time line to demonstrate how discounting to find present value can be used to measure all cash flows at time zero.
2. Use the basic formula for future value along with the given interest rate, *i,* and number of periods, *n,* to calculate the future value interest factor (FVIF) in each of the following cases.

Case	*Interest Rate,* i	*Number of Periods,* n
A	10%	2
B	8	4
C	6	3
D	4	2

3. For each of the following cases, calculate the future value of the single cash flow deposited today that will be available at the end of the deposit period if the interest is compounded annually at the rate specified over the given period.

Case	*Single Cash Flow*	*Interest Rate*	*Deposit Period (yr)*
A	\$ 100	6%	30
B	5,000	10	25
C	12,500	12	7
D	23,200	14	10

4. Calculate the balance you would have in an account after 4 years assuming $6,000 was deposited today at 16% compounded annually, semiannually, and quarterly.
5. For each of the following cases, find the future value at the end of the deposit period, assuming that interest is compounded semiannually at the given nominal interest rate.

Case	Initial Deposit	Nominal Interest Rate	Deposit Period (yr)
A	$500	8%	2
B	300	6	5
C	300	10	7

6. Calculate the future value of the annuity, assuming that each case is an
   a. Ordinary annuity
   b. Annuity due

Case	Annuity	Interest Rate	Deposit Period (yr)
A	$4,000	10%	7
B	2,500	8	30

7. For the following mixed stream of cash flows, determine the future value at the end of the final year if deposits are made at the beginning of each year into an account paying annual interest of 10%, assuming that no withdrawals are made during the period.

Year	Cash Flow Stream
1	$1,000
2	500
3	2,000

8. Using the basic formula for present value along with the given discount rate, *i*, and number of periods, *n*, calculate the present value interest factor (PVIF) in each of the following cases.

Case	Discount Rate, i	Number of Periods, n
A	5%	5
B	10	3
C	12	2

9. Ted Roberts has been offered a $1,000 future payment 3 years from today. If his opportunity cost is 7% compounded annually, what value would he place on this opportunity?
10. Find the present value of the following mixed stream of cash flows using a 10% discount rate. (Assume deposits are made at the end of each year.)

Year	Cash Flow Stream
1	$20,000
2	15,000
3	10,000

11. For each of the following cases, calculate the present value of the annuity, assuming that the annuity cash flows occur at the end of each year.

Case	Annuity	Interest Rate	Periods (yr)
A	$10,000	10%	5
B	20,000	12	10

12. In the following case the mixed end-of-period cash flow stream has an annuity embedded within it. Calculate the present value of the cash flow stream, assuming a 10% discount rate.

Year	Cash Flow
1	$10,000
2	8,000
3	5,000
4	5,000
5	5,000
6	5,000
7	3,000

13. Determine the present value of each of the following perpetuities.

Perpetuities	Annual Amount	Discount Rate
A	$10,000	10%
B	20,000	5
C	50,000	8

14. A retirement home at Westbrook Manor Estates now costs $80,000. Inflation is expected to cause this price to increase at 5% per year over the next 30 years before Chris O'Neal retires. How large an equal annual end-of-the-year deposit must be made each year into an account paying an annual interest rate of 10% for Chris to have the cash to purchase a home at retirement?
15. Determine the equal annual end-of-year payment required for each year over the life of the following loans to repay them fully during the stated term.

Loan	Principal	Interest Rate	Term of Loan (yr)
A	$10,000	14%	10
B	15,000	10	5

16. Calculate the average annual compound growth rate associated with each cash flow stream.

	Cash Flows		
Year	A	B	C
1	$ 500	$200	$2,000
2	600	300	2,500
3	800	400	2,600
4	1,000	500	2,800
5	1,200	600	3,000
6		700	3,250
7		800	
8		900	

17. You are saving money for your retirement. How much will you have on deposit in 40 years if you invest at 12% as follows:
    a. At the end of each of the next 8 years you deposit $2,000. After 8 years you do not make any more deposits, but the balance continues to compound at 12% for the next 32 years.
    b. Your first $2,000 deposit is made at the end of the eighth year and at the end of each subsequent year for the next 32 years.
    c. How much was invested in total in part a? In part b? Which investment plan appears to be superior?
18. Suppose your credit card balance is $1,125. The minimum payment amount is $22 and the annual percentage rate is 18%.
    a. If you make a constant monthly payment of $22, how long will it take you to pay off the credit card balance? (Note: You will need a financial calculator to solve this problem because monthly compounding requires that the 18% annual rate be divided by 12.)
    b. How much interest will you pay if you elect to make the minimum payment?
19. You bought a house 2 years ago with a $100,000 mortgage. This loan requires only annual payments for 15 years. You are now preparing your taxes and want to know how much interest you paid during the second year of the loan. If the loan interest rate is 8%, prepare an amortization schedule that shows second-year total interest paid and the end-of-year loan balance.
20. You have just taken out an installment loan for $100,000. Assume that the loan will be repaid in 12 equal monthly installments of $9,456 and that the first payment will be due 1 month from today. How much of your third monthly payment will go toward the repayment of principal?
21. You have graduated from college and landed a good job. You want to replace the junk car you are driving but do not want to take out a car loan. Instead, you decide to invest $350 per month. You will put the money in the stock market and hope to earn 12%. If the market performs as you are hoping (a very optimistic assumption), how long will it take to accumulate $20,000?

SELF-TEST PROBLEMS

1. What is the value today of $1,250 to be received 3 years from now, assuming a 12% discount rate?
2. What is the future value of a 5-year ordinary annuity with annual payments of $200, evaluated at a 15% interest rate?
3. What would be the present value today of $1,250 to be received 3 years from now, assuming a 12% discount rate and quarterly compounding?
4. What is the present value of a 5-year ordinary annuity with annual payments of $200, evaluated at a 15% interest rate?
5. What is the present value of a series of $200 payments to begin at the end of the current year and to continue for 20 years, assuming a 12% discount rate?
6. What is the present value of a series of $100 payments made monthly for 5 years, assuming a 12% discount rate?
7. What is the present value of a series of payments in which $100 is received at the end of the current year, $200 is received at the end of the following year, and $300 is received at the end of the next year, assuming a 10% discount rate?

8. If a 5-year ordinary annuity has a present value of $1,000 and if the interest rate is 10%, what is the amount of each annuity payment?
9. You have the opportunity to buy a perpetuity that pays $1,000 annually. Your required return on this investment is 15%. You should be essentially indifferent to buying or not buying the investment if it were offered at what price?
10. How much will be in an account earning 10% at the end of 5 years if $100 is deposited today?
11. How much will be in an account earning 8% at the end of 5 years if $100 is deposited today, assuming quarterly compounding?
12. What is the annual payment due on a $1,000 loan payable in equal annual payments over 3 years at 10%?
13. What is the present value of a $100 payment that will continue to be paid annually forever, assuming a 10% interest rate?
14. How much will be in a retirement account if $2,000 is deposited at the end of each year for 20 years and the account earns 12%?
15. Assume that you will receive $2,000 a year in years 1 through 5, $3,000 a year in years 6 through 8, and $4,000 in year 9, with all cash flows to be received at the end of the year. If you require a 14% rate of return, what is the present value of these cash flows?
16. Suppose the present value of a 2-year ordinary annuity is $100. If the discount rate is 10%, what must be the annual cash flow?
17. If $100 is placed in an account that earns a nominal 4%, compounded quarterly, what will it be worth in 5 years?
18. You are financing a new car with a loan of $10,000 to be repaid in five annual end-of-year installments of $2,504.56. What annual interest rate are you paying?
19. You are given the following cash flow information. The appropriate discount rate is 12% for years 1–5 and 10% for years 6–10. Payments are received at the end of the year. What should you be willing to pay right now to receive the following income stream? {Hint: First discount years 6–10 at 10% back to the end of year 5. Then discount this amount back to year 0 at 12%. Finally discount years 1–5 back to year 0 at 12%.)

Year	*Amount*
1–5	$20,000
6–10	25,000

20. What is the present value of the following cash flow stream? All cash flows are annual and received at the end of the period. Assume a 10% interest rate. $200 is received at the end of years 1–5, $300 is received at the end of years 6–10.
21. What is the future value of a 10-year ordinary annuity of $15,000 per year, assuming an interest rate of 10% per year?

*22. What is the future value of a 2-year ordinary annuity of $500 per month, assuming an annual interest rate of 12% per year?

23. What is the future value of a $1,000 deposit made today that will be left on deposit for 5 years earning an annual rate of 12%?
24. What is the present value of an ordinary annuity of $1,500 per year that will be received for 5 years, assuming an interest rate of 12%?
25. What is the present value of an annuity due of $1,500 per year that will be received for 5 years, assuming an interest rate of 12%?
26. What is the present value of a lump sum of $300 that will be paid at the end of the fifth year, assuming that the interest rate is 10%?
27. What is the future value at the end of year 3 of the following cash flow stream, assuming a 10% interest rate? $100 will be received at the end of year 1, $200 will be received at the end of year 2, and $300 will be received at the end of year 3.
28. What will be the present value of the cash flow in problem 27?
29. If the present value of a 5-year ordinary annuity is $500, assuming that the discount rate is 10%, what are the annual payments?
30. What is the future value after 5 years of $1,000 that is placed in a savings account today that pays interest quarterly, assuming an annual interest rate of 8%?
31. What is the effective annual interest rate for an account that pays interest quarterly at the annual rate of 8%?
32. In today's dollars you need $50,000 per year to retire. If you will retire in 30 years and if inflation will average 4% per year between now and when you retire, how much will you need during the first year of your retirement?

*33. What will be the monthly payment on a loan of $15,000 that is amortized over 2 years at an annual interest rate of 12%?

*34. What is the interest rate on a loan of $20,000 due in 5 years and requiring an annual payment of $5,686.29?

35. What is the present value of a perpetuity that pays $2 per quarter if the discount rate is 12% per year?

*Requires financial calculator or the use of equation method.

WEB EXPLORATION

1. There are many sites on the Web to help you compute whether you are properly preparing for your retirement. One of the better ones is offered by Quicken. You will find it at www.quicken.com/retirement/planner/.

   Have you set aside enough retirement money to last your lifetime? The earlier you start, the easier it will be. In general, your retirement funds will come from four sources:
   a. Pension Plans
   b. Social Security
   c. Tax-deferred savings
   d. Basic (taxable) savings

   Use the Retirement Planner to predict the income from the first two, and to determine how much you will need to save to make up the balance for your retirement goals.

2. An alternative to the financial goals calculation in problem 1 are sites that offer calculators that let you input figures to compute your goals. Go to library.thinkquest.org/10326/other_features/calc.html. Use both the financial calculator provided and your own financial calculator to answer the following questions.

   Solve for the length of time and total gains in the following problems (do not input $ or ,) by plugging in each number in the following sequence, as per part a below:
   a. Your Goals = $1,000,000; Initial Capital = $1,000; Monthly Invested = $400; ROI = 12%.
   b. $2,000,000; $1,000; $600; 15%.
   c. $1,500,000; $10,000; $0; 15%.

   Verify the calculator figures with your figures you have calculated manually. What have you observed? Why?

MINI CASE

You have graduated and obtained the job of your dreams. It is now time to evaluate what you must do to look out for your retirement. You consider contacting the financial planner provided by your firm but decide to make an initial stab at the calculations first. You decide to do the calculations on a spreadsheet program such Excel or Lotus 1-2-3 so that it will be easy to change your assumptions depending on what the financial planner advises. You try to be as complete as possible so that the results are useful to you and so that you will know what to ask the planner.

If you are married, use joint income and joint expenses. Be sure to consider Social Security and company pension plans.

a. Determine what your annual retirement expenses will be. Use current dollars. Show each major element. Include everything. Do not cut yourself short. Allow enough funds to pursue recreational activities. Do not forget that for most retired people medical expenses are a major expense.
b. Estimate what you believe inflation will average between now and when you retire. Determine how many years you will work between now and when you retire. Then estimate how long you will live after retirement. Clearly state these assumptions and justify them in your write-up. The Web Exploration assignments in Chapters 1 to 5 contain numerous references to sites that can be used to compute life expectancy and inflation.
c. Estimate what average rate of return you believe is possible given the types of investments you plan to make. Justify your estimate. This may require contacting brokerage firms. Be conservative.
d. Determine what alternative sources of funds are available to you to offset your retirement costs. Again be conservative. Do not count on anything unless it is assured.
e. Compute how much you will need each year during your retirement years. (Hint: Take the amount you need in current dollars and find the future value using the inflation rate as the discount factor.) Compute how large a retirement balance is needed to retire. Be sure to adjust for inflation. (Hint: This will be the present value of the stream of annual withdrawals you will need to make during your retirement years. Note that each year while you are retired the amount you need will continue to grow by the inflation rate.) Now compute the size of your retirement nest egg by computing

the present value of the annual cash plans you need during retirement.

f. Compute how much must be saved every month to reach your retirement goal. (Hint: This will be a future value of an annuity calculation where the future amount is the nest egg needed from part e and you compute the PMT amount.)

g. Is the above reasonable? If not, devise an uneven savings retirement plan that is more rational. That is, design a savings program where you save less at the beginning, then increase your savings amount later when you can afford it more.