Formulas

2.3

Evaluating Formulas • Solving for a Letter

Many applications of mathematics involve relationships among two or more quantities. An equation that represents such a relationship will use two or more letters and is known as a **formula**. Although most of the letters in this book represent variables, some—like c in $E = mc^2$ or π in $C = \pi d$—represent constants.

Evaluating Formulas

Example 1

Furnace output. Contractors in the Northeast use the formula $B = 30a$ to determine the minimum furnace output B, in British thermal units (Btu's), for a well-insulated house with a square feet of flooring (*Source*: U.S. Department of Energy). Determine the minimum furnace output for an 1800-ft^2 house that is well insulated.

Solution We substitute 1800 for a and calculate B:

$$B = 30a = 30(1800) = 54{,}000.$$

The furnace should be able to provide at least 54,000 Btu's of heat.

technology connection

Sometimes we may wish to recall and modify a calculation. For example, suppose that after calculating 30 · 1800 we wish to find 30 · 1870. Pressing **2nd** **ENTRY** gives the following

```
30 * 1800
                    54000
30 * 1800
```

Moving the cursor left, we change 1800 to 1870 and press **ENTER**.

```
30 * 1800
                    54000
30 * 1870
                    56100
```

1. Verify the work above and then use **2nd** **ENTRY** to find 39 · 1870. 72,930

Solving for a Letter

Suppose that a contractor has an extra furnace and wants to determine the size of the largest (well-insulated) house in which it can be used. The contractor can substitute the amount of the furnace's output in Btu's—say, 63,000—for B, and then solve for a:

$63{,}000 = 30a$ Replacing B with 63,000

$2100 = a.$ Dividing both sides by 30:

$$\begin{array}{r} 2\,100 \\ 30\overline{)63{,}000} \\ \underline{60} \\ 30 \\ \underline{30} \\ 000 \end{array}$$

Were these calculations to be performed for a variety of furnaces, the contractor would find it easier to first solve $B = 30a$ for a, and *then* substitute values for B. This can be done in much the same way that we solved equations in Sections 2.1 and 2.2.

Example 2

Solve for a: $B = 30a$.

Solution We have

$$B = 30a \qquad \text{We want this letter alone.}$$

$$\frac{B}{30} = a. \qquad \text{Dividing both sides by 30}$$

The equation $a = B/30$ gives a quick, easy way to determine the floor area of the largest (well-insulated) house that a furnace with B Btu's could heat.

To see how the addition and multiplication principles apply to formulas, compare the following. In (A), we solve as usual; in (B), we do not simplify; and in (C), we *cannot* simplify since a, b, and c are unknown.

A. $5x + 2 = 12$

$5x = 12 - 2$

$5x = 10$

$x = \frac{10}{5} = 2$

B. $5x + 2 = 12$

$5x = 12 - 2$

$x = \frac{12 - 2}{5}$

C. $ax + b = c$

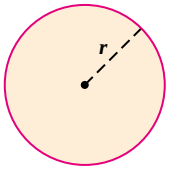
$ax = c - b$

$x = \frac{c - b}{a}$

Example 3

Circumference of a circle. The formula $C = 2\pi r$ gives the *circumference* C of a circle with radius r. Solve for r.

Solution The **circumference** is the distance around a circle.



Given a radius r, we can use this equation to find a circle's circumference C.

$$C = 2\pi r \qquad \text{We want this letter alone.}$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \qquad \text{Dividing both sides by } 2\pi$$

Given a circle's circumference C, we can use this equation to find the radius r.

$$\frac{C}{2\pi} = r$$

Example 4

Nutrition. The number of calories K needed each day by a moderately active woman who weighs w pounds, is h inches tall, and is a years old, can be estimated using the formula

$$K = 917 + 6(w + h - a).*$$

Solve for h.

*Based on information from M. Parker (ed.), *She Does Math*! (Washington DC: Mathematical Association of America, 1995), p. 96.

Study Tip

The *Student's Solutions Manual* is an excellent resource if you need additional help with an exercise in the exercise sets. It contains worked-out solutions to the odd-numbered exercises in each exercise set.

Solution We reverse the order in which the operations occur on the right side:

$$K = 917 + 6(w + h - a) \qquad \text{We want } h \text{ alone.}$$

$$K - 917 = 6(w + h - a) \qquad \text{Subtracting 917 from both sides}$$

$$\frac{K - 917}{6} = w + h - a \qquad \text{Dividing both sides by 6}$$

$$\frac{K - 917}{6} + a - w = h. \qquad \text{Adding } a \text{ and subtracting } w \text{ on both sides}$$

This formula can be used to estimate a woman's height, if we know her age, weight, and caloric needs.

The above steps are similar to those used in Section 2.2 to solve equations. We use the addition and multiplication principles just as before. The main difference is the need to factor when combining like terms.

To Solve a Formula for a Given Letter

1. If the letter for which you are solving appears in a fraction, use the multiplication principle to clear fractions.
2. Get all terms with the letter for which you are solving on one side of the equation and all other terms on the other side.
3. Combine like terms, if necessary. This may require factoring.
4. Multiply or divide to solve for the letter in question.

Example 5 Solve for x: $y = ax + bx - 4$.

Solution We solve as follows:

$$y = ax + bx - 4 \qquad \text{We want this letter alone.}$$

$$y + 4 = ax + bx \qquad \text{Adding 4 to both sides}$$

$$y + 4 = x(a + b) \qquad \text{Combining like terms by factoring out } x$$

$$\frac{y + 4}{a + b} = x. \qquad \text{Dividing both sides by } a + b, \text{ or multiplying both sides by } 1/(a + b)$$

We can also write this as

$$x = \frac{y + 4}{a + b}.$$

Caution! Had we performed the following steps in Example 5, we would *not* have solved for x:

$$y = ax + bx - 4$$

$$y - ax + 4 = bx \quad \text{Subtracting } ax \text{ and adding 4 to both sides}$$

Two occurrences of x

$$\frac{y - ax + 4}{b} = x. \quad \text{Dividing both sides by } b$$

The mathematics of each step is correct, but since x occurs on both sides of the formula, *we have not solved the formula for x*. Remember that the letter being solved for should be alone on one side of the equation, with no occurrence of that letter on the other side!

Exercise Set 2.3

FOR EXTRA HELP


Digital Video Tutor CD 1
Videotape 3


InterAct Math


Math Tutor Center

MathXL


MyMathLab.com

1. *Distance from a storm.* The formula $M = \frac{1}{5}t$ can be used to determine how far M, in miles, you are from lightning when its thunder takes t seconds to reach your ears. If it takes 10 sec for the sound of thunder to reach you after you have seen the lightning, how far away is the storm? 2 mi

2. *Electrical power.* The power rating P, in watts, of an electrical appliance is determined by

$$P = I \cdot V,$$

where I is the current, in amperes, and V is the voltage, measured in volts. If a kitchen requires 30 amps of current and the voltage in the house is 115 volts, what is the wattage of the kitchen? 3450 watts

3. *College enrollment.* At many colleges, the number of "full-time-equivalent" students f is given by

$$f = \frac{n}{15},$$

where n is the total number of credits for which students have enrolled in a given semester. Determine the number of full-time-equivalent students on a campus in which students registered for a total of 21,345 credits. 1423 students

4. *Surface area of a cube.* The surface area A of a cube with side s is given by

$$A = 6s^2.$$

Find the surface area of a cube with sides of 3 in. 54 in^2

5. *Calorie density.* The calorie density D, in calories per ounce, of a food that contains c calories and weighs w ounces is given by

$$D = \frac{c}{w}.*$$

Eight ounces of fat-free milk contains 84 calories. Find the calorie density of fat-free milk. 10.5 cal/oz

6. *Wavelength of a musical note.* The wavelength w, in meters per cycle, of a musical note is given by

$$w = \frac{r}{f},$$

where r is the speed of the sound, in meters per second, and f is the frequency, in cycles per second. The speed of sound in air is 344 m/sec. What is the

*Source: *Nutrition Action Healthletter*, March 2000, p. 9. Center for Science in the Public Interest, Suite 300; 1875 Connecticut Ave NW, Washington, D.C. 20008.

wavelength of a note whose frequency in air is 24 cycles per second? $14.\overline{3}$ m/cycle, or $\frac{43}{3}$ m/cycle

7. *Absorption of ibuprofen.* When 400 mg of the painkiller ibuprofen is swallowed, the number of milligrams n in the bloodstream t hours later (for $0 \le t \le 6$) is estimated by

$$n = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t.$$

How many milligrams of ibuprofen remain in the blood 1 hr after 400 mg has been swallowed? 255 mg

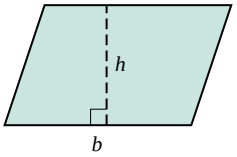
8. *Size of a league schedule.* When all n teams in a league play every other team twice, a total of N games are played, where

$$N = n^2 - n.$$

If a soccer league has 7 teams and all teams play each other twice, how many games are played? 42

Solve each formula for the indicated letter.

9. $A = bh$, for b
(Area of parallelogram with base b and height h)



$b = \frac{A}{h}$

10. $A = bh$, for h $h = \frac{A}{b}$

11. $d = rt$, for r
(A distance formula, where d is distance, r is speed, and t is time) $r = \frac{d}{t}$

12. $d = rt$, for t $t = \frac{d}{r}$

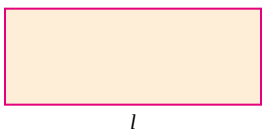
13. $I = Prt$, for P
(Simple-interest formula, where I is interest, P is principal, r is interest rate, and t is time) $P = \frac{I}{rt}$

14. $I = Prt$, for t $t = \frac{I}{Pr}$

15. $H = 65 - m$, for m
(To determine the number of heating degree days H for a day with m degrees Fahrenheit as the average temperature) $m = 65 - H$

16. $d = h - 64$, for h
(To determine how many inches d above average an h-inch-tall woman is) $h = d + 64$

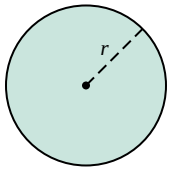
17. $P = 2l + 2w$, for l
(Perimeter of a rectangle of length l and width w)



$l = \frac{P - 2w}{2}$, or $l = \frac{P}{2} - w$

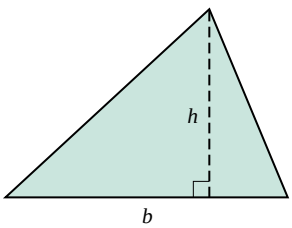
18. $P = 2l + 2w$, for w $w = \frac{P - 2l}{2}$, or $w = \frac{P}{2} - l$

19. $A = \pi r^2$, for π
(Area of a circle with radius r) $\pi = \frac{A}{r^2}$



20. $A = \pi r^2$, for r^2 $r^2 = \frac{A}{\pi}$

21. $A = \frac{1}{2}bh$, for h
(Area of a triangle with base b and height h) $h = \frac{2A}{b}$



22. $A = \frac{1}{2}bh$, for b $b = \frac{2A}{h}$

23. $E = mc^2$, for m
(A relativity formula from physics) $m = \frac{E}{c^2}$

24. $E = mc^2$, for c^2 $c^2 = \frac{E}{m}$

25. $Q = \frac{c + d}{2}$, for d $d = 2Q - c$

26. $Q = \frac{p - q}{2}$, for p $p = 2Q + q$

27. $A = \frac{a + b + c}{3}$, for b $b = 3A - a - c$

28. $A = \frac{a + b + c}{3}$, for c $c = 3A - a - b$

29. $M = \frac{A}{s}$, for A
(To compute the Mach number M for speed A and speed of sound s) $A = Ms$

30. $P = \frac{ab}{c}$, for b $b = \frac{Pc}{a}$

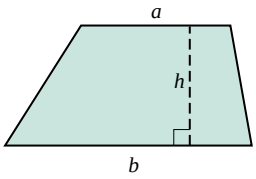
31. $A = at + bt$, for t $t = \frac{A}{a + b}$

32. $S = rx + sx$, for x $x = \frac{S}{r + s}$

33. *Area of a trapezoid.* The formula

$$A = \tfrac{1}{2}ah + \tfrac{1}{2}bh$$

can be used to find the area A of a trapezoid with bases a and b and height h. Solve for h. (*Hint*: First clear fractions.) $h = \dfrac{2A}{a + b}$



34. *Compounding interest.* The formula

$$A = P + Prt$$

is used to find the amount A in an account when simple interest is added to an investment of P dollars (see Exercise 13). Solve for P. $P = \dfrac{A}{1 + rt}$

35. *Chess rating.* The formula

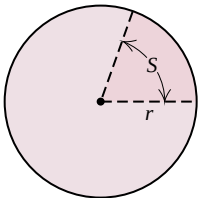
$$R = r + \frac{400(W - L)}{N}$$

is used to establish a chess player's rating R after that player has played N games, won W of them, and lost L of them. Here r is the average rating of the opponents (*Source*: The U.S. Chess Federation). Solve for L. $L = W - \dfrac{N(R - r)}{400}$, or $L = \dfrac{400W - NR + Nr}{400}$

36. *Angle measure.* The angle measure S, of a sector of a circle, is given by

$$S = \frac{360A}{\pi r^2},$$

where r is the radius, A is the area of the sector, and S is in degrees. Solve for r^2. $r^2 = \dfrac{360A}{S\pi}$



37. Naomi has a formula that allows her to convert Celsius temperatures to Fahrenheit temperatures. She needs a formula for converting Fahrenheit temperatures to Celsius temperatures. What advice can you give her?

38. Under what circumstances would it be useful to solve $d = rt$ for r? (See Exercise 11.)

SKILL MAINTENANCE

Multiply.

Aha! **39.** $0.79(38.4)0$ [1.7] 0

40. $(0.085)(108)$ [1.7] 9.18

Simplify.

41. $20 \div (-4) \cdot 2 - 3$ [1.8] -13

42. $5|8 - (2 - 7)|$ [1.8] 65

SYNTHESIS

43. The equations

$$P = 2l + 2w \quad \text{and} \quad w = \frac{P}{2} - l$$

are equivalent formulas involving the perimeter P, length l, and width w of a rectangle. Devise a problem for which the second of the two formulas would be more useful.

44. Describe a circumstance for which the answer to Exercise 34 would be useful.

45. The number of calories K needed each day by a moderately active man who weighs w kilograms, is h centimeters tall, and is a years old, can be determined by

$$K = 19.18w + 7h - 9.52a + 92.4.*$$

If Janos is moderately active, weighs 82 kg, is 185 cm tall, and needs to consume 2627 calories a day, how old is he? 35 yr

46. *Altitude and temperature.* Air temperature drops about 1° Celsius (C) for each 100-m rise above ground level, up to 12 km (*Source*: *A Sourcebook of School Mathematics*, Mathematical Association of America, 1980). If the ground level temperature is t°C, find a formula for the temperature T at an elevation of h meters. $T = t - \dfrac{h}{100}$°C

47. *Dosage size.* Clark's rule for determining the size of a particular child's medicine dosage c is

$$c = \frac{w}{a} \cdot d,$$

where w is the child's weight, in pounds, and d is the usual adult dosage for an adult weighing a pounds. (*Source*: Olsen, June Looby, et al., *Medical Dosage Calculations*. Redwood City, CA: Addison Wesley, 1995). Solve for a. $a = \dfrac{w}{c} \cdot d$

*Based on information from M. Parker (ed.), *She Does Math!* (Washington DC: Mathematical Association of America, 1995), p. 96.

48. *Weight of a fish.* An ancient fisherman's formula for estimating the weight of a fish is

$$w = \frac{lg^2}{800},$$

where w is the weight, in pounds, l is the length, in inches, and g is the girth (distance around the midsection), in inches. Estimate the girth of a 700-lb yellow tuna that is 8 ft long. About 76.4 in.

Solve each formula for the given letter.

49. $\frac{y}{z} \div \frac{z}{t} = 1$, for y $y = \frac{z^2}{t}$

50. $ac = bc + d$, for c $c = \frac{d}{a - b}$

51. $qt = r(s + t)$, for t $t = \frac{rs}{q - r}$

52. $3a = c - a(b + d)$, for a $a = \frac{c}{3 + b + d}$

53. *Furnace output.* The formula

$$B = 50a$$

is used in New England to estimate the minimum furnace output B, in Btu's, for an old, poorly insulated house with a square feet of flooring. Find an equation for determining the number of Btu's saved by insulating an old house. (*Hint*: See Example 1.) $S = 20a$, where S is the number of Btu's saved

54. Revise the formula in Example 4 so that a woman's weight in kilograms (2.2046 lb = 1 kg) and her height in centimeters (0.3937 in. = 1 cm) are used. $K = 917 + 13.2276w + 2.3622h - 6a$

55. Revise the formula in Exercise 45 so that a man's weight in pounds (2.2046 lb = 1 kg) and his height in inches (0.3937 in. = 1 cm) are used. $K = 8.70w + 17.78h - 9.52a + 92.4$

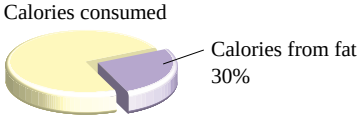
2.4 Applications with Percent

Converting Between Percent Notation and Decimal Notation • Solving Percent Problems

Recently Middlesex Toy and Hobby installed a new cash register and the sales clerks inadvertently set up the machine to print out "totals" on each receipt without separating each transaction into "merchandise" and the five-percent "sales tax." For tax purposes, the shop needs a formula for separating each total into the amount spent on merchandise and the amount spent on tax. Before developing such a formula, we need to review the basics of percent problems.

Converting Between Percent Notation and Decimal Notation

Nutritionists recommend that no more than 30% of the calories in a person's diet come from fat. This means that of every 100 calories consumed, no more than 30 should come from fat. Thus, 30% is a ratio of 30 to 100.



The percent symbol % means "per hundred." We can regard the percent symbol as part of a name for a number. For example,

$$30\% \text{ is defined to mean } \frac{30}{100}, \text{ or } 30 \times \frac{1}{100}, \text{ or } 30 \times 0.01.$$

Percent Notation

$$n\% \text{ means } \frac{n}{100}, \text{ or } n \times \frac{1}{100}, \text{ or } n \times 0.01.$$

Example 1 Convert to decimal notation: **(a)** 78%; **(b)** 1.3%.

Solution

a) $78\% = 78 \times 0.01$ Replacing % with $\times 0.01$
$= 0.78$

b) $1.3\% = 1.3 \times 0.01$ Replacing % with $\times 0.01$
$= 0.013$

As shown above, multiplication by 0.01 simply moves the decimal point two places to the left.

To convert from percent notation to decimal notation, move the decimal point two places to the left and drop the percent symbol.

Example 2 Convert 43.67% to decimal notation.

Solution

43.67% 0.43.67 $43.67\% = 0.4367$

Move the decimal point two places to the left.

The procedure used in Examples 1 and 2 can be reversed:

$0.38 = 38 \times 0.01$
$= 38\%.$ Replacing $\times 0.01$ with %

To convert from decimal notation to percent notation, move the decimal point two places to the right and write a percent symbol.

Example 3 Convert to percent notation: **(a)** 1.27; **(b)** $\frac{1}{4}$; **(c)** 0.3.

Solution

a) We first move the decimal point two places to the right: 1.27.

and then write a % symbol: 127% This is the same as multiplying 1.27 by 100 and writing %

b) Note that $\frac{1}{4} = 0.25$. We move the decimal point two places to the right: $0.25.$

and then write a % symbol: 25% Multiplying by 100 and writing %

c) We first move the decimal point two places to the right (recall that $0.3 = 0.30$): $0.30.$

and then write a % symbol: 30% Multiplying by 100 and writing %

Solving Percent Problems

To solve problems involving percents, we translate to mathematical language and then solve an equation.

Example 4

What is 11% of 49?

Solution

Translate: What is 11% of 49?

$$a = 0.11 \cdot 49$$ "of" means multiply; 11% = 0.11

$$a = 5.39$$

A way of checking answers is by estimating as follows:

$$11\% \times 49 \approx 10\% \times 50$$
$$= 0.10 \times 50 = 5.$$

Since 5 is close to 5.39, our answer is reasonable.

Thus, 5.39 is 11% of 49. The answer is 5.39.

Example 5

3 is 16 percent of what?

Solution

Translate: 3 is 16 percent of what?

$$3 = 0.16 \cdot y$$

$$\frac{3}{0.16} = y$$ Dividing both sides by 0.16

$$18.75 = y$$

Thus, 3 is 16 percent of 18.75. The answer is 18.75.

Example 6

What percent of \$50 is \$16?

Solution

Translate: What percent of \$50 is \$16?

$$n \cdot 50 = 16$$

$$n = \frac{16}{50} \qquad \text{Dividing both sides by 50}$$

$$n = 0.32 = 32\% \qquad \text{Converting to percent notation}$$

Thus, 32% of \$50 is \$16. The answer is 32%.

Examples 4–6 represent the three basic types of percent problems.

Example 7

Retail sales. Recently, receipts from Middlesex Toy and Hobby indicated the total amount paid (including tax), but not the price of the merchandise. Given that the sales tax was 5%, find the following.

a) The cost of the merchandise when the total read \$31.50
b) A formula for the cost of the merchandise c when the total reads T dollars

Solution

a) When tax is added to the cost of an item, the customer actually pays more than 100% of the item's price. When sales tax is 5%, the total paid is 105% of the price of the merchandise. Thus if c = the cost of the merchandise, we have

\$31.50 is 105% of c

$$31.50 = 1.05 \cdot c$$

$$\frac{31.50}{1.05} = c \qquad \text{Dividing both sides by 1.05}$$

$$30 = c. \qquad \text{Simplifying}$$

The merchandise cost \$30 before tax.

b) When the total is T dollars, we modify the approach used in part (a):

$$T = 1.05c$$

$$\frac{T}{1.05} = c. \qquad \text{Dividing both sides by 1.05}$$

As a check, note that when T is \$31.50, we have \$31.50 ÷ 1.05 = \$30. Since this matches the result of part (a), our formula is probably correct.

The formula $c = T/1.05$ can be used to find the cost of the merchandise when the total T is known and the sales tax is 5%.

Exercise Set 2.4

FOR EXTRA HELP


Digital Video Tutor CD 1
Videotape 3


InterAct Math


Math Tutor Center


MathXL


MyMathLab.com

Find decimal notation.

1. 82% 0.82 **2.** 49% 0.49 **3.** 9% 0.09 **4.** 91.3% 0.913

5. 43.7% 0.437 **6.** 2% 0.02 **7.** 0.46% 0.0046 **8.** 4.8% 0.048

Find percent notation.

9. 0.29 29% **10.** 0.78 78% **11.** 0.998 99.8%

12. 0.358 35.8% **13.** 1.92 192% **14.** 1.39 139%

15. 2.1 210% **16.** 9.2 920% **17.** 0.0068 0.68%

18. 0.0095 0.95% **19.** $\frac{3}{8}$ 37.5% **20.** $\frac{3}{4}$ 75%

21. $\frac{7}{25}$ 28% **22.** $\frac{4}{5}$ 80% **23.** $\frac{2}{3}$ $66\frac{2}{3}\%$ **24.** $\frac{5}{6}$ $83\frac{1}{3}\%$

Solve.

25. What percent of 68 is 17? 25%

26. What percent of 150 is 39? 26%

27. What percent of 125 is 30? 24%

28. What percent of 300 is 57? 19%

29. 14 is 30% of what number? $46\frac{2}{3}$, or $\frac{140}{3}$

30. 54 is 24% of what number? 225

31. 0.3 is 12% of what number? 2.5

32. 7 is 175% of what number? 4

33. What number is 35% of 240? 84

34. What number is 1% of one million? 10,000

35. What percent of 60 is 75? 125%

Aha! **36.** What percent of 70 is 70? 100%

37. What is 2% of 40? 0.8

38. What is 40% of 2? 0.8

Aha! **39.** 25 is what percent of 50? 50%

40. 8 is 2% of what number? 400

41. *Student loans.* To finance her community college education, Sarah takes out a Stafford loan for $3500. After a year, Sarah decides to pay off the interest, which is 8% of $3500. How much will she pay? $280

42. *Student loans.* Paul takes out a subsidized federal Stafford loan for $2400. After a year, Paul decides to pay off the interest, which is 7% of $2400. How much will he pay? $168

43. *Votes for president.* In 2000, Al Gore received 48.62 million votes. This accounted for 48.36% of all votes cast. How many people voted in the 2000 presidential election? 100 million voters

44. *Lotteries and education.* In 1997, $6.2 billion of state lottery money was used for education. This accounted for 52% of all lottery proceeds for the year (*Source*: *Statistical Abstract of the United States*, 1999). What was the total amount of lottery proceeds in 1997? (Round to the nearest tenth of a billion.) About $11.9 billion

45. *Infant health.* In a study of 300 pregnant women with "poor" diets, 8% had babies in good or excellent health. How many women in this group had babies in good or excellent health? 24 women

46. *Infant health.* In a study of 300 pregnant women with "good-to-excellent" diets, 95% had babies in good or excellent health. How many women in this group had babies in good or excellent health? 285 women

47. *Nut consumption.* Each American consumes, on average, 2.25 lb of tree nuts each year (*Source*: *USA Today*, 2/17/00). Of this amount, 25% is almonds. How many pounds of almonds does the average American consume each year? 0.5625 lb

48. *Junk mail.* The U.S. Postal Service reports that we open and read 78% of the junk mail that we receive. A business sends out 9500 advertising brochures. How many of them can the business expect to be opened and read? 7410 brochures

49. *Left-handed bowlers.* It has been determined by sociologists that 17% of the population is left-handed. Each week 160 bowlers enter a tournament conducted by the Professional Bowlers Association. How many would you expect to be left-handed? (Round to the nearest one.) 27 bowlers

50. *Kissing and colds.* In a medical study, it was determined that if 800 people kiss someone else who has a cold, only 56 will actually catch the cold. What percent is this? 7%

51. On a test of 88 items, a student got 76 correct. What percent were correct? $86\frac{4}{11}\%$

52. A baseball player had 13 hits in 25 times at bat. What percent were hits? 52%

53. A bill at Officeland totaled $37.80. How much did the merchandise cost if the sales tax is 5%? $36

54. Doreen's checkbook shows that she wrote a check for $987 for building materials. What was the price of the materials if the sales tax is 5%? $940

55. *Deducting sales tax.* A tax-exempt school group received a bill of $157.41 for educational software. The bill incorrectly included sales tax of 6%. How much should the school group pay? $148.50

56. *Deducting sales tax.* A tax-exempt charity received a bill of $145.90 for a sump pump. The bill incorrectly included sales tax of 5%. How much does the charity owe? $138.95

57. *Cost of self-employment.* Because of additional taxes and fewer benefits, it has been estimated that a self-employed person must earn 20% more than a non–self-employed person performing the same task(s). If Roy earns $15 an hour working for Village Copy, how much would he need to earn on his own for a comparable income? $18/hr

58. Refer to Exercise 57. Clara earns $12 an hour working for Round Edge stairbuilders. How much would Clara need to earn on her own for a comparable income? $14.40/hr

59. *Calorie content.* Pepperidge Farm Light Style 7 Grain Bread® has 140 calories in a 3-slice serving. This is 15% less than the number of calories in a serving of regular bread. How many calories are in a serving of regular bread? 165 calories

60. *Fat content.* Peek Freans Shortbread Reduced Fat Cookies® contain 35 calories of fat in each serving. This is 40% less than the fat content in the leading imported shortbread cookie. How many calories of fat are in a serving of the leading shortbread cookie? 58 calories

61. Campus Bookbuyers pays $30 for a book and sells it for $60. Is this a 100% markup or a 50% markup? Explain.

62. If Julian leaves a $12 tip for a $90 dinner, is he being generous, stingy, or neither? Explain.

SKILL MAINTENANCE

Translate to an algebraic expression.

63. 5 more than some number [1.1] Let n represent the number; $5 + n$

64. 4 less than Tino's weight [1.1] Let t represent Tino's weight; $t - 4$

65. The product of 8 and twice a [1.1] $8 \cdot 2a$

66. 1 more than the product of two numbers [1.1] Let x and y represent the two numbers; $1 + xy$

SYNTHESIS

67. Does the following advertisement provide a convincing argument that summertime is when most burglaries occur? Why or why not?

68. Erin is returning a tent that she bought during a 25%-off storewide sale that has ended. She is offered store credit for 125% of what she paid (not to be used on sale items). Is this fair to Erin? Why or why not?

69. The community of Bardville has 1332 left-handed females. If 48% of the community is female and 15% of all females are left-handed, how many people are in the community? 18,500 people

70. Rollie's Music charges $11.99 for a compact disc. Sound Warp charges $13.99 but you have a coupon for $2 off. In both cases, a 7% sales tax is charged on the *regular* price. How much does the disc cost at each store? Rollie's: $12.83; Sound Warp: $12.97

71. The new price of a car is 25% higher than the old price. The old price is what percent lower than the new price? 20%

72. Claude pays 26% of his pretax earnings in taxes. What percentage of his *post*-tax earnings is this? $35\frac{5}{37}\%$

73. *U.S. birth rate.* There were 3.88 million births in 1997 and 3.94 million births in 1998 (*Source*: *Burlington Free Press*, page 2A, March 29, 2000). By what percentage did the number of births increase? About 1.55%

Aha!

74. Would it be better to receive a 5% raise and then an 8% raise or the other way around? Why?

75. Herb is in the 30% tax bracket. This means that 30¢ of each dollar earned goes to taxes. Which would cost him the least: contributing $50 that is tax-deductible or contributing $40 that is not tax-deductible? Explain.

COLLABORATIVE CORNER

Sales and Discounts

Focus: Applications and models using percent

Time: 15 minutes

Group size: 3

Materials: Calculators are optional.

Often a store will reduce the price of an item by a fixed percentage. When the sale ends, the items are returned to their original prices. Suppose a department store reduces all sporting goods 20%, all clothing 25%, and all electronics 10%.

ACTIVITY

1. Each group member should select one of the following items: a $50 basketball, an $80 jacket, or a $200 portable sound system. Fill in the first three columns of the first three rows of the chart below.
2. Apply the appropriate discount and determine the sale price of your item. Fill in the fourth column of the chart.
3. Next, find a multiplier that can be used to convert the sale price back to the original price and fill in the remaining column of the chart. Does this multiplier depend on the price of the item?
4. Working as a group, compare the results of part (3) for all three items. Then develop a formula for a multiplier that will restore a sale price to its original price, p, after a discount r has been applied. Complete the fourth row of the table and check that your formula will duplicate the results of part (3).
5. Use the formula from part (4) to find the multiplier that a store would use to return an item to its original price after a "30% off" sale expires. Fill in the last line on the chart.
6. Inspect the last column of your chart. How can these multipliers be used to determine the percentage by which a sale price is increased when a sale ends?

Original Price, p	Discount, r	$1 - r$	Sale Price	Multiplier to convert back to p
p	r	$1 - r$		
	.30			

Problem Solving

2.5

Five Steps for Problem Solving • Applying the Five Steps

Probably the most important use of algebra is as a tool for problem solving. In this section, we develop a problem-solving approach that will be used throughout the remainder of the text.

Five Steps for Problem Solving

In Section 2.4, we solved a problem in which Middlesex Toy and Hobby needed a formula. To solve the problem, we *familiarized* ourselves with percent notation so that we could then *translate* the problem into an equation. At the end of the section, we *solved* the equation, *checked* the solution, and *stated* the answer.

Five Steps for Problem Solving in Algebra

1. *Familiarize* yourself with the problem.
2. *Translate* to mathematical language. (This often means writing an equation.)
3. *Carry out* some mathematical manipulation. (This often means *solving* an equation.)
4. *Check* your possible answer in the original problem.
5. *State* the answer clearly.

Of the five steps, the most important is probably the first one: becoming familiar with the problem. Here are some hints for familiarization.

To Become Familiar with a Problem

1. Read the problem carefully. Try to visualize the problem.
2. Reread the problem, perhaps aloud. Make sure you understand all important words.
3. List the information given and the question(s) to be answered. Choose a variable (or variables) to represent the unknown and specify what the variable represents. For example, let L = length in centimeters, d = distance in miles, and so on.
4. Look for similarities between the problem and other problems you have already solved.
5. Find more information. Look up a formula in a book, at a library, or on-line. Consult a reference librarian or an expert in the field.

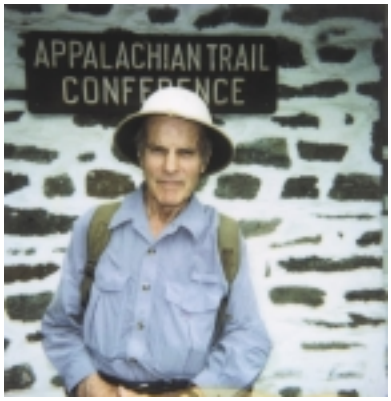
(continued)

6. Make a table that uses all the information you have available. Look for patterns that may help in the translation.
7. Make a drawing and label it with known and unknown information, using specific units if given.
8. Think of a possible answer and check the guess. Observe the manner in which the guess is checked.

Applying the Five Steps

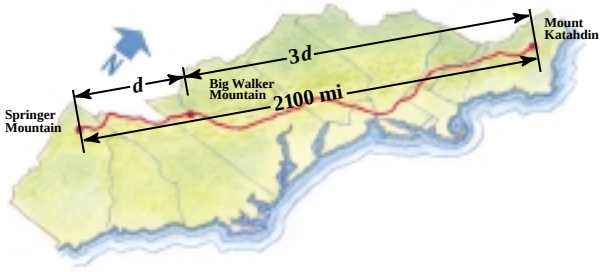
Example 1

Hiking. In 1998, at age 79, Earl Shaffer became the oldest person to hike all 2100 miles of the Appalachian Trail—from Springer Mountain, Georgia, to Mount Katahdin, Maine (*Source*: Appalachian Trail Conference). At one point, Shaffer stood atop Big Walker Mountain, Virginia, which is three times as far from the northern end as from the southern end. How far was Shaffer from each end of the trail?



Solution

1. Familiarize. It may be helpful to make a drawing.



To gain some familiarity, let's suppose that Shaffer stood 600 mi from Springer Mountain. Three times 600 mi is 1800 mi. Since 600 mi + 1800 mi = 2400 mi and 2400 mi > 2100 mi, we see that our guess is too large. Rather than guess again, we let

d = the distance, in miles, to the southern end,

and

$3d$ = the distance, in miles, to the northern end.

(We could also let x = the distance to the northern end and $\frac{1}{3}x$ = the distance to the southern end.)

2. Translate. From the drawing, we see that the lengths of the two parts of the trail must add up to 2100 mi. This leads to our translation.

Rewording: Distance to southern end plus distance to northern end is 2100 mi

Translating: $d + 3d = 2100$

3. Carry out. We solve the equation:

$$\begin{aligned} d + 3d &= 2100 \\ 4d &= 2100 \quad \text{Combining like terms} \\ d &= 525. \quad \text{Dividing both sides by 4} \end{aligned}$$

4. Check. As expected, d is less than 600 mi. If $d = 525$ mi, then $3d = 1575$ mi. Since 525 mi + 1575 mi = 2100 mi, we have a check.

5. State. Atop Big Walker Mountain, Shaffer stood 525 mi from Springer Mountain and 1575 mi from Mount Katahdin.

Example 2

Page numbers. The sum of two consecutive page numbers is 305. Find the page numbers.

Solution

1. Familiarize. If the meaning of the word consecutive is unclear, we should consult a dictionary or someone who might know. Consecutive numbers are integers that are one unit apart. Thus, 18 and 19 are consecutive numbers, as are -24 and -23. Let's "guess and check": If the first page number is 40, the next would be 41. Since $40 + 41 = 81$ and $81 < 305$, our guess is much too small. Suppose the first page number is 130. The next page would then be 131. Since $130 + 131 = 261$ and $261 < 305$, our guess is still a bit too small. We could continue guessing, but algebra offers a more direct approach. Let's have

$x =$ the first page number

and, since the two numbers must be one unit apart,

$x + 1 =$ the next page number.

2. Translate. We reword the problem and translate as follows.

Rewording:	First page number	plus	next page number	is	305	
Translating:	x	$+$	$(x + 1)$	$=$	305	

3. Carry out. We solve the equation:

$$\begin{aligned} x + (x + 1) &= 305 \\ 2x + 1 &= 305 \quad \text{Using an associative law and combining like terms} \\ 2x &= 304 \quad \text{Subtracting 1 from both sides} \\ x &= 152. \quad \text{Dividing both sides by 2} \end{aligned}$$

If x is 152, then $x + 1$ is 153.

4. Check. Our possible answers are 152 and 153. These are consecutive integers and their sum is 305, so the answers check in the original problem.

5. State. The page numbers are 152 and 153.

Study Tip

Do not be surprised if your success rate drops some as you work through the exercises in this section. *This is normal.* Your success rate will increase as you gain experience with these types of problems and use some of the study tips already listed.

Example 3

Taxi rates. In Bermuda, a taxi ride costs \$4.80 plus \$1.68 for each mile traveled. Debbie and Alex have budgeted \$18 for a taxi ride (excluding tip). How far can they travel on their \$18 budget?

Solution

1. **Familiarize.** Suppose the taxi takes Debbie and Alex 5 mi. Such a ride would cost \$4.80 + 5(\$1.68), or \$4.80 + \$8.40 = \$13.20. Since 13.20 < 18, we know that the actual answer exceeds 5 mi. Rather than guess again, we let

   s = the distance, in miles, driven by the taxi for \$18.

2. **Translate.** We rephrase the problem and translate.

   Rewording: The initial charge plus the mileage charge is \$18.

   Translating: $4.80 + s(\$1.68) = \$18$

3. **Carry out.** We solve as follows:

$$\begin{aligned} 4.80 + 1.68s &= 18 \\ 1.68s &= 13.20 && \text{Subtracting 4.80 from both sides} \\ s &= \frac{13.20}{1.68} && \text{Dividing both sides by 1.68} \\ s &\approx 7.8. && \text{Simplifying} \end{aligned}$$

   In the work above, the symbol ≈ means *is approximately equal to*. We use it here because we rounded off our result.

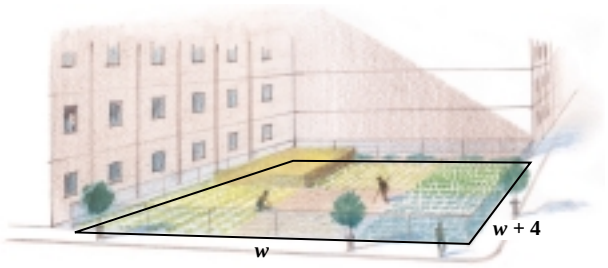
4. **Check.** A 7.8-mi taxi ride would cost \$4.80 + 7.8(\$1.68), or \$17.90. Since we rounded down when finding s, the check will not be precise.

5. **State.** Debbie and Alex can take a 7.8-mi taxi ride and stay within their budget.

The division in Example 3 would normally be rounded *up* to 7.9. In the circumstances of the problem, however, this would lead to a taxi fare slightly in excess of \$18. When solving problems, be careful whenever you round off to round in the appropriate direction.

Example 4

Gardening. A rectangular community garden is to be enclosed with 92 m of fencing. In order to allow for compost storage, the garden must be 4 m longer than it is wide. Determine the dimensions of the garden.



Solution

1. **Familiarize.** Recall that the perimeter of a rectangle is twice the length plus twice the width. Suppose the garden were 30 m wide. The length would then be 30 + 4 m, or 34 m, and the perimeter would be $2 \cdot 30$ m + $2 \cdot 34$ m, or 128 m. This shows that for the perimeter to be 92 m, the width must be less than 30 m. Instead of guessing again, we let w = the width of the garden, in meters. Since the garden is "4 m longer than it is wide," we let $w + 4$ = the length of the garden, in meters.

2. **Translate.** To translate, we use $w + 4$ as the length and 92 as the perimeter.

Rewording: Twice the length plus twice the width is 92 m.

Translating: $2(w + 4) + 2w = 92$

To double the length, $w + 4$, parentheses are essential.

3. **Carry out.** We solve the equation:

$$\begin{aligned} 2(w + 4) + 2w &= 92 \\ 2w + 8 + 2w &= 92 \quad \text{Using the distributive law} \\ 4w + 8 &= 92 \quad \text{Combining like terms} \\ 4w &= 84 \\ w &= 21. \end{aligned}$$

The dimensions appear to be $w = 21$ m and l, or $w + 4$, $= 25$ m.

4. **Check.** If the width is 21 m and the length 25 m, then the garden is 4 m longer than it is wide. The perimeter is 2(25 m) + 2(21 m), or 92 m, and since 92 m of fencing is available, we have a check.

5. **State.** The garden should be 21 m wide and 25 m long.

Caution! Always be sure to answer the original problem completely. For instance, in Example 1 we need to find *two* numbers: the distances from *each* end of the trail to the hiker. Similarly, in Example 2 we needed to find two page numbers and in Example 4 we needed to find two dimensions, not just the width.

Example 5

Selling a home. The McCanns are planning to sell their home. If they want to be left with $117,500 after paying 6% of the selling price to a realtor as a commission, for how much must they sell the house?

Solution

1. Familiarize. Suppose the McCanns sell the house for $120,000. A 6% commission can be determined by finding 6% of $120,000:

$$6\% \text{ of } \$120{,}000 = 0.06(\$120{,}000) = \$7200.$$

Subtracting this commission from $120,000 would leave the McCanns with

$$\$120{,}000 - \$7200 = \$112{,}800.$$

This shows that in order for the McCanns to clear $117,500, the house must sell for more than $120,000. To determine what the sale price must be, we could check more guesses. Instead, we let $x =$ the selling price, in dollars. With a 6% commission, the realtor would receive $0.06x$.

2. Translate. We reword the problem and translate as follows.

Rewording: Selling price less commission is amount remaining.

Translating: $x \quad - \quad 0.06x \quad = \quad 117{,}500$

3. Carry out. We solve the equation:

$$x - 0.06x = 117{,}500$$
$$1x - 0.06x = 117{,}500$$
$$0.94x = 117{,}500 \qquad \text{Combining like terms. Had we noted that after the commission has been paid, 94\% remains, we could have begun with this equation.}$$
$$x = \frac{117{,}500}{0.94} \qquad \text{Dividing both sides by 0.94}$$
$$x = 125{,}000.$$

4. Check. To check, we first find 6% of \$125,000:

$$6\% \text{ of } \$125{,}000 = 0.06(\$125{,}000) = \$7500. \qquad \text{This is the commission.}$$

Next, we subtract the commission to find the remaining amount:

$$\$125{,}000 - \$7500 = \$117{,}500.$$

Since, after the commission, the McCanns are left with \$117,500, our answer checks. Note that the \$125,000 sale price is greater than \$120,000, as predicted in the *Familiarize* step.

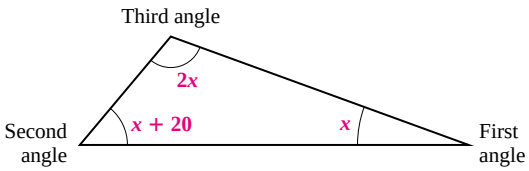
5. State. To be left with \$117,500, the McCanns must sell the house for \$125,000.

Example 6

Angles in a triangle. The second angle of a triangle is 20° greater than the first. The third angle is twice as large as the first. How large are the angles?

Solution

1. Familiarize. We make a drawing. In this case, the measure of the first angle is x, the measure of the second angle is $x + 20$, and the measure of the third angle is $2x$.



2. Translate. To translate, we need to recall that the sum of the measures of the angles in a triangle is 180°.

Rewording: Measure of first angle + measure of second angle + measure of third angle is 180°

Translating: $x + (x + 20) + 2x = 180$

3. Carry out. We solve:

$$\begin{aligned} x + (x + 20) + 2x &= 180 \\ 4x + 20 &= 180 \\ 4x &= 160 \\ x &= 40. \end{aligned}$$

The measures for the angles appear to be:

First angle: $x = 40°$,
Second angle: $x + 20 = 40 + 20 = 60°$,
Third angle: $2x = 2(40) = 80°$.

4. Check. Consider 40°, 60°, and 80°. The second angle is 20° greater than the first, the third is twice the first, and the sum is 180°. These numbers check.

5. State. The measures of the angles are 40°, 60°, and 80°.

We close this section with some tips to aid you in problem solving.

Problem-Solving Tips

1. The more problems you solve, the more your skills will improve.
2. Look for patterns when solving problems. Each time you study an example in a text, you may observe a pattern for problems that you will encounter later in the exercise sets or in other practical situations.
3. When translating in mathematics, consider the dimensions of the variables and constants in the equation. The variables that represent length should all be in the same unit, those that represent money should all be in dollars or all in cents, and so on.
4. Make sure that units appear in the answer whenever appropriate and that you have completely answered the original problem.

Exercise Set 2.5

FOR EXTRA HELP


Digital Video Tutor CD 2
Videotape 4


InterAct Math


Math Tutor Center

MathXL


MyMathLab.com

Solve. Even though you might find the answer quickly in some other way, practice using the five-step problem-solving process.

1. Three less than twice a number is 19. What is the number? 11
2. Two fewer than ten times a number is 78. What is the number? 8
3. Five times the sum of 3 and some number is 70. What is the number? 11
4. Twice the sum of 4 and some number is 34. What is the number? 13
5. *Price of sneakers.* Amy paid $63.75 for a pair of New Balance 903 running shoes during a 15%-off sale. What was the regular price? $75

6. *Price of a CD player.* Doug paid $72 for a shockproof portable CD player during a 20%-off sale. What was the regular price? $90

7. *Price of a textbook.* Evelyn paid $89.25, including 5% tax, for her biology textbook. How much did the book itself cost? $85

8. *Price of a printer.* Jake paid $100.70, including 6% tax, for a color printer. How much did the printer itself cost? $95

9. *Running.* In 1997, Yiannis Kouros of Australia set the record for the greatest distance run in 24 hr by running 188 mi (*Source*: *Guinness World Records 2000 Millennium Edition*). After 8 hr, he was approximately twice as far from the finish line as he was from the start. How far had he run? Approximately $62\frac{2}{3}$ mi

10. *Sled-dog racing.* The Iditarod sled-dog race extends for 1049 mi from Anchorage to Nome. If a musher is twice as far from Anchorage as from Nome, how many miles has the musher traveled? $699\frac{1}{3}$ mi

11. The sum of three consecutive page numbers is 60. Find the numbers. 19, 20, 21

12. The sum of three consecutive page numbers is 99. Find the numbers. 32, 33, 34

13. The sum of two consecutive odd numbers is 60. Find the numbers. (*Hint*: Odd numbers, like even numbers, are separated by two units.) 29, 31

14. The sum of two consecutive odd integers is 108. What are the integers? 53, 55

15. The sum of two consecutive even integers is 126. What are the integers? 62, 64

16. The sum of two consecutive even numbers is 50. Find the numbers. 24, 26

17. *Oldest groom.* The world's oldest groom was 19 yr older than his bride (*Source*: *Guinness World Records 2000 Millennium Edition*). Together, their ages totaled 187 yr. How old were the bride and the groom? Bride: 84 yr; groom: 103 yr

18. *Oldest divorcees.* In the world's oldest divorcing couple, the woman was 6 yr younger than the man (*Source*: *Guinness World Records 2000 Millennium Edition*). Together, their ages totaled 188 yr. How old were the man and the woman? Man: 97 yr; woman: 91 yr

19. *Angles of a triangle.* The second angle of a triangle is three times as large as the first. The third angle is 30° more than the first. Find the measure of each angle. 30°, 90°, 60°

20. *Angles of a triangle.* The second angle of a triangle is four times as large as the first. The third angle is 45° less than the sum of the other two angles. Find the measure of each angle. 22.5°, 90°, 67.5°

21. *Angles of a triangle.* The second angle of a triangle is three times as large as the first. The third angle is 10° more than the sum of the other two angles. Find the measure of the third angle. 95°

22. *Angles of a triangle.* The second angle of a triangle is four times as large as the first. The third angle is 5° more than the sum of the other two angles. Find the measure of the second angle. 70°

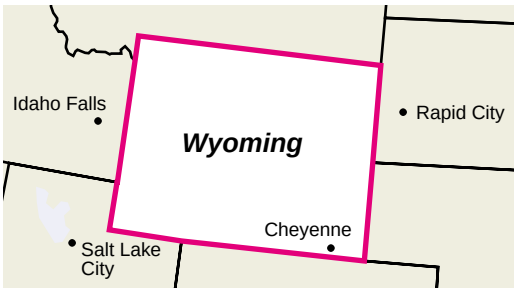
23. *Page numbers.* The sum of the page numbers on the facing pages of a book is 385. What are the page numbers? 192, 193

24. *Page numbers.* The sum of the page numbers on the facing pages of a book is 281. What are the page numbers? 140, 141

25. *Perimeter of a triangle.* The perimeter of a triangle is 195 mm. If the lengths of the sides are consecutive odd integers, find the length of each side. 63 mm, 65 mm, 67 mm

26. *Hancock Building dimensions.* The top of the John Hancock Building in Chicago is a rectangle whose length is 60 ft more than the width. The perimeter is 520 ft. Find the width and the length of the rectangle. Find the area of the rectangle. Width: 100 ft; length: 160 ft; area: 16,000 ft^2

27. *Dimensions of a state.* The perimeter of the state of Wyoming is 1280 mi. The width is 90 mi less than the length. Find the width and the length. Width: 275 mi; length: 365 mi



28. *Copier paper.* The perimeter of standard-size copier paper is 99 cm. The width is 6.3 cm less than the length. Find the length and the width. Length: 27.9 cm; width: 21.6 cm

29. *Stock prices.* Sarah's investment in America Online stock grew 28% to $448. How much did she invest? $350

30. *Savings interest.* Sharon invested money in a savings account at a rate of 6% simple interest. After 1 yr, she has $6996 in the account. How much did Sharon originally invest? $6600

31. *Credit cards.* The balance in Will's Mastercard® account grew 2%, to $870, in one month. What was his balance at the beginning of the month? $852.94

32. *Loan interest.* Alvin borrowed money from a cousin at a rate of 10% simple interest. After 1 yr, $7194 paid off the loan. How much did Alvin borrow? $6540

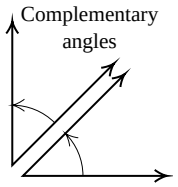
33. *Taxi fares.* In Beniford, taxis charge $3 plus 75¢ per mile for an airport pickup. How far from the airport can Courtney travel for $12? 12 mi

34. *Taxi fares.* In Cranston, taxis charge $4 plus 90¢ per mile for an airport pickup. How far from the airport can Ralph travel for $17.50? 15 mi

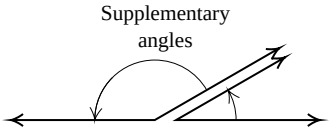
35. *Truck rentals.* Truck-Rite Rentals rents trucks at a daily rate of $49.95 plus 39¢ per mile. Concert Productions has budgeted $100 for renting a truck to haul equipment to an upcoming concert. How far can they travel in one day and stay within their budget? $128\frac{1}{3}$ mi

36. *Truck rentals.* Fine Line Trucks rents an 18-ft truck for $42 plus 35¢ per mile. Judy needs a truck for one day to deliver a shipment of plants. How far can she drive and stay within a budget of $70? 80 mi

37. *Complementary angles.* The sum of the measures of two *complementary* angles is 90°. If one angle measures 15° more than twice the measure of its complement, find the measure of each angle. 65°, 25°



38. *Supplementary angles.* The sum of the measures of two *supplementary* angles is 180°. If one angle measures 45° less than twice the measure of its supplement, find the measure of each angle. 105°, 75°



39. *Cricket chirps and temperature.* The equation $T = \frac{1}{4}N + 40$ can be used to determine the temperature T, in degrees Fahrenheit, given the number of times N a cricket chirps per minute. Determine the number of chirps per minute for a temperature of 80°F. 160

40. *Race time.* The equation $R = -0.028t + 20.8$ can be used to predict the world record in the 200-m dash, where R is the record in seconds and t is the number of years since 1920. In what year will the record be 18.0 sec? 2020

41. Sean claims he can solve most of the problems in this section by guessing. Is there anything wrong with this approach? Why or why not?

42. When solving Exercise 17, Beth used a to represent the bride's age and Ben used a to represent the groom's age. Is one of these approaches preferable to the other? Why or why not?

SKILL MAINTENANCE

Write a true sentence using either $<$ or $>$.

43. $-9 \quad 5$ [1.4] $<$

44. $1 \quad 3$ [1.4] $<$

45. $-4 \quad 7$ [1.4] $<$

46. $-9 \quad -12$ [1.4] $>$

SYNTHESIS

47. Write a problem for a classmate to solve. Devise it so that the problem can be translated to the equation $x + (x + 2) + (x + 4) = 375$.

48. Write a problem for a classmate to solve. Devise it so that the solution is "Audrey can drive the rental truck for 50 mi without exceeding her budget."

49. *Discounted dinners.* Kate's "Dining Card" entitles her to $10 off the price of a meal after a 15% tip has been added to the cost of the meal. If, after the discount, the bill is $32.55, how much did the meal originally cost? $37

50. *Test scores.* Pam scored 78 on a test that had 4 fill-ins worth 7 points each and 24 multiple-choice questions worth 3 points each. She had one fill-in wrong. How many multiple-choice questions did Pam get right? 19

51. *Gettysburg Address.* Abraham Lincoln's 1863 Gettysburg Address refers to the year 1776 as "four *score* and seven years ago." Determine what a score is. 20

52. One number is 25% of another. The larger number is 12 more than the smaller. What are the numbers? 4, 16

53. *Perimeter of a rectangle.* The width of a rectangle is three fourths of the length. The perimeter of the rectangle becomes 50 cm when the length and the width are each increased by 2 cm. Find the length and the width. Length: 12 cm; width: 9 cm

54. *Angles in a quadrilateral.* The measures of the angles in a quadrilateral are consecutive odd numbers. Find the measure of each angle. 87°, 89°, 91°, 93°

55. *Angles in a pentagon.* The measures of the angles in a pentagon are consecutive even numbers. Find the measure of each angle. 104°, 106°, 108°, 110°, 112°

56. *Sharing fruit.* Apples are collected in a basket for six people. One third, one fourth, one eighth, and one fifth of the apples are given to four people, respectively. The fifth person gets ten apples, and one apple remains for the sixth person. Find the original number of apples in the basket. 120

57. *Discounts.* In exchange for opening a new credit account, Filene's Department Stores® subtracts 10% from all purchases made the day the account is established. Julio is opening an account and has a coupon for which he receives 10% off the first day's reduced price of a camera. If Julio's final price is $77.75, what was the price of the camera before the two discounts? $95.99

58. *Winning percentage.* In a basketball league, the Falcons won 15 of their first 20 games. In order to win 60% of the total number of games, how many more games will they have to play, assuming they win only half of the remaining games? 30

59. *Music-club purchases.* During a recent sale, BMG Music Service® charged $8.49 for the first CD ordered and $3.99 for all others. For shipping and handling, BMG charged $2.47 for the first CD, $2.28 for the second CD, and $1.99 for all others. The total cost of a shipment (excluding tax) was $65.07. How many CD's were in the shipment? 10

60. *Test scores.* Ella has an average score of 82 on three tests. Her average score on the first two tests is 85. What was the score on the third test? 76

61. *Taxi fares.* In New York City, a taxi ride costs $2 plus 30¢ per $\frac{1}{5}$ mile and 20¢ per minute stopped in traffic. Due to traffic, Glenda's taxi took 20 min to complete what is usually a 10-min drive. If she is charged $13 for the ride, how far did Glenda travel? 6 mi

62. A school purchases a piano and must choose between paying $2000 at the time of purchase or $2150 at the end of one year. Which option should the school select and why?

63. Annette claims the following problem has no solution: "The sum of the page numbers on facing pages is 191. Find the page numbers." Is she correct? Why or why not?

Aha!

64. The perimeter of a rectangle is 101.74 cm^2. If the length is 4.25 cm longer than the width, find the dimensions of the rectangle. Width: 23.31 cm; length: 27.56 cm

65. The second side of a triangle is 3.25 cm longer than the first side. The third side is 4.35 cm longer than the second side. If the perimeter of the triangle is 26.87 cm, find the length of each side. 5.34 cm, 8.59 cm, 12.94 cm

Solving Inequalities

2.6

Solutions of Inequalities • Graphs of Inequalities • Solving Inequalities Using the Addition Principle • Solving Inequalities Using the Multiplication Principle • Using the Principles Together

Many real-world situations translate to *inequalities*. For example, a student might need to register for *at least* 12 credits; an elevator might be designed to hold *at most* 2000 pounds; a tax credit might be allowable for families with incomes of *less than* $25,000; and so on. Before solving applications of this type, we must adapt our equation-solving principles to the solving of inequalities.

Solutions of Inequalities

Recall from Section 1.4 that an inequality is a number sentence containing $>$ (is greater than), $<$ (is less than), $\geq$ (is greater than or equal to), or $\leq$ (is less than or equal to). Inequalities like

$$-7 > x, \qquad t < 5, \qquad 5x - 2 \geq 9, \quad \text{and} \quad -3y + 8 \leq -7$$

are true for some replacements of the variable and false for others.

Example 1 Determine whether the given number is a solution of $x < 2$: **(a)** -3; **(b)** 2.

Solution

a) Since $-3 < 2$ is true, -3 is a solution.
b) Since $2 < 2$ is false, 2 is not a solution.

Example 2 Determine whether the given number is a solution of $y \geq 6$: **(a)** 6; **(b)** -4.

Solution

a) Since $6 \geq 6$ is true, 6 is a solution.
b) Since $-4 \geq 6$ is false, -4 is not a solution.

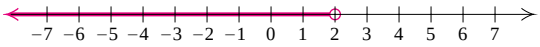
Graphs of Inequalities

Because the solutions of inequalities like $x < 2$ are too numerous to list, it is helpful to make a drawing that represents all the solutions. The **graph** of an inequality is such a drawing. Graphs of inequalities in one variable can be drawn on a number line by shading all points that are solutions. Open dots are used to indicate endpoints that are *not* solutions and closed dots indicate endpoints that *are* solutions.

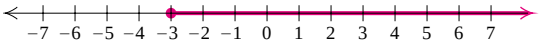
Example 3 Graph each inequality: **(a)** $x < 2$; **(b)** $y \geq -3$; **(c)** $-2 < x \leq 3$.

Solution

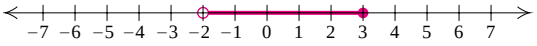
a) The solutions of $x < 2$ are those numbers less than 2. They are shown on the graph by shading all points to the left of 2. The open dot at 2 and the shading to its left indicates that 2 is *not* part of the graph, but numbers like 1.2 and 1.99 are.



b) The solutions of $y \geq -3$ are shown on the number line by shading the point for -3 and all points to the right of -3. The closed dot at -3 indicates that -3 *is* part of the graph.

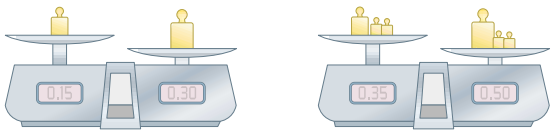


c) The inequality $-2 < x \leq 3$ is read "-2 is less than x *and* x is less than or equal to 3," or "x is greater than -2 *and* less than or equal to 3." To be a solution of $-2 < x \leq 3$, a number must be a solution of both $-2 < x$ *and* $x \leq 3$. The number 1 is a solution, as are -0.5, 1.9, and 3. The open dot indicates that -2 is *not* a solution, whereas the closed dot indicates that 3 *is* a solution. The other solutions are shaded.



Solving Inequalities Using the Addition Principle

Consider a balance similar to one that appears in Section 2.1. When one side of the balance holds more weight than the other, the balance tips in that direction. If equal amounts of weight are then added to or subtracted from both sides of the balance, the balance remains tipped in the same direction.



The balance illustrates the idea that when a number, such as 2, is added to (or subtracted from) both sides of a true inequality, such as $3 < 7$, we get another true inequality:

$$3 + 2 < 7 + 2, \quad \text{or} \quad 5 < 9.$$

Similarly, if we add -4 to both sides of $x + 4 < 10$, we get an *equivalent* inequality:

$$x + 4 + (-4) < 10 + (-4), \quad \text{or} \quad x < 6.$$

We say that $x + 4 < 10$ and $x < 6$ are **equivalent**, which means that both inequalities have the same solution set.

The Addition Principle for Inequalities

For any real numbers a, b, and c:

$a < b$ is equivalent to $a + c < b + c$;
$a \le b$ is equivalent to $a + c \le b + c$;
$a > b$ is equivalent to $a + c > b + c$;
$a \ge b$ is equivalent to $a + c \ge b + c$.

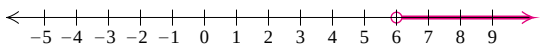
As with equations, our goal is to isolate the variable on one side.

Example 4 Solve $x + 2 > 8$ and then graph the solution.

Solution We use the addition principle, subtracting 2 from both sides:

$$x + 2 - 2 > 8 - 2 \qquad \text{Subtracting 2 from, or adding } -2 \text{ to, both sides}$$
$$x > 6.$$

From the inequality $x > 6$, we can determine the solutions easily. Any number greater than 6 makes $x > 6$ true and is a solution of that inequality as well as the inequality $x + 2 > 8$. The graph is as follows:



Because most inequalities have an infinite number of solutions, we cannot possibly check them all. A partial check can be made using one of the possible solutions. For this example, we can substitute any number greater than 6—say, 6.1—into the original inequality:

$$\begin{array}{r|l} x + 2 > 8 & \\ \hline 6.1 + 2 \stackrel{?}{} 8 & \\ 8.1 \mid 8 & \text{TRUE} \end{array} \qquad 8.1 > 8 \text{ is a true statement.}$$

Since $8.1 > 8$ is true, 6.1 is a solution. Any number greater than 6 is a solution.

Although the inequality $x > 6$ is easy to solve (we merely replace x with numbers greater than 6), it is worth noting that $x > 6$ is an *inequality*, not a *solution*. In fact, the solutions of $x > 6$ are numbers. To describe the set of all solutions, we will use **set-builder notation** to write the *solution set* of Example 4 as

$$\{x \mid x > 6\}.$$

This notation is read

"The set of all x such that x is greater than 6."

Thus a number is in $\{x \mid x > 6\}$ if that number is greater than 6. From now on, solutions of inequalities will be written using set-builder notation.

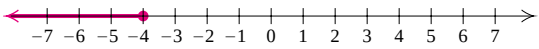
Example 5

Solve $3x - 1 \le 2x - 5$ and then graph the solution.

Solution We have

$$\begin{aligned} 3x - 1 &\le 2x - 5 \\ 3x - 1 + 1 &\le 2x - 5 + 1 && \text{Adding 1 to both sides} \\ 3x &\le 2x - 4 && \text{Simplifying} \\ 3x - 2x &\le 2x - 4 - 2x && \text{Subtracting } 2x \text{ from both sides} \\ x &\le -4. && \text{Simplifying} \end{aligned}$$

The graph is as follows:



Any number less than or equal to -4 is a solution, so the solution set is $\{x \mid x \le -4\}$.

technology connection

As a partial check of Example 5, we can let $y_1 = 3x - 1$ and $y_2 = 2x - 5$. By scrolling up or down, you can note that for $x \le -4$, we have $y_1 \le y_2$.

X	Y1	Y2
−5	−16	−15
−4	−13	−13
−3	−10	−11
−2	−7	−9
−1	−4	−7
0	−1	−5
1	2	−3

X = −5

Solving Inequalities Using the Multiplication Principle

There is a multiplication principle for inequalities similar to that for equations, but it must be modified when multiplying both sides by a negative number. Consider the true inequality

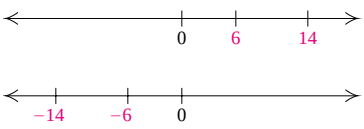
$$3 < 7.$$

If we multiply both sides by a *positive* number, say 2, we get another true inequality:

$$3 \cdot 2 < 7 \cdot 2, \quad \text{or} \quad 6 < 14. \qquad \text{TRUE}$$

If we multiply both sides by a negative number, say -2, we get a *false* inequality:

$$3 \cdot (-2) < 7 \cdot (-2), \quad \text{or} \quad -6 < -14. \qquad \text{FALSE}$$



The fact that $6 < 14$ is true, but $-6 < -14$ is false, stems from the fact that the negative numbers, in a sense, mirror the positive numbers. Whereas 14 is to the *right* of 6, the number -14 is to the *left* of -6. Thus if we reverse the inequality symbol in $-6 < -14$, we get a true inequality:

$$-6 > -14. \qquad \text{TRUE}$$

The Multiplication Principle for Inequalities

For any real numbers a and b, and for any *positive* number c:

$a < b$ is equivalent to $ac < bc$, and
$a > b$ is equivalent to $ac > bc$.

For any real numbers a and b, and for any *negative* number c:

$a < b$ is equivalent to $ac > bc$, and
$a > b$ is equivalent to $ac < bc$.

Similar statements hold for $\leq$ and $\geq$.

Example 6 Solve and graph each inequality: **(a)** $\frac{1}{4}x < 7$; **(b)** $-2y < 18$.

Solution

a) $\frac{1}{4}x < 7$

$4 \cdot \frac{1}{4}x < 4 \cdot 7$ Multiplying both sides by 4, the reciprocal of $\frac{1}{4}$

The symbol stays the same, since 4 is positive.

$x < 28$ Simplifying

The solution set is $\{x \mid x < 28\}$. The graph is as follows:

b) $-2y < 18$

$\frac{-2y}{-2} > \frac{18}{-2}$ Multiplying both sides by $-\frac{1}{2}$, or dividing both sides by -2

At this step, we reverse the inequality, because $-\frac{1}{2}$ is negative.

$y > -9$ Simplifying

As a partial check, we substitute a number greater than -9, say -8, into the original inequality:

$$\begin{array}{r|l} -2y < 18 & \\ \hline -2(-8) \stackrel{?}{|} 18 & \\ 16 \mid 18 & \text{TRUE} \end{array}$$

$16 < 18$ is a true statement.

The solution set is $\{y \mid y > -9\}$. The graph is as follows:

Using the Principles Together

We use the addition and multiplication principles together to solve inequalities much as we did when solving equations.

Example 7

Solve: **(a)** $6 - 5y > 7$; **(b)** $2x - 9 \le 7x + 1$.

Solution

a)

$$6 - 5y > 7$$

$$-6 + 6 - 5y > -6 + 7 \quad \text{Adding } -6 \text{ to both sides}$$

$$-5y > 1 \quad \text{Simplifying}$$

$$-\tfrac{1}{5} \cdot (-5y) < -\tfrac{1}{5} \cdot 1 \quad \text{Multiplying both sides by } -\tfrac{1}{5}\text{, or dividing both sides by } -5$$

Remember to reverse the inequality symbol!

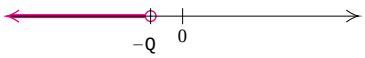
$$y < -\tfrac{1}{5} \quad \text{Simplifying}$$

As a check, we substitute a number smaller than $-\frac{1}{5}$, say -1, into the original inequality:

$6 - 5y > 7$		
$6 - 5(-1)$	?	7
$6 - (-5)$		
11		7 TRUE

$11 > 7$ is a true statement.

The solution set is $\{y \mid y < -\frac{1}{5}\}$. We show the graph in the margin for reference.



b)

$$2x - 9 \le 7x + 1$$

$$2x - 9 - 1 \le 7x + 1 - 1 \quad \text{Subtracting 1 from both sides}$$

$$2x - 10 \le 7x \quad \text{Simplifying}$$

$$2x - 10 - 2x \le 7x - 2x \quad \text{Subtracting } 2x \text{ from both sides}$$

$$-10 \le 5x \quad \text{Simplifying}$$

$$\frac{-10}{5} \le \frac{5x}{5} \quad \text{Dividing both sides by 5}$$

$$-2 \le x \quad \text{Simplifying}$$

The solution set is $\{x \mid -2 \le x\}$, or $\{x \mid x \ge -2\}$.

All of the equation-solving techniques used in Sections 2.1 and 2.2 can be used with inequalities provided we remember to reverse the inequality symbol when multiplying or dividing both sides by a negative number.

Example 8 Solve: **(a)** $16.3 - 7.2p \le -8.18$; **(b)** $3(x - 2) - 1 < 2 - 5(x + 6)$.

Solution

a) The greatest number of decimal places in any one number is *two*. Multiplying both sides by 100 will clear decimals. Then we proceed as before.

$$16.3 - 7.2p \le -8.18$$

$100(16.3 - 7.2p) \le 100(-8.18)$ Multiplying both sides by 100

$100(16.3) - 100(7.2p) \le 100(-8.18)$ Using the distributive law

$1630 - 720p \le -818$ Simplifying

$-720p \le -818 - 1630$ Subtracting 1630 from both sides

$-720p \le -2448$ Simplifying

$p \ge \dfrac{-2448}{-720}$ Dividing both sides by -720

Remember to reverse the symbol.

$$p \ge 3.4$$

The solution set is $\{p \mid p \ge 3.4\}$.

b) $3(x - 2) - 1 < 2 - 5(x + 6)$

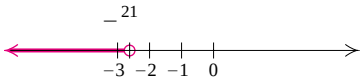
$3x - 6 - 1 < 2 - 5x - 30$ Using the distributive law to remove parentheses

$3x - 7 < -5x - 28$ Simplifying

$3x + 5x < -28 + 7$ Adding $5x$ and also 7 to both sides. This isolates the x-terms on one side.

$8x < -21$ Simplifying

$x < -\dfrac{21}{8}$ Dividing both sides by 8



The solution set is $\left\{x \mid x < -\frac{21}{8}\right\}$.

Exercise Set 2.6

FOR EXTRA HELP

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Determine whether each number is a solution of the given inequality.

1. $x > -2$

a) 5 Yes **b)** 0 Yes **c)** -1.9 Yes
d) -7.3 No **e)** 1.6 Yes

2. $y < 5$

a) 0 Yes **b)** 5 No **c)** 4.99 Yes
d) -13 Yes **e)** $7\frac{1}{4}$ No

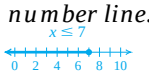
3. $x \geq 6$

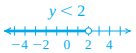
a) -6 No **b)** 0 No **c)** 6 Yes
d) 6.01 Yes **e)** $-3\frac{1}{2}$ No

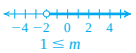
4. $x \leq 10$

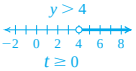
a) 4 Yes **b)** -10 Yes **c)** 0 Yes
d) 10.2 No **e)** -4.7 Yes

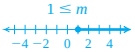
Graph on a number line.

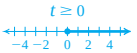
5. $x \leq 7$ 

6. $y < 2$ 

7. $t > -2$ 

8. $y > 4$ 

9. $1 \leq m$ 

10. $0 \leq t$ 

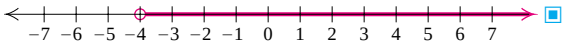
11. $-3 < x \leq 5$ ▣

12. $-5 \leq x < 2$ ▣

13. $0 < x < 3$ ▣

14. $-5 \leq x \leq 0$ ▣

Describe each graph using set-builder notation.

15. 
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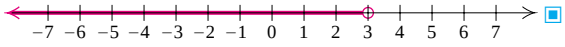
16. 
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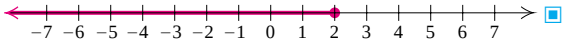
17. 
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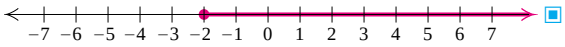
18. 
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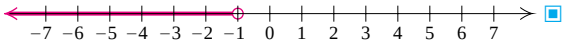
19. 
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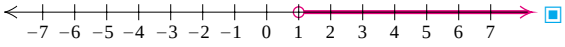
20. 
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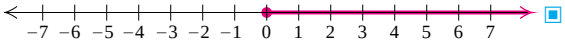
21. 
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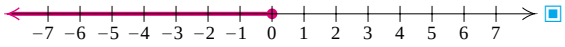
22. 
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Solve using the addition principle. Graph and write set-builder notation for the answers.

23. $y + 2 > 9$ ▣

24. $y + 6 > 9$ ▣

25. $x + 8 \leq -10$ ▣

26. $x + 9 \leq -12$ ▣

27. $x - 3 < 7$ ▣

28. $x - 3 < 14$ ▣

29. $5 \leq t + 8$ ▣

30. $4 \leq t + 9$ ▣

31. $y - 7 > -12$ ▣

32. $y - 10 > -16$ ▣

33. $2x + 4 \leq x + 9$ ▣

34. $2x + 4 \leq x + 1$ ▣

Solve using the addition principle. Write the answers in set-builder notation.

35. $5x - 6 \geq 4x - 1$ $\{x \mid x \geq 5\}$

36. $3x - 9 \geq 2x + 11$ $\{x \mid x \geq 20\}$

37. $y + \frac{1}{3} \leq \frac{5}{6}$ $\{y \mid y \leq \frac{1}{2}\}$

38. $x + \frac{1}{4} \leq \frac{1}{2}$ $\{x \mid x \leq \frac{1}{4}\}$

39. $t - \frac{1}{8} > \frac{1}{2}$ $\{t \mid t > \frac{5}{8}\}$

40. $y - \frac{1}{3} > \frac{1}{4}$ $\{y \mid y > \frac{7}{12}\}$

41. $-9x + 17 > 17 - 8x$ $\{x \mid x < 0\}$

42. $-8n + 12 > 12 - 7n$ $\{n \mid n < 0\}$

Aha!

43. $-23 < -t$ $\{t \mid t < 23\}$

44. $19 < -x$ $\{x \mid x < -19\}$

Solve using the multiplication principle. Graph and write set-builder notation for the answers.

45. $5x < 35$ ▣

46. $8x \geq 32$ ▣

47. $9y \leq 81$ ▣

48. $350 > 10t$ ▣

49. $-7x < 13$ ▣

50. $8y < 17$ ▣

51. $-24 > 8t$ ▣

52. $-16x < -64$ ▣

Solve using the multiplication principle. Write the answers in set-builder notation.

53. $7y \geq -2$ $\{y \mid y \geq -\frac{2}{7}\}$

54. $5x > -3$ $\{x \mid x > -\frac{3}{5}\}$

55. $-2y \leq \frac{1}{5}$ $\{y \mid y \geq -\frac{1}{10}\}$

56. $-2x \geq \frac{1}{5}$ $\{x \mid x \leq -\frac{1}{10}\}$

57. $-\frac{8}{5} > -2x$ $\{x \mid x > \frac{4}{5}\}$

58. $-\frac{5}{8} < -10y$ $\{y \mid y < \frac{1}{16}\}$

Solve using the addition and multiplication principles.

59. $7 + 3x < 34$ $\{x \mid x < 9\}$

60. $5 + 4y < 37$ $\{y \mid y < 8\}$

61. $6 + 5y \geq 26$ $\{y \mid y \geq 4\}$

62. $7 + 8x \geq 71$ $\{x \mid x \geq 8\}$

63. $4t - 5 \leq 23$ $\{t \mid t \leq 7\}$

64. $5y - 9 \leq 21$ $\{y \mid y \leq 6\}$

65. $13x - 7 < -46$ $\{x \mid x < -3\}$

66. $8y - 4 < -52$ $\{y \mid y < -6\}$

67. $16 < 4 - 3y$ $\{y \mid y < -4\}$

68. $22 < 6 - 8x$ $\{x \mid x < -2\}$

69. $39 > 3 - 9x$ $\{x \mid x > -4\}$

70. $40 > 5 - 7y$ $\{y \mid y > -5\}$

71. $5 - 6y > 25$ $\{y \mid y < -\frac{10}{3}\}$

72. $8 - 2y > 14$ $\{y \mid y < -3\}$

73. $-3 < 8x + 7 - 7x$ $\{x \mid x > -10\}$

74. $-5 < 9x + 8 - 8x$ $\{x \mid x > -13\}$

75. $6 - 4y > 4 - 3y$ $\{y \mid y < 2\}$

76. $7 - 8y > 5 - 7y$ $\{y \mid y < 2\}$

77. $7 - 9y \leq 4 - 8y$ $\{y \mid y \geq 3\}$

78. $6 - 13y \leq 4 - 12y$ $\{y \mid y \geq 2\}$

79. $33 - 12x < 4x + 97$ $\{x \mid x > -4\}$

80. $27 - 11x > 14x - 18$ $\{x \mid x < \frac{9}{5}\}$

81. $2.1x + 43.2 > 1.2 - 8.4x$ $\{x \mid x > -4\}$

82. $0.96y - 0.79 \leq 0.21y + 0.46$ $\{y \mid y \leq \frac{5}{3}\}$

83. $0.7n - 15 + n \geq 2n - 8 - 0.4n$ $\{n \mid n \geq 70\}$

84. $1.7t + 8 - 1.62t < 0.4t - 0.32 + 8$ $\{t \mid t > 1\}$

85. $\frac{x}{3} - 4 \leq 1$ $\{x \mid x \leq 15\}$

86. $\frac{2}{3} - \frac{x}{5} < \frac{4}{15}$ $\{x \mid x > 2\}$

▣ Answers to Exercises 11–34 and 45–52 can be found on p. A-40.

87. $3 < 5 - \frac{t}{7}$ $\{t \mid t < 14\}$

88. $2 > 9 - \frac{x}{5}$ $\{x \mid x > 35\}$

89. $4(2y - 3) < 36$ $\{y \mid y < 6\}$

90. $3(2y - 3) > 21$ $\{y \mid y > 5\}$

91. $3(t - 2) \geq 9(t + 2)$ $\{t \mid t \leq -4\}$

92. $8(2t + 1) > 4(7t + 7)$ $\left\{t \mid t < -\frac{5}{3}\right\}$

93. $3(r - 6) + 2 < 4(r + 2) - 21$ $\{r \mid r > -3\}$

94. $5(t + 3) + 9 > 3(t - 2) + 6$ $\{t \mid t > -12\}$

95. $\frac{2}{3}(2x - 1) \geq 10$ $\{x \mid x \geq 8\}$

96. $\frac{4}{5}(3x + 4) \leq 20$ $\{x \mid x \leq 7\}$

97. $\frac{3}{4}\left(3x - \frac{1}{2}\right) - \frac{2}{3} < \frac{1}{3}$ $\left\{x \mid x < \frac{11}{18}\right\}$

98. $\frac{2}{3}\left(\frac{7}{8} - 4x\right) - \frac{5}{8} < \frac{3}{8}$ $\left\{x \mid x > -\frac{5}{32}\right\}$

99. Are the inequalities $x > -3$ and $3 > -x$ equivalent? Why or why not?

100. Are the inequalities $t > -7$ and $7 < -t$ equivalent? Why or why not?

SKILL MAINTENANCE

Translate to an algebraic expression.

101. The sum of 3 and some number [1.1] Let n represent "some number"; $3 + n$

102. Twice the sum of two numbers [1.1] Let x and y represent the two numbers; $2(x + y)$

103. Three less than twice a number [1.1] Let x represent the number; $2x - 3$

104. Five more than twice a number [1.1] Let y represent the number; $5 + 2y$

SYNTHESIS

105. Explain in your own words why it is necessary to reverse the inequality symbol when multiplying both sides of an inequality by a negative number.

106. Explain how it is possible for the graph of an inequality to consist of just one number. (*Hint*: See Example 3c.)

Solve.

107. $6[4 - 2(6 + 3t)] > 5[3(7 - t) - 4(8 + 2t)] - 20$ $\left\{t \mid t > -\frac{27}{19}\right\}$

108. $27 - 4[2(4x - 3) + 7] \geq 2[4 - 2(3 - x)] - 3$ $\left\{x \mid x \leq \frac{5}{6}\right\}$

Solve for x.

109. $-(x + 5) \geq 4a - 5$ $\{x \mid x \leq -4a\}$

110. $\frac{1}{2}(2x + 2b) > \frac{1}{3}(21 + 3b)$ $\{x \mid x > 7\}$

111. $y < ax + b$ (Assume $a > 0$.) ▣

112. $y < ax + b$ (Assume $a < 0$.) ▣

113. Determine whether each number is a solution of the inequality $|x| < 3$.

a) 3.2 No **b)** -2 Yes **c)** -3 No
d) -2.9 Yes **e)** 3 No **f)** 1.7 Yes

114. Graph the solutions of $|x| < 3$ on a number line.

Aha! **115.** Determine the solution set of $|x| > -3$. $\{x \mid x$ is a real number$\}$, or $(-\infty, \infty)$

116. Determine the solution set of $|x| < 0$. $\varnothing$

114. $\{x \mid -3 < x < 3\}$ 

▣ Answers to Exercises 111 and 112 can be found on p. A-40.

2.7 Solving Applications with Inequalities

Translating to Inequalities • Solving Problems

The five steps for problem solving can be used for problems involving inequalities.

Translating to Inequalities

Before solving problems that involve inequalities, we list some important phrases to look for. Sample translations are listed as well.

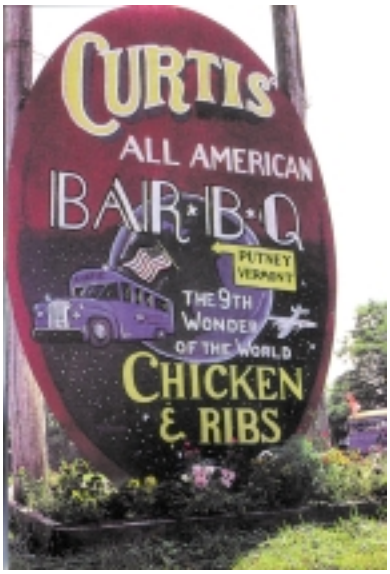
Study Tip

Make an effort to do your homework as soon as possible after each class. Make this part of your routine, choosing a time and a place where you can focus with a minimum of interruptions.

Important Words	Sample Sentence	Translation
is at least	Bill is at least 21 years old.	$b \geq 21$
is at most	At most 5 students dropped the course.	$n \leq 5$
cannot exceed	To qualify, earnings cannot exceed $12,000.	$r \leq 12{,}000$
must exceed	The speed must exceed 15 mph.	$s > 15$
is less than	Tucker's weight is less than 50 lb.	$w < 50$
is more than	Boston is more than 200 miles away.	$d > 200$
is between	The film is between 90 and 100 minutes long.	$90 < t < 100$
no more than	Bing weighs no more than 90 lb.	$w \leq 90$
no less than	Valerie scored no less than 8.3.	$s \geq 8.3$

Solving Problems

Example 1



Catering costs. To cater a party, Curtis' Barbeque charges a $50 setup fee plus $15 per person. The cost of Hotel Pharmacy's end-of-season softball party cannot exceed $450. How many people can attend the party?

Solution

1. **Familiarize.** Suppose that 20 people were to attend the party. The cost would then be $\$50 + \$15 \cdot 20$, or $350. This shows that more than 20 people could attend without exceeding $450. Instead of making another guess, we let n represent the number of people in attendance.

2. **Translate.** The cost of the party will be $50 for the setup fee plus $15 times the number of people attending. We can reword as follows:

Rewording: The setup fee | plus | the cost of the meals | cannot exceed | $450.

Translating: $50 + 15 \cdot n \leq 450$

3. **Carry out.** We solve for n:

$$\begin{aligned} 50 + 15n &\leq 450 \\ 15n &\leq 400 && \text{Subtracting 50 from both sides} \\ n &\leq \frac{400}{15} && \text{Dividing both sides by 15} \\ n &\leq 26\frac{2}{3}. && \text{Simplifying} \end{aligned}$$

4. **Check.** Although the solution set of the inequality is all numbers less than or equal to $26\frac{2}{3}$, since n represents the number of people in attendance, we round *down* to 26. If 26 people attend, the cost will be $\$50 + \$15 \cdot 26$, or $440, and if 27 attend, the cost will exceed $450.

5. **State.** At most 26 people can attend the party.

Caution! Solutions of problems should always be checked using the original wording of the problem. In some cases, answers might need to be whole numbers or integers or rounded off in a particular direction.

Example 2

Nutrition. The U.S. Department of Health and Human Services and the Department of Agriculture recommend that for a typical 2000-calorie daily diet, no more than 65 g of fat be consumed. In the first three days of a four-day vacation, Phil consumed 70 g, 62 g, and 80 g of fat. Determine (in terms of an inequality) how many grams of fat Phil can consume on the fourth day if he is to average no more than 65 g of fat per day.

Solution

1. Familiarize. Suppose Phil consumed 64 g of fat on the fourth day. His daily average for the vacation would then be

$$\frac{70\text{ g} + 62\text{ g} + 80\text{ g} + 64\text{ g}}{4} = 69\text{ g}.$$

This shows that Phil cannot consume 64 g of fat on the fourth day, if he is to average no more than 65 g of fat per day. Let's have x represent the number of grams of fat that Phil consumes on the fourth day.

2. Translate. We reword the problem and translate as follows:

Rewording: The average consumption of fat should be no more than 65 g.

Translating: $\dfrac{70 + 62 + 80 + x}{4} \quad \leq \quad 65$

3. Carry out. Because of the fraction, it is convenient to use the multiplication principle first:

$$\frac{70 + 62 + 80 + x}{4} \leq 65$$

$$4\left(\frac{70 + 62 + 80 + x}{4}\right) \leq 4 \cdot 65 \qquad \text{Multiplying both sides by 4}$$

$$70 + 62 + 80 + x \leq 260$$

$$212 + x \leq 260 \qquad \text{Simplifying}$$

$$x \leq 48. \qquad \text{Subtracting 212 from both sides}$$

4. Check. As a partial check, we show that Phil can consume 48 g of fat on the fourth day and not exceed a 65-g average for the four days:

$$\frac{70 + 62 + 80 + 48}{4} = \frac{260}{4} = 65.$$

5. State. Phil's average fat intake for the vacation will not exceed 65 g per day if he consumes no more than 48 g of fat on the fourth day.

Exercise Set 2.7

FOR EXTRA HELP

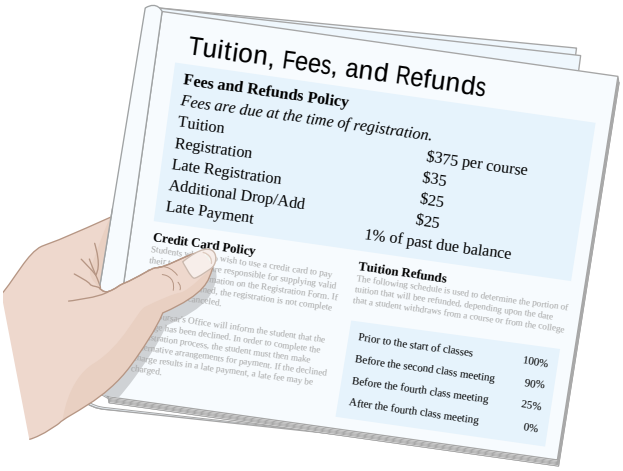
Digital Video Tutor CD 2
Videotape 4 | InterAct Math | Math Tutor Center | MathXL | MyMathLab.com

Translate to an inequality.

1. A number is at least 7. ▣
2. A number is greater than or equal to 5. ▣
3. The baby weighs more than 2 kilograms (kg). ▣
4. Between 75 and 100 people attended the concert. ▣
5. The speed of the train was between 90 and 110 mph. ▣
6. At least 400,000 people attended the Million Man March. ▣
7. At most 1,200,000 people attended the Million Man March. ▣
8. The amount of acid is not to exceed 40 liters (L). ▣
9. The cost of gasoline is no less than $1.50 per gallon. ▣
10. The temperature is at most $-2°C$. ▣

Use an inequality and the five-step process to solve each problem.

11. *Blueprints.* To make copies of blueprints, Vantage Reprographics charges a $5 setup fee plus $4 per copy. Myra can spend no more than $65 for the copying. What numbers of copies will allow her to stay within budget? 15 or fewer copies
12. *Banquet costs.* The women's volleyball team can spend at most $450 for its awards banquet at a local restaurant. If the restaurant charges a $40 setup fee plus $16 per person, at most how many can attend? 25 persons
13. *Truck rentals.* Ridem rents trucks at a daily rate of $42.95 plus $0.46 per mile. The Letsons want a one-day truck rental, but must stay within a budget of $200. What mileages will allow them to stay within budget? Round to the nearest tenth of a mile. Mileages less than or equal to 341.4 mi
14. *Phone costs.* Simon claims that it costs him at least $3.00 every time he calls an overseas customer. If his typical call costs 75¢ plus 45¢ for each minute, how long do his calls typically last? 5 min or more
15. *Parking costs.* Laura is certain that every time she parks in the municipal garage it costs her at least $2.20. If the garage charges 45¢ plus 25¢ for each half hour, for how long is Laura's car generally parked? 3.5 hr or more
16. *Furnace repairs.* RJ's Plumbing and Heating charges $25 plus $30 per hour for emergency service. Gary remembers being billed over $100 for an emergency call. How long was RJ's there? More than 2.5 hr
17. *College tuition.* Angelica's financial aid stipulates that her tuition not exceed $1000. If her local community college charges a $35 registration fee plus $375 per course, what is the greatest number of courses for which Angelica can register? 2



18. *Van rentals.* Atlas rents a cargo van at a daily rate of $44.95 plus $0.39 per mile. A business has budgeted $250 for a one-day van rental. What mileages will allow the business to stay within budget? (Round to the nearest tenth of a mile.) Mileages less than or equal to 525.8 mi
19. *Grade average.* Nadia is taking a literature course in which four tests are given. To get a B, a student must average at least 80 on the four tests. Nadia scored 82, 76, and 78 on the first three tests. What scores on the last test will earn her at least a B? Scores greater than or equal to 84
20. *Quiz average.* Rod's quiz grades are 73, 75, 89, and 91. What scores on a fifth quiz will make his average quiz grade at least 85? Scores greater than or equal to 97
21. *Nutrition.* Following the guidelines of the Food and Drug Administration, Dale tries to eat at least 5 servings of fruits or vegetables each day. For the first six days of one week, she had 4, 6, 7, 4, 6, and 4 servings. How many servings of fruits or

▣ Answers to Exercises 1–10 can be found on p. A-40.

vegetables should Dale eat on Saturday, in order to average at least 5 servings per day for the week? 4 servings or more

22. *College course load.* To remain on financial aid, Millie needs to complete an average of at least 7 credits per quarter each year. In the first three quarters of 2001, Millie completed 5, 7, and 8 credits. How many credits of course work must Millie complete in the fourth quarter if she is to remain on financial aid? 8 credits or more

23. *Music lessons.* Band members at Colchester Middle School are expected to average at least 20 min of practice time per day. One week Monroe practiced 15 min, 28 min, 30 min, 0 min, 15 min, and 25 min. How long must he practice on the seventh day if he is to meet expectations? 27 min or more

24. *Electrician visits.* Dot's Electric made 17 customer calls last week and 22 calls this week. How many calls must be made next week in order to maintain an average of at least 20 for the three-week period? 21 calls or more

25. *Perimeter of a rectangle.* The width of a rectangle is fixed at 8 ft. What lengths will make the perimeter at least 200 ft? at most 200 ft? Lengths greater than or equal to 92 ft; lengths less than or equal to 92 ft

26. *Perimeter of a triangle.* One side of a triangle is 2 cm shorter than the base. The other side is 3 cm longer than the base. What lengths of the base will allow the perimeter to be greater than 19 cm? Lengths greater than 6 cm

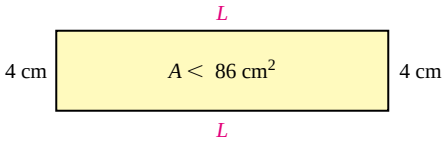
27. *Perimeter of a pool.* The perimeter of a rectangular swimming pool is not to exceed 70 ft. The length is to be twice the width. What widths will meet these conditions? Widths less than or equal to $11\frac{2}{3}$ ft

28. *Volunteer work.* George and Joan do volunteer work at a hospital. Joan worked 3 more hr than George, and together they worked more than 27 hr. What possible numbers of hours did each work? George: more than 12 hr; Joan: more than 15 hr

29. *Cost of road service.* Rick's Automotive charges $50 plus $15 for each (15-min) unit of time when making a road call. Twin City Repair charges $70 plus $10 for each unit of time. Under what circumstances would it be more economical for a motorist to call Rick's? Times less than 4 units, or 1 hr

30. *Cost of clothes.* Angelo is shopping for a new pair of jeans and two sweaters of the same kind. He is determined to spend no more than $120.00 for the clothes. He buys jeans for $21.95. How much can Angelo spend for each sweater? At most $49.02

31. *Area of a rectangle.* The width of a rectangle is fixed at 4 cm. For what lengths will the area be less than 86 cm^2? Lengths less than 21.5 cm



32. *Area of a rectangle.* The width of a rectangle is fixed at 16 yd. For what lengths will the area be at least 264 yd^2? Lengths greater than or equal to 16.5 yd

33. *Insurance-covered repairs.* Most insurance companies will replace a vehicle if an estimated repair exceeds 80% of the "blue-book" value of the vehicle. Michelle's insurance company paid $8500 for repairs to her Subaru after an accident. What can be concluded about the blue-book value of the car? Blue-book value is greater than or equal to $10,625

34. *Insurance-covered repairs.* Following an accident, Jeff's Ford pickup was replaced by his insurance company because the damage was so extensive. Before the damage, the blue-book value of the truck was $21,000. How much would it have cost to repair the truck? (See Exercise 33.) More than $16,800

35. *Body temperature.* A person is considered to be feverish when his or her temperature is higher than 98.6°F. The formula $F = \frac{9}{5}C + 32$ can be used to convert Celsius temperatures C to Fahrenheit temperatures F. For which Celsius temperatures is a person considered feverish? Temperatures greater than 37°C

36. *Melting butter.* Butter stays solid at Fahrenheit temperatures below 88°. Use the formula in Exercise 35 to determine those Celsius temperatures for which butter stays solid. Temperatures less than 31.1°C

37. *Fat content in foods.* Reduced Fat Skippy® peanut butter contains 12 g of fat per serving. In order for a food to be labeled "reduced fat," it must have at least 25% less fat than the regular item. What can you conclude about the number of grams of fat in a serving of the regular Skippy peanut butter? It contains at least 16 g of fat per serving.

38. *Fat content in foods.* Reduced Fat Chips Ahoy!® cookies contain 5 g of fat per serving. What can you conclude about the number of grams of fat in regular Chips Ahoy! cookies (see Exercise 37)? They contain at least $6\frac{2}{3}$ g of fat per serving.

39. *Well drilling.* All Seasons Well Drilling offers two plans. Under the "pay-as-you-go" plan, they charge \$500 plus \$8 a foot for a well of any depth. Under their "guaranteed-water" plan, they charge a flat fee of \$4000 for a well that is guaranteed to provide adequate water for a household. For what depths would it save a customer money to use the pay-as-you-go plan? Depths less than 437.5 ft

40. *Running.* In the course of a week, Tony runs 6 mi, 3 mi, 5 mi, 5 mi, 4 mi, and 4 mi. How far should his next run be if he is to average at least 5 mi per day? 8 mi or more

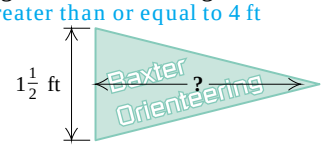
41. *Track records.* The formula $R = -0.012t + 20.8$, where R is in seconds, can be used to predict the world record in the 200-m dash t years after 1920. For what years will the world record be less than 19.8 sec? Years after 2003

42. *Track records.* The formula $R = -0.0084t + 3.85$, where R is in minutes, can be used to predict the world record in the 1500-m run t years after 1930. For what years will the world record be less than 3.22 min? Years after 2005

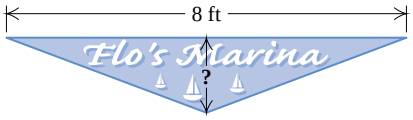
43. *Pond depth.* On July 1, Garrett's Pond was 25 ft deep. Since that date, the water level has dropped $\frac{2}{3}$ ft per week. For what dates will the water level not exceed 21 ft? Dates at least 6 weeks after July 1

44. *Weight gain.* A 9-lb puppy is gaining weight at a rate of $\frac{3}{4}$ lb per week. When will the puppy's weight exceed $22\frac{1}{2}$ lb? When the puppy is more than 18 weeks old

45. *Area of a triangular flag.* As part of an outdoor education course, Wanda needs to make a bright-colored triangular flag with an area of at least 3 ft^2. What heights can the triangle be if the base is $1\frac{1}{2}$ ft? Heights greater than or equal to 4 ft



46. *Area of a triangular sign.* Zoning laws in Harrington prohibit displaying signs with areas exceeding 12 ft^2. If Flo's Marina is ordering a triangular sign with an 8-ft base, how tall can the sign be? Heights less than or equal to 3 ft



47. *Toll charges.* The equation $y = 0.027x + 0.19$ can be used to determine the approximate cost y, in dollars, of driving x miles on the Indiana toll road. For what mileages x will the cost be at most \$6? Mileages less than or equal to 215.2 mi

48. *Price of a movie ticket.* The average price of a movie ticket can be estimated by the equation $P = 0.1522Y - 298.592$, where Y is the year and P is the average price, in dollars. The price is lower than what might be expected due to senior-citizen discounts, children's prices, and special volume discounts. For what years will the average price of a movie ticket be at least \$6? (Include the year in which the \$6 ticket first occurs.) Years 2001 and beyond

49. If f represents Fran's age and t represents Todd's age, write a sentence that would translate to $t + 3 < f$.

50. Explain how the meanings of "Five more than a number" and "Five is more than a number" differ.

SKILL MAINTENANCE

Simplify.

51. $\dfrac{9-5}{6-4}$ [1.8] 2

52. $\dfrac{8-5}{12-6}$ [1.8] $\frac{1}{2}$

53. $\dfrac{8-(-2)}{1-4}$ [1.8] $-\frac{10}{3}$

54. $\dfrac{7-9}{4-(-6)}$ [1.8] $-\frac{1}{5}$

SYNTHESIS

55. Write a problem for a classmate to solve. Devise the problem so the answer is "The Rothmans can drive 90 mi without exceeding their truck rental budget."

56. Write a problem for a classmate to solve. Devise the problem so the answer is "At most 18 passengers can go on the boat." Design the problem so that at least one number in the solution must be rounded down.

57. *Parking fees.* Mack's Parking Garage charges $4.00 for the first hour and $2.50 for each additional hour. For how long has a car been parked when the charge exceeds $16.50? More than 6 hr

58. *Ski wax.* Green ski wax works best between 5° and 15° Fahrenheit. Determine those Celsius temperatures for which green ski wax works best. (See Exercise 35.) Temperatures between $-15°C$ and $-9\frac{4}{9}°C$

Aha! **59.** The area of a square can be no more than 64 cm^2. What lengths of a side will allow this? Lengths less than or equal to 8 cm

Aha! **60.** The sum of two consecutive odd integers is less than 100. What is the largest pair of such integers? 47 and 49

61. *Nutritional standards.* In order for a food to be labeled "lowfat," it must have fewer than 3 g of fat per serving. Reduced fat Tortilla Pops® contain 60% less fat than regular nacho cheese tortilla chips, but still cannot be labeled lowfat. What can you conclude about the fat content of a serving of nacho cheese tortilla chips? They contain at least 7.5 g of fat per serving.

62. *Parking fees.* When asked how much the parking charge is for a certain car (see Exercise 57), Mack replies "between 14 and 24 dollars." For how long has the car been parked? Between 5 and 9 hr

63. Alice's Books allows customers to select one free book for every 10 books purchased. The price of that book cannot exceed the average cost of the 10 books. Neoma has bought 9 books that average $12 per book. How much should her tenth book cost if she wants to select a $15 book for free? At least $42

64. After 9 quizzes, Blythe's average is 84. Is it possible for Blythe to improve her average by two points with the next quiz? Why or why not?

65. Arnold and Diaz Booksellers offers a preferred-customer card for $25. The card entitles a customer to a 10% discount on all purchases for a period of one year. Under what circumstances would an individual save money by purchasing a card?

COLLABORATIVE

CORNER

Calling Plans

Focus: Problem solving and inequalities

Time: 20 minutes

Group size: 4

Materials: Calculators

A recent ad for "Five Line" is represented below.

	Rate per minute	Monthly fee	Distance	Restrictions
MCI 5¢ Everyday	25¢ in the day, 5¢ evening and weekends	$1.95	State-to-state	Minimum monthly bill of $5.00
Five Line	5¢	None	State-to-state AND in-state	Each completed call costs a minimum of 50¢.
AT&T Seven Sense	7¢	$5.95 ($4.95 with internet billing)	State-to-state	
Sprint Nickel Nights	10¢ in the day, 5¢ in the evening	$5.95	State-to-state	

ACTIVITY

The following table lists one month of Kate's calls. There are 34 calls for a total of 173 minutes.

Length of call (in minutes)	Number of calls of given length
1	13
2	6
3	5
4	3
5	2
9	1
16	1
18	1
33	1
35	1

1. Assume all calls are state-to-state calls. Each group member should choose a different one of the four plans and compute the bill (not including taxes and other fees) for each of the following situations.
   a) All 34 calls are evening (off-peak) calls.
   b) All 34 calls are daytime (peak) calls.
2. Is there a best plan for evening phone use? Is there a best plan for daytime phone use?
3. For each plan, describe the type of caller, if one exists, for whom that plan works best.
4. How can inequalities be used to compare these plans?