2.2 Introduction to Fractions

OBJECTIVES

- To read and write fractions and mixed numbers
- To write improper fractions as mixed numbers and mixed numbers as improper fractions
- To find equivalent fractions and to write fractions in simplest form
- To compare fractions
- To solve word problems with fractions

What Fractions Are and Why They Are Important

A fraction can mean *part of a whole*. Just as a whole number answers the question, How many?, a fraction answers the question, What part of? Every day we use fractions in this sense. For example, we can speak of *two-thirds* of a class (meaning two of every three students) or *three-fourths* of a dollar (indicating that we have split a dollar into four equal parts and have taken three of these parts).

A fraction can also mean *the quotient of two whole numbers*. In this sense, the fraction $\frac{3}{4}$ tells us what we get when we divide the whole number 3 by the whole number 4.

Point out to students that the words *fraction* and *fragment* have a common origin. Ask them to explain how the meanings of the two words are connected.

Definition

A **fraction** is any number that can be written in the form $\frac{a}{b}$, where a and b are whole numbers and b is not zero.

Ask students to explain why in this definition b cannot be 0.

From this definition, $\frac{1}{2}$, $\frac{3}{9}$, $\frac{6}{5}$, $\frac{8}{2}$, and $\frac{0}{1}$ are all fractions.

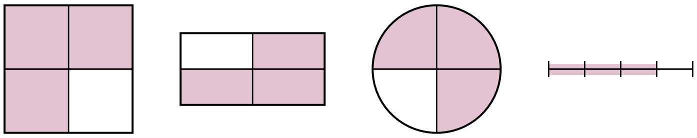
When written as $\frac{a}{b}$, a fraction has three components. $\frac{\text{Numerator}}{\text{Denominator}}$ ← **Fraction line**

- The **denominator** (on the bottom) stands for the number of parts into which the whole is divided.
- The **numerator** (on top) tells us how many parts of the whole the fraction contains.
- The **fraction line** separates the numerator from the denominator, and stands for "out of" or "divided by."

Alternatively, a fraction can be represented as either a decimal or a percent. We discuss decimals and percents in Chapters 3 and 6.

Fraction Diagrams and Proper Fractions

Diagrams help us work with fractions. Each diagram represents the fraction three-fourths, or $\frac{3}{4}$.



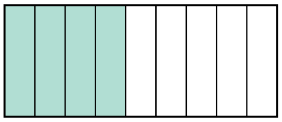
Note that in each diagram the whole has been divided into 4 *equal* parts, with 3 of the parts shaded.

The number $\frac{3}{4}$ is an example of a **proper fraction** because its numerator is smaller than its denominator. Let's consider some other examples of proper fractions.

Ask students if they can explain why a fraction whose numerator is smaller than its denominator must have a value less than 1.

EXAMPLE 1

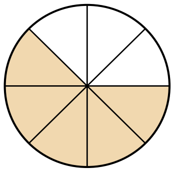
What fraction does the diagram represent?



Solution In this diagram, the whole is divided into 9 equal parts, so the denominator of the illustrated fraction is 9. Four of these parts are shaded, so the numerator is 4. The diagram illustrates the fraction $\frac{4}{9}$.

PRACTICE 1

The diagram illustrates what fraction? $\frac{5}{8}$



EXAMPLE 2

A manufacturing company plans to lay off 71 of its 230 workers. What fraction of its employees does the company plan to lay off?

Solution There are 230 workers altogether, so the denominator of our fraction is 230. Because we are concerned with 71 of these workers, 71 is the numerator. The company plans to lay off $\frac{71}{230}$ of its employees.

PRACTICE 2

The annual tuition at a college is $2,451. If a student paid $1,000 toward this tuition, what fraction of the tuition did the student pay? $\frac{1{,}000}{2{,}451}$

EXAMPLE 3

The U.S. Senate approved a foreign aid spending bill by a vote of 83 to 17. What fraction of the senators voted against the bill?

Solution First, we find the total number of senators. Because 83 senators voted for the bill and 17 voted against it, the total number of senators is 83 + 17, or 100. So $\frac{17}{100}$ of the senators voted against the bill.

PRACTICE 3

You have read 125 pages of a novel assigned by your English instructor. If 39 pages remain, what fraction of the book have you read? $\frac{125}{164}$

Mixed Numbers and Improper Fractions

Ask students why a fraction whose numerator is larger than or equal to its denominator must have a value greater than or equal to 1.

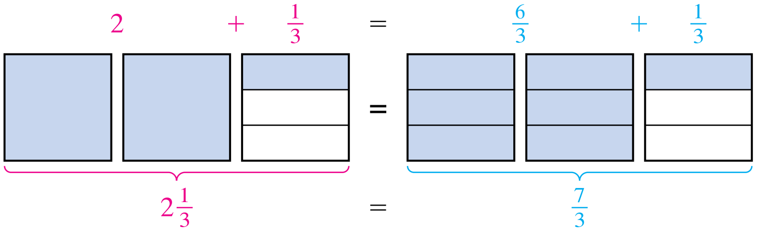
On many jobs if you work overtime, the rate of pay increases to one-and-a-half times the regular rate. A number such as $1\frac{1}{2}$, with a whole number part and a proper fraction part, is called a **mixed number**. A mixed number can also be expressed as an **improper fraction**, that is, a fraction whose numerator is larger than or equal to its denominator. The number $\frac{3}{2}$ is an example of an improper fraction.

Diagrams help us understand that mixed numbers and improper fractions are different forms of the same numbers, as Example 4 illustrates.

EXAMPLE 4

Draw diagrams to show that $2\frac{1}{3} = \frac{7}{3}$.

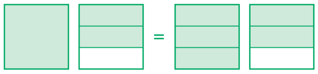
Solution First, represent the mixed number and the improper fraction in diagrams.



Both diagrams represent $2 + \frac{1}{3}$, so the numbers $2\frac{1}{3}$ and $\frac{7}{3}$ must be equal.

PRACTICE 4

By means of diagrams, explain why $1\frac{2}{3} = \frac{5}{3}$.



In Example 4 each unit (or square) corresponds to one whole, which is also three-thirds. That is why the total number of *thirds* in $2\frac{1}{3}$ is $(2 \times 3) + 1$, or 7. The number of *wholes* in $\frac{7}{3}$ is 2 wholes, with $\frac{1}{3}$ of a whole left over. We can generalize these observations into two rules.

To Change a Mixed Number to an Improper Fraction

- multiply the denominator of the fraction by the whole number part of the mixed number,
- add the numerator of the fraction to this product, and
- write this sum over the original denominator to form the improper fraction.

EXAMPLE 5

Write each of the following mixed numbers as an improper fraction.

a. $3\frac{2}{9}$ **b.** $12\frac{1}{4}$

Solution

a. We first multiply the denominator 9 by the whole number 3, getting 27. We next add the numerator 2. We then write the sum 29 over the original denominator to get $\frac{29}{9}$.

b. $$12\frac{1}{4} = \frac{(4 \times 12) + 1}{4}$$
$$= \frac{48 + 1}{4} = \frac{49}{4}$$

PRACTICE 5

Express each mixed number as an improper fraction.

a. $5\frac{1}{3}$ $\frac{16}{3}$ **b.** $20\frac{2}{5}$ $\frac{102}{5}$

To Change an Improper Fraction to a Mixed Number

- divide the numerator by the denominator, and
- if there is a remainder, write it over the denominator.

EXAMPLE 6

Write each improper fraction as a mixed or whole number.

a. $\frac{11}{2}$ **b.** $\frac{20}{20}$ **c.** $\frac{42}{5}$

Solution

a. $\frac{11}{2} = \begin{array}{r} 5\text{ R1} \\ 2\overline{)11} \end{array}$ **Divide the numerator by the denominator.**

$\frac{11}{2} = 5\frac{1}{2}$ **Write the remainder over the denominator.**

In other words, 5 R1 means that in $\frac{11}{2}$ there are 5 wholes with $\frac{1}{2}$ of a whole left over.

b. $\frac{20}{20} = 1$ **c.** $\frac{42}{5} = 8\frac{2}{5}$

PRACTICE 6

Express as a whole or mixed number.

a. $\frac{4}{2}$ 2 **b.** $\frac{50}{9}$ $5\frac{5}{9}$ **c.** $\frac{8}{3}$ $2\frac{2}{3}$

Changing an improper fraction to a mixed number is important when we are dividing whole numbers: It allows us to express any remainder as a fraction. Previously, we would have said that the problems $2\overline{)7}$ and $4\overline{)13}$ both have the answer 3 R1. But by interpreting these problems as improper fractions, we see that their answers are different.

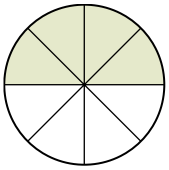
$$\frac{7}{2} = 3\frac{1}{2} \quad \text{but} \quad \frac{13}{4} = 3\frac{1}{4}$$

When a number is expressed as a mixed number, we know its size more readily than when it is expressed as an improper fraction. For instance, consider the mixed number $11\frac{7}{8}$. We immediately see that it is larger than 11 and smaller than 12 (that is, between 11 and 12). We could not reach this conclusion so easily if we were to examine only $\frac{95}{8}$, its improper form. However, there are situations—when we multiply or divide fractions—in which the use of improper fractions is preferable.

Equivalent Fractions

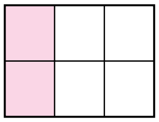
Some fractions that at first glance appear to be different from one another are really the same.

For instance, suppose that we cut a pizza into 8 equal slices, and then eat 4 of the slices. The shaded portion of the diagram at the right represents the amount eaten. Do you see in this diagram that the fractions $\frac{4}{8}$ and $\frac{1}{2}$ describe the same part of the whole pizza? We say that these fractions are **equivalent**.

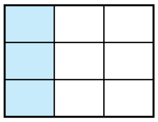


Any fraction has infinitely many equivalent fractions. To see why, let's consider the fraction $\frac{1}{3}$. We can draw different diagrams representing one-third of a whole.

$\frac{1}{3}$



$\frac{2}{6}$



$\frac{3}{9}$

All the shaded portions of the diagrams are identical, so $\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$.

A faster way to generate fractions equivalent to $\frac{1}{3}$ is to multiply both its numerator and denominator by the *same* whole number. Any whole number except 0 will do.

$$\frac{1}{3} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$$

$$\frac{1}{3} = \frac{1 \cdot 3}{3 \cdot 3} = \frac{3}{9}$$

$$\frac{1}{3} = \frac{1 \cdot 4}{3 \cdot 4} = \frac{4}{12}$$

$$\frac{1}{3} = \frac{1 \cdot 5}{3 \cdot 5} = \frac{5}{15}$$

So $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \cdots$.

Can you explain how you would generate fractions equivalent to $\frac{3}{5}$?

Have students write their answers in a journal.

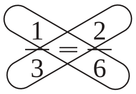
To Find an Equivalent Fraction

for $\frac{a}{b}$ (b not 0), multiply the numerator and denominator by the same whole number.

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n}, \qquad n \text{ not } 0$$

Ask students why neither *b* nor *n* can be equal to 0 here.

An important property of equivalent fractions is that their **cross products** are always equal.



In this case, $1 \cdot 6 = 2 \cdot 3 = 6$.

EXAMPLE 7

Find two fractions equivalent to $\frac{1}{7}$.

Solution Let's multiply the numerator and denominator by 2 and then by 6.

$$\frac{1}{7} = \frac{1 \cdot 2}{7 \cdot 2} = \frac{2}{14} \quad \text{and} \quad \frac{1}{7} = \frac{1 \cdot 6}{7 \cdot 6} = \frac{6}{42}$$

We use cross products to check.

$$\frac{1}{7} \overset{?}{=} \frac{2}{14}$$

$$2 \cdot 7 \overset{?}{=} 1 \cdot 14$$

$$14 \overset{\checkmark}{=} 14$$

So $\frac{1}{7}$ and $\frac{2}{14}$ are equivalent.

$$\frac{1}{7} \overset{?}{=} \frac{6}{42}$$

$$6 \cdot 7 \overset{?}{=} 1 \cdot 42$$

$$42 \overset{\checkmark}{=} 42$$

So $\frac{1}{7}$ and $\frac{6}{42}$ are equivalent.

PRACTICE 7

Identify three fractions equivalent to $\frac{2}{5}$.

Possible answer: $\frac{4}{10}, \frac{6}{15}, \frac{8}{20}$

EXAMPLE 8

Write $\frac{3}{7}$ as an equivalent fraction whose denominator is 35.

Solution

$$\frac{3}{7} = \frac{n}{35}$$

The question is: What number *n* makes the fractions equivalent?

$$\frac{3}{7} = \frac{n}{7 \cdot 5}$$

Express 35 as 7 · 5.

$$\frac{3 \cdot 5}{7 \cdot 5} = \frac{n}{7 \cdot 5}$$

Multiply both the numerator and denominator of $\frac{3}{7}$ by 5.

$$\frac{3}{7} = \frac{15}{35}$$

So *n* must be 3 · 5, or 15.

Therefore $\frac{15}{35}$ is equivalent to $\frac{3}{7}$. To check, we find the cross products: Both $3 \cdot 35$ and $7 \cdot 15$ equal 105.

PRACTICE 8

Express $\frac{5}{8}$ as a fraction whose denominator is 72. $\frac{45}{72}$

Writing a Fraction in Simplest Form

In the preceding section, we showed that $\frac{4}{8}$ and $\frac{1}{2}$ are equivalent fractions. Note that we could have written $\frac{4}{8}$ in its **simplest form** by dividing its numerator and denominator by 4.

$$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2} \leftarrow \text{1 and 2 have no common factor except 1.}$$

To Simplify (Reduce) a Fraction

such as $\frac{a}{b}$, divide its numerator and denominator by the same number.

$$\frac{a}{b} = \frac{a \div n}{b \div n}, \qquad \text{neither } b \text{ nor } n \text{ is } 0$$

A fraction is said to be in **simplest form** (or **reduced to lowest terms**) when the only common factor of its numerator and its denominator is 1.

EXAMPLE 9

Express $\frac{3}{15}$ in simplest form.

Solution To reduce this fraction, we can divide both its numerator and its denominator by 3.

$$\frac{3}{15} = \frac{3 \div 3}{15 \div 3} = \frac{1}{5}$$

To be sure that we have not made an error, let's check whether the cross products are equal: $3 \cdot 5 = 15$ and $1 \cdot 15 = 15$.

PRACTICE 9

Reduce $\frac{14}{21}$ to lowest terms. $\frac{2}{3}$

To reduce a fraction to lowest terms, we divide the numerator and denominator by all the factors that they have in common. To find these common factors, it is often helpful to express both the numerator and denominator as the product of prime factors. We can then divide out or *cancel* all common factors.

EXAMPLE 10

VIDEO

Write $\frac{42}{28}$ in lowest terms.

Solution $\frac{42}{28} = \frac{2 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 7}$ Express the numerator and denominator as the product of primes.

$= \frac{\overset{1}{\cancel{2}} \cdot 3 \cdot \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{7}}}$ Cancel the common factors, noting that 1 remains.

$= \frac{3}{2}$ Multiply the remaining factors.

PRACTICE 10

Simplify $\frac{45}{18}$. $\frac{5}{2}$

YOU TRY IT

EXAMPLE 11

Suppose that your annual income is \$39,000. If you pay \$9,000 for rent and \$3,000 for food per year, rent and food account for what fraction of your income? Simplify your answer.

Solution You must first find the total part of the income that you pay for rent and food per year.

$$\underset{\text{Rent}}{\underset{\downarrow}{\$9{,}000}} + \underset{\text{Food}}{\underset{\downarrow}{\$3{,}000}} = \underset{\text{Total part}}{\underset{\downarrow}{\$12{,}000}}$$

The total part is \$12,000 and the whole is \$39,000, so the fraction is $\frac{12{,}000}{39{,}000}$. You can simplify this fraction in the following way.

$$\frac{12{,}000}{39{,}000} = \frac{12{,}0\cancel{00}}{39{,}0\cancel{00}} = \frac{12}{39}$$

Note that canceling a 0 is the same as dividing by 10.

$$= \frac{3 \cdot 4}{3 \cdot 13} = \frac{\overset{1}{\cancel{3}} \cdot 4}{\underset{1}{\cancel{3}} \cdot 13} = \frac{4}{13}$$

Therefore $\frac{4}{13}$ of your income goes for rent and food.

PRACTICE 11

An acre is a unit of area approximately equal to 4,900 square yards. You own a parcel of land 50 yards by 30 yards. What fraction of an acre is this? $\frac{15}{49}$

Comparing Fractions

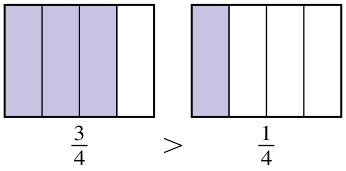
Some situations require us to *compare* fractions, that is, to rank them in order of size.

For instance, suppose that $\frac{5}{8}$ of one airline's flights arrive on time, in contrast to $\frac{3}{5}$ of another airline's flights. To decide which airline has a better record for on-time arrivals, we need to compare the fractions.

Or to take another example, suppose that the drinking water in your home, according to a lab report, has 2 parts per million (ppm) of lead. Is the water safe to drink? If the federal limit on lead in drinking water is 15 parts per billion (ppb), again you need to compare fractions.

One way to handle such problems is to draw diagrams corresponding to the fractions in question. The larger fraction corresponds to the larger shaded region.

For instance, the diagrams to the right show that $\frac{3}{4}$ is greater than $\frac{1}{4}$. The symbol $>$ stands for "greater than."



Both $\frac{3}{4}$ and $\frac{1}{4}$ have the same denominator, so we can rank them simply by comparing their numerators.

$$\frac{3}{4} > \frac{1}{4} \qquad \text{because} \qquad 3 > 1$$

For **like fractions**, the fraction with the larger numerator is the larger fraction.

Have students note that the symbols < and > always point to the smaller number.

Definitions

Like fractions are fractions with the same denominator.

Unlike fractions are fractions with different denominators.

To Compare Fractions

- compare the numerators of like fractions, and
- express unlike fractions as equivalent fractions having the same denominator and then compare their numerators.

EXAMPLE 12

Compare $\frac{7}{15}$ and $\frac{4}{9}$.

Solution These fractions are unlike because they have different denominators. Therefore we need to express them as equivalent fractions having the same denominator. But what should that denominator be?

One common denominator that we can use is the *product of the denominators:* $15 \cdot 9 = 135$.

$$\frac{7}{15} = \frac{63}{135} \qquad \textbf{135 = 15 · 9, so the new numerator is 7 · 9, or 63.}$$

$$\frac{4}{9} = \frac{60}{135} \qquad \textbf{135 = 9 · 15, so the new numerator is 4 · 15, or 60.}$$

Next, we compare the numerators of the like fractions that we just found.

Because $63 > 60$, $\frac{63}{135} > \frac{60}{135}$. Therefore $\frac{7}{15} > \frac{4}{9}$.

Another common denominator that we can use is the least common multiple of the denominators.

$$15 = 3 \times 5 \qquad 9 = 3 \times 3 = 3^2$$

The LCM is $3^2 \times 5 = 9 \times 5 = 45$. We then compute the equivalent fractions.

$$\begin{array}{ccl} \frac{7}{15} & \frac{4}{9} & \\ \downarrow & \downarrow & \textbf{is equivalent to} \\ \frac{21}{45} & \frac{20}{45} & \end{array}$$

Because $\frac{21}{45} > \frac{20}{45}$, we know that $\frac{7}{15} > \frac{4}{9}$.

PRACTICE 12

Which is larger, $\frac{13}{24}$ or $\frac{11}{16}$? $\frac{11}{16}$

Point out that another approach here is to say that $135 \div 15$ is 9 and that $9 \times 7 = 63$.

Note that in Example 12 we computed the LCM of the two denominators. This type of computation is used frequently in working with fractions.

Definition

For any set of fractions, their **least common denominator** (LCD) is the least common multiple of their denominators.

In Example 13, pay particular attention to how we use the LCD.

EXAMPLE 13

Order from smallest to largest: $\frac{3}{4}, \frac{7}{10}$, and $\frac{29}{40}$.

Solution Because these fractions are unlike, we need to find equivalent fractions with a common denominator. Let's use their LCD as that denominator.

$$4 = 2 \times 2 = 2^2$$

$$10 = 2 \times 5$$

$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$$

The LCD $= 2^3 \times 5 = 8 \times 5 = 40$. Check: 4 and 10 are both factors of 40.

We write each fraction with a denominator of 40.

$$\frac{3}{4} = \frac{3 \cdot 10}{4 \cdot 10} = \frac{30}{40} \qquad \frac{7}{10} = \frac{7 \cdot 4}{10 \cdot 4} = \frac{28}{40} \qquad \frac{29}{40} = \frac{29}{40}$$

Then we order the fractions from smallest to largest. (The symbol $<$ stands for "less than.")

$$\frac{28}{40} < \frac{29}{40} < \frac{30}{40}, \quad \text{or} \quad \frac{7}{10} < \frac{29}{40} < \frac{3}{4}$$

PRACTICE 13

Arrange $\frac{9}{10}, \frac{23}{30}$, and $\frac{8}{15}$ from smallest to largest.

$\frac{8}{15}, \frac{23}{30}, \frac{9}{10}$

EXAMPLE 14

About $\frac{7}{10}$ of Earth's surface is covered by water and $\frac{1}{20}$ is covered by desert. Does water or desert cover more of Earth?

Solution We need to compare $\frac{7}{10}$ with $\frac{1}{20}$.

$$\frac{7}{10} = \frac{14}{20}$$

$$\frac{1}{20} = \frac{1}{20}$$

Since $\frac{14}{20} > \frac{1}{20}, \frac{7}{10} > \frac{1}{20}$. Therefore water covers more of Earth than desert does.

PRACTICE 14

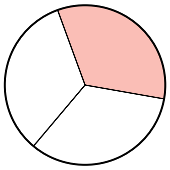
You work $\frac{1}{3}$ of a day and sleep $\frac{7}{24}$ of a day. Do you spend more time working or sleeping?

Working; $\frac{1}{3} > \frac{7}{24}$

Exercises 2.2

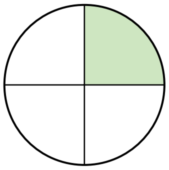
Identify a fraction or mixed number that represents the shaded part of each figure.

1.



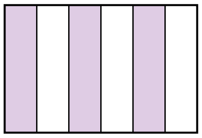
$\frac{1}{3}$

2.



$\frac{1}{4}$

3.

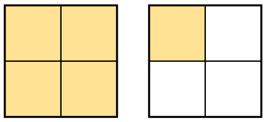


$\frac{3}{6}$

4.

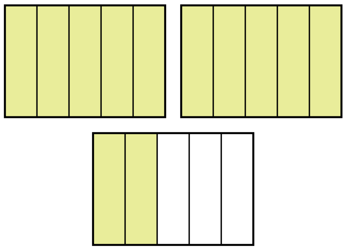
$\frac{4}{5}$

5.



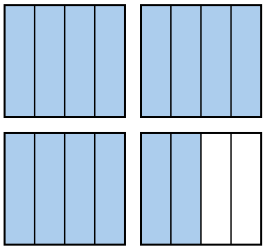
$1\frac{1}{4}$

6.



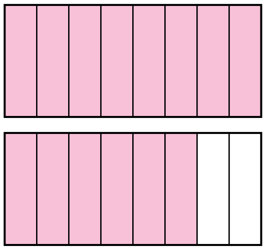
$2\frac{2}{5}$

7.



$3\frac{2}{4}$

8.



$1\frac{6}{8}$

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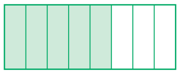
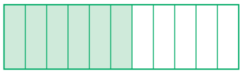
Manual

Draw a diagram to represent each fraction or mixed number.

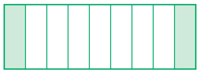
9. $\frac{5}{8}$



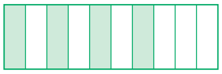
10. $\frac{6}{11}$



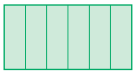
11. $\frac{2}{9}$



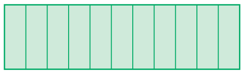
12. $\frac{4}{10}$



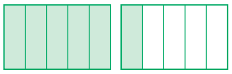
13. $\frac{6}{6}$



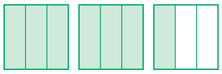
14. $\frac{11}{11}$



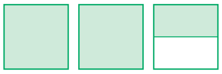
15. $\frac{6}{5}$



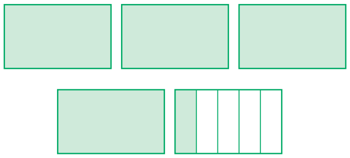
16. $\frac{7}{3}$



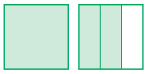
17. $2\frac{1}{2}$



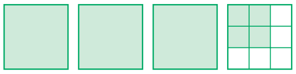
18. $4\frac{1}{5}$



19. $1\frac{2}{3}$



20. $3\frac{4}{9}$



Indicate whether each number is a proper fraction, improper fraction, or mixed number.

21. $\frac{3}{4}$
Proper

22. $\frac{7}{12}$
Proper

23. $\frac{10}{9}$
Improper

24. $\frac{11}{10}$
Improper

25. $16\frac{2}{3}$
Mixed

26. $12\frac{1}{2}$
Mixed

27. $\frac{5}{5}$
Improper

28. $\frac{4}{4}$
Improper

29. $\frac{5}{8}$
Proper

30. $\frac{5}{6}$
Proper

31. $66\frac{2}{3}$
Mixed

32. $10\frac{3}{4}$
Mixed

Write each number as an improper fraction.

33. $2\frac{3}{5}$
$\frac{13}{5}$

34. $1\frac{1}{3}$
$\frac{4}{3}$

35. $6\frac{1}{9}$
$\frac{55}{9}$

36. $10\frac{2}{3}$
$\frac{32}{3}$

37. $11\frac{2}{5}$
$\frac{57}{5}$

38. $12\frac{3}{4}$
$\frac{51}{4}$

39. 5
$\frac{5}{1}$

40. 8
$\frac{8}{1}$

41. $7\frac{3}{8}$
$\frac{59}{8}$

42. $6\frac{5}{6}$
$\frac{41}{6}$

43. $9\frac{7}{9}$
$\frac{88}{9}$

44. $10\frac{1}{2}$
$\frac{21}{2}$

45. $12\frac{2}{3}$
$\frac{38}{3}$

46. $20\frac{1}{8}$
$\frac{161}{8}$

47. $19\frac{3}{5}$
$\frac{98}{5}$

48. $11\frac{5}{7}$
$\frac{82}{7}$

49. 14
$\frac{14}{1}$

50. 10
$\frac{10}{1}$

51. $4\frac{10}{11}$
$\frac{54}{11}$

52. $2\frac{7}{13}$
$\frac{33}{13}$

53. $8\frac{3}{14}$
$\frac{115}{14}$

54. $4\frac{1}{6}$
$\frac{25}{6}$

55. $8\frac{2}{25}$
$\frac{202}{25}$

56. $14\frac{1}{10}$
$\frac{141}{10}$

Express each fraction as a mixed or whole number.

57. $\frac{4}{3}$
$1\frac{1}{3}$

58. $\frac{6}{5}$
$1\frac{1}{5}$

59. $\frac{10}{9}$
$1\frac{1}{9}$

60. $\frac{12}{5}$
$2\frac{2}{5}$

61. $\frac{9}{3}$

3

62. $\frac{12}{12}$

1

63. $\frac{15}{15}$

1

64. $\frac{62}{3}$

$20\frac{2}{3}$

65. $\frac{99}{5}$

$19\frac{4}{5}$

66. $\frac{31}{2}$

$15\frac{1}{2}$

67. $\frac{82}{9}$

$9\frac{1}{9}$

68. $\frac{100}{100}$

1

69. $\frac{45}{45}$

1

70. $\frac{40}{3}$

$13\frac{1}{3}$

71. $\frac{74}{9}$

$8\frac{2}{9}$

72. $\frac{41}{8}$

$5\frac{1}{8}$

73. $\frac{27}{2}$

$13\frac{1}{2}$

74. $\frac{58}{11}$

$5\frac{3}{11}$

75. $\frac{100}{9}$

$11\frac{1}{9}$

76. $\frac{19}{1}$

19

77. $\frac{27}{1}$

27

78. $\frac{72}{9}$

8

79. $\frac{56}{7}$

8

80. $\frac{38}{3}$

$12\frac{2}{3}$

Find two fractions equivalent to each fraction.

81. $\frac{1}{8}$

Possible answer: $\frac{2}{16}, \frac{3}{24}$

82. $\frac{2}{5}$

Possible answer: $\frac{4}{10}, \frac{6}{15}$

83. $\frac{2}{11}$

Possible answer: $\frac{4}{22}, \frac{6}{33}$

84. $\frac{1}{10}$

Possible answer: $\frac{2}{20}, \frac{3}{30}$

85. $\frac{3}{4}$

Possible answer: $\frac{6}{8}, \frac{9}{12}$

86. $\frac{5}{6}$

Possible answer: $\frac{10}{12}, \frac{15}{18}$

87. $\frac{1}{7}$

Possible answer: $\frac{2}{14}, \frac{3}{21}$

88. $\frac{3}{5}$

Possible answer: $\frac{6}{10}, \frac{9}{15}$

Find n and check.

89. $\frac{3}{4} = \frac{n}{12}$

9

90. $\frac{2}{9} = \frac{n}{18}$

4

91. $\frac{5}{8} = \frac{n}{24}$

15

92. $\frac{7}{10} = \frac{n}{20}$

14

93. $4 = \frac{n}{10}$

40

94. $5 = \frac{n}{15}$

75

95. $\frac{3}{5} = \frac{n}{60}$

36

96. $\frac{4}{9} = \frac{n}{63}$

28

97. $\frac{5}{8} = \frac{n}{64}$

40

98. $\frac{3}{10} = \frac{n}{40}$

12

99. $3 = \frac{n}{18}$

54

100. $2 = \frac{n}{21}$

42

101. $\frac{4}{9} = \frac{n}{81}$ 36

102. $\frac{7}{8} = \frac{n}{24}$ 21

103. $\frac{6}{7} = \frac{n}{49}$ 42

104. $\frac{5}{6} = \frac{n}{48}$ 40

105. $\frac{2}{17} = \frac{n}{51}$ 6

106. $\frac{1}{3} = \frac{n}{90}$ 30

107. $\frac{7}{12} = \frac{n}{84}$ 49

108. $\frac{1}{4} = \frac{n}{100}$ 25

109. $\frac{2}{3} = \frac{n}{48}$ 32

110. $\frac{7}{8} = \frac{n}{56}$ 49

111. $\frac{3}{10} = \frac{n}{100}$ 30

112. $\frac{5}{6} = \frac{n}{144}$ 120

Simplify each fraction.

113. $\frac{2}{4}$ $\frac{1}{2}$

114. $\frac{6}{8}$ $\frac{3}{4}$

115. $\frac{6}{9}$ $\frac{2}{3}$

116. $\frac{9}{12}$ $\frac{3}{4}$

117. $\frac{10}{10}$ 1

118. $\frac{21}{21}$ 1

119. $\frac{5}{15}$ $\frac{1}{3}$

120. $\frac{4}{24}$ $\frac{1}{6}$

121. $\frac{42}{10}$ $\frac{21}{5}$ or $4\frac{1}{5}$

122. $\frac{40}{25}$ $\frac{8}{5}$ or $1\frac{3}{5}$

123. $\frac{66}{99}$ $\frac{2}{3}$

124. $\frac{25}{49}$ $\frac{25}{49}$

125. $\frac{25}{100}$ $\frac{1}{4}$

126. $\frac{75}{100}$ $\frac{3}{4}$

127. $\frac{125}{1{,}000}$ $\frac{1}{8}$

128. $\frac{875}{1{,}000}$ $\frac{7}{8}$

129. $\frac{20}{16}$ $\frac{5}{4}$ or $1\frac{1}{4}$

130. $\frac{15}{9}$ $\frac{5}{3}$ or $1\frac{2}{3}$

131. $\frac{66}{32}$ $\frac{33}{16}$ or $2\frac{1}{16}$

132. $\frac{30}{18}$ $\frac{5}{3}$ or $1\frac{2}{3}$

133. $\frac{18}{32}$ $\frac{9}{16}$

134. $\frac{36}{45}$ $\frac{4}{5}$

135. $\frac{50}{1{,}000}$ $\frac{1}{20}$

136. $\frac{25}{1{,}000}$ $\frac{1}{40}$

137. $\frac{36}{28}$ $\frac{9}{7}$ or $1\frac{2}{7}$

138. $\frac{7}{24}$ $\frac{7}{24}$

139. $\frac{19}{51}$ $\frac{19}{51}$

140. $\frac{10}{2}$ 5

141. $\frac{27}{9}$ 3

142. $\frac{36}{144}$ $\frac{1}{4}$

143. $\frac{12}{84}$ $\frac{1}{7}$

144. $\frac{21}{36}$ $\frac{7}{12}$

145. $\frac{36}{48}$ $\frac{3}{4}$

146. $\frac{75}{20}$ $\frac{15}{4}$ or $3\frac{3}{4}$

147. $\frac{375}{1{,}000}$ $\frac{3}{8}$

148. $\frac{168}{1{,}000}$ $\frac{21}{125}$

149. $4\frac{71}{142}$ $4\frac{1}{2}$

150. $3\frac{38}{57}$ $3\frac{2}{3}$

151. $5\frac{200}{300}$ $5\frac{2}{3}$

152. $10\frac{400}{1{,}600}$ $10\frac{1}{4}$

153. $7\frac{6}{15}$ $7\frac{2}{5}$

154. $11\frac{51}{102}$ $11\frac{1}{2}$

155. $2\frac{100}{100}$ 3

156. $1\frac{144}{144}$ 2

Between each pair of numbers, insert the appropriate sign: <, =, or >.

157. $\frac{7}{20} < \frac{11}{20}$

158. $\frac{5}{10} > \frac{3}{10}$

159. $\frac{1}{8} > \frac{1}{9}$

160. $\frac{5}{6} < \frac{7}{8}$

161. $\frac{2}{3} = \frac{6}{9}$

162. $\frac{9}{12} = \frac{3}{4}$

163. $2\frac{1}{3} < 2\frac{9}{15}$

164. $2\frac{3}{7} > 1\frac{1}{2}$

For each group of three numbers, choose the number whose value is between the other two.

165. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ $\frac{1}{3}$

166. $\frac{3}{2}, \frac{3}{3}, \frac{3}{4}$ $\frac{3}{3}$

167. $\frac{2}{3}, \frac{7}{12}, \frac{5}{6}$ $\frac{2}{3}$

168. $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$ $\frac{5}{6}$

169. $\frac{3}{5}, \frac{2}{3}, \frac{8}{9}$ $\frac{2}{3}$

170. $\frac{5}{8}, \frac{1}{2}, \frac{4}{11}$ $\frac{1}{2}$

Applications

Solve. Write your answer in simplest form.

171. In a session of Congress, there were 98 male senators and 2 female senators.

a. What fraction of the senators were women? $\frac{1}{50}$

b. What fraction of the senators were men? $\frac{49}{50}$

172. At a party, 6 of the guests are men and 8 are women.

a. What fraction of the guests are men? $\frac{3}{7}$

b. If 1 man leaves, what fraction of the guests are men? $\frac{5}{13}$

173. Suppose that your college newspaper accepts 30 of every 50 articles submitted. What fraction of the articles are rejected? $\frac{2}{5}$

174. A baseball player got 16 hits in 40 times at bat. What fraction of his times at bat did he not get a hit? $\frac{3}{5}$

175. The gutter on your roof overflows whenever more than $\frac{1}{4}$ inch of rain falls. Yesterday $\frac{23}{100}$ inch of rain fell. Did the gutter overflow? Explain.

No; $\frac{1}{4} = \frac{25}{100}$ which is greater than $\frac{23}{100}$.

176. You learn in a course on probability and statistics that, when you roll a pair of dice, the probability of rolling a 5 is $\frac{1}{9}$ and that the probability of rolling a 6 is $\frac{5}{36}$. Does rolling a 5 or a 6 have a greater probability? Explain.

Rolling a 6, because $\frac{5}{36}$ is greater than $\frac{1}{9}$.

177. You drilled three holes in a piece of wood. The diameters of the three holes are $\frac{1}{8}$ in., $\frac{3}{16}$ in., and $\frac{3}{8}$ in.

a. Which hole is largest? $\frac{3}{8}$ in.

b. If you have a bolt with a $\frac{5}{32}$ inch diameter, will the bolt fit through all three holes? No

178. When fog hit the New York City area, visibility was reduced to one-sixteenth mile at Kennedy Airport, one-eighth mile at LaGuardia Airport and one-half mile at Newark Airport.

a. Which of the three airports had the best visibility? Newark

b. Which of the three airports had the worst visibility? Kennedy

179. The following chart gives the rainfall for each of the past 4 days.

M	Tu	W	Th
2 in.	1 in.	2 in.	0 in.

What was the average rainfall for the 4 days? $1\frac{1}{4}$ in.

180. The following chart gives the age of the first six American presidents at the time of their inauguration.

President	Washington	J. Adams	Jefferson	Madison	Monroe	J. Q. Adams
Age	57	61	57	57	58	57

What was their average age at inauguration? (***Source:*** *Significant American Presidents of the United States*) $57\frac{5}{6}$ yr

■ *Check your answers on page A-3.*

MINDSTRETCHERS

WRITING

1. Do you think that there is anything improper about an improper fraction? Explain.

Yes. The part is greater than the whole, thus the definition of a fraction as "part of a whole" is invalid.

GROUPWORK

2. Working with a partner, determine how many fractions there are between the numbers 1 and 2. There are an infinite number of fractions $\left(\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots\right)$.

CRITICAL THINKING

3. Consider the three equivalent fractions shown. Note that the numerators and denominators are made up of the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9—each appearing once.

$$\frac{3}{6} = \frac{7}{14} = \frac{29}{58}$$

a. Verify that these fractions are equivalent by making sure that their cross products are equal. $42 = 42$; $174 = 174$; $406 = 406$

b. Complete the following fractions to form another trio of equivalent fractions that use the same nine digits only once.

$$\frac{2}{4} = \frac{3}{6} = \frac{79}{158}$$