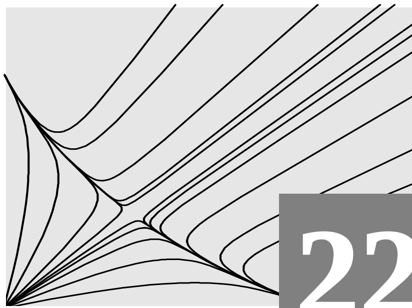


22 Competing Species Population Models

Tools Used in Lab 22
Competitive Exclusion

Two species, x and y, are competing for the same resources. Given initial populations, the carrying capacities of the habitat, the intrinsic growth rates, and the competitiveness of each species, what are the possible outcomes? Which parameters are most important in determining an outcome? Can an outcome be predicted?

1. The Competition Model

Competition between two species with populations x and y is typically described algebraically by equations of the form

$$\begin{aligned} \frac{dx}{dt} &= x(a_1 - b_1 x - c_1 y) \\ \frac{dy}{dt} &= y(a_2 - b_2 y - c_2 x). \end{aligned} \tag{1}$$

1.1 Explain the relationship of each of the six terms to the populations and their interaction.

$a_1 x$

$b_1 x^2$

$c_1 xy$

$a_2 y$

$b_2 y^2$

$c_2 xy$

The **Competitive Exclusion** tool gives the equations in a form that provides a clearer geometric indication of the meanings of the constants. This is *not* an obvious fact, but it will become clear in Exercise **1.3**.

$$\frac{dN_1}{dt} = r_1 N_1 \frac{K_1 - N_1 - B_1 N_2}{K_1}$$
$$\frac{dN_2}{dt} = r_2 N_2 \frac{K_2 - N_2 - B_2 N_1}{K_2} \qquad (2)$$

1.2 Show that the equations in Equation (2) are equivalent to the equations in Equation (1).

1.3 Experiment with the constants in Equations (2) and describe the effect of changing each.

K_1

K_2

B_1

B_2

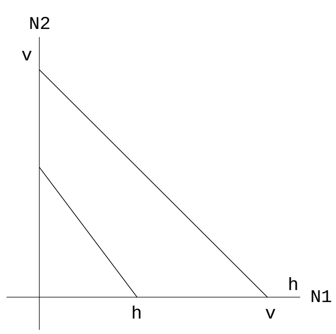
r_1

r_2

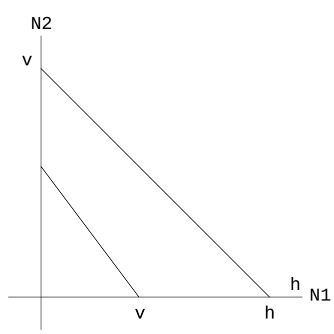
2. Possible Behaviors of Solutions

2.1 As shown below, there are four possible nullcline configurations in the first quadrant (h for horizontal and v for vertical). In each region carved out by the nullclines, draw arrows to indicate the general direction of the solutions in that region. Mark on each sketch all stable equilibria as • and all unstable equilibria as ∘ . Finally, in a different color, sketch some typical phase plane trajectories for each of these cases.

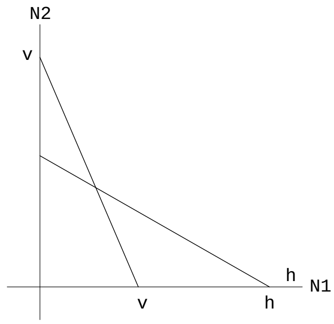
Case 1



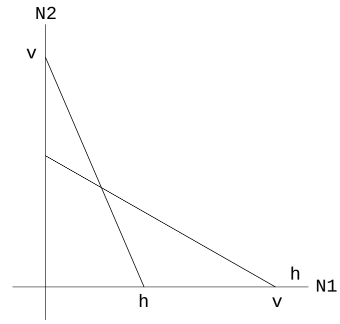
Case 2



Case 3



Case 4



2.2 For each case, sketch a typical time series in two colors, one color for N_1, another for N_2.

Case 1

Case 2

Case 3

Case 4

3. Connecting with the Biology

3.1 Select the best case for each of the situations described below and justify your choice.

a. You are working with a regional wetlands ecological commission to save frogs, of which N_1 and N_2 are populations of two species.

b. You are working for a pest control company to eradicate nasty species N_1 (you name it).

3.2 If you are in a situation where the graph looks like Case 3:

a. What parameters would you change to achieve the goal in Exercise **3.1b**?

b. Give a possible “real life” scenario that describes how you might change the environment to accomplish this goal.

Lab 22: Tool Instructions

Competitive Exclusion Tool

Setting Initial Conditions

The initial conditions for the active trajectory are displayed to the right of the graphing plane. They are set using either the mouse or the keyboard.

1. Click the mouse on the N_1N_2 graphing plane to set $N1(0)$ and $N2(0)$.
2. Click the mouse on the value for one of the initial conditions displayed beside the N_1N_2 graph to activate a keyboard editor. Set a new number using the number keys, the right and left arrow keys, and the **[Delete]** key. Press the **[Return]** key or click the mouse away from the number to leave the editor, set the initial value, and start a trajectory.
3. Clicking while a trajectory is being drawn will cancel the trajectory.

Parameter Sliders

The constants for carrying capacity *K*, competitiveness *B*, and growth rate *r*, for each species are set using the horizontal sliders on the right side of the screen.

Press the mouse down on the slider knob for the parameter you want to change and drag the mouse back and forth, or click the mouse in the slider channel at the desired value for the parameter.

Buttons

Click the mouse on the **[Clear]** button to remove all output from the graphs.

Click the mouse on the **[Pause]** button to stop a trajectory without canceling it.

Click the mouse on the **[Continue]** button to resume the motion of the paused trajectory.