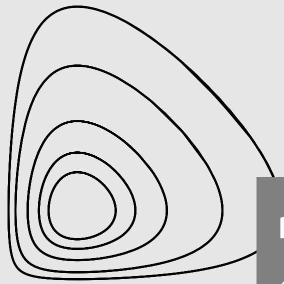

# 21 Predator-Prey Population Models

**Tools Used in Lab 21**

- Hudson Bay Data (Hare-Lynx)
- Lotka-Volterra
- Lotka-Volterra with Harvest

*How can we model the interaction between a species of predators and their prey, a species of herbivores? Using the well-known Lotka-Volterra Predator-Prey system, we explore the effects of the various constants and the effect of harvesting on the numbers of each species.*

## 1. Basic Model

The **Lotka-Volterra** tool gives the predator-prey equations in the form

$$\begin{aligned} \frac{dH}{dt} &= aH - bHP \\ \frac{dP}{dt} &= -cP + d(bHP) \end{aligned} \qquad (1)$$

where $H$ and $P$ are, respectively, the prey (herbivore) and predator populations.

During World War I, the biologist D'Ancona wondered why the decrease in fishing in the Mediterranean Sea brought on by the war caused an increase in the percentage of the catch that was shark. The mathematician Vito Volterra used the preceding model to provide the answer.

The Lotka-Volterra equations have since been applied to many similar problems. The **Hudson Bay Data (Hare-Lynx)** tool displays data on kills of lynx and hares in the Arctic over a number of years. Compare the graphs given in the **Hudson Bay Data (Hare-Lynx)** tool to those predicted by the model in the **Lotka-Volterra** tool.

Use the **Lotka-Volterra** tool to answer the following questions.

**1.1** Explain the biological meaning of each parameter.

*a*:

*b*:

*c*:

*d*:

**1.2** In terms of the constants, what is the value of the equilibrium $(H_E, P_E)$ for the system?

**1.3** In the absence of predators, what happens to the population of herbivores? Why? Is this reasonable?

**1.4** In the absence of herbivores, what happens to the population of predators? Why? Is this reasonable?

**1.5** Experiment with the **Lotka-Volterra** tool to find the effect of changing each parameter and describe the effect in words.

## 2. Additional Exercise

In the Lotka-Volterra equation, take $a = 1.10$, $b = 0.90$, $c = 1.00$, $d = 1.00$. Set

$$\frac{dP}{dH} = \frac{dP \ / \ dt}{dH \ / \ dt} = \frac{H(1.10 - 0.90P)}{P(0.90H - 1.00)}$$

Use separation of variables followed by an appropriate substitution to find a function $f(H,P)$ so that the solution curves of the system lie on the level curves of the function $f$ and we can conclude that the solution curves are indeed closed curves. Finding an argument for the conclusion is challenging!

## 3. Adding Harvesting

What happens if you harvest one or both species?

With harvesting, the Lotka-Volterra equations become

$$\begin{aligned} \frac{dH}{dt} &= aH - bHP - h_h H \\ \frac{dP}{dt} &= -cP + d(bHP) - h_p P \end{aligned} \tag{2}$$

Use the **Lotka-Volterra with Harvest** tool to answer the following questions.

**3.1** Explain the meaning of each parameter.

$h_h$:

$h_p$:

**3.2** In terms of the constants, what is the value of the equilibrium $(H_E, P_E)$ when there is harvesting?

**3.3** What is the effect of harvesting herbivores? Does the model allow the possibility of extinction of the herbivores with harvesting?

**3.4** What is the effect of harvesting predators? Does the model allow for the extinction of the predators with harvesting?

**3.5** What is the effect of harvesting both species equally? Do both species become extinct? If not, who wins? When is pest control a bad idea?

# Lab 21: Tool Instructions

## Hudson Bay Data (Hare-Lynx) Tool

This tool displays data on hare and lynx populations from the Hudson-Bay Trading Company. It is not meant to be interactive.

## Lotka-Volterra Tool

### Setting Initial Conditions

The initial conditions for the active trajectory are displayed to the right of the graphing plane. They are set using either the mouse or the keyboard.

1. Click the mouse on the *HP* graph to set $H(0)$ and $P(0)$.
2. Click the mouse on the value for one of the initial conditions displayed beside the *HP* graph to activate a keyboard editor. Set a new number using the number keys, the right and left arrow keys, and the Delete key. Press the **[Return]** key or click the mouse away from the number to leave the editor, set the initial value, and start the trajectory.
3. Clicking while a trajectory is being drawn will stop the trajectory.

### Parameter Sliders

Use the sliders to set the growth rate *a*, the predation rate *b*, the predator mortality rate *c*, and the food conversion rate *d*.
Press the mouse down on the slider knob for the parameter you want to change and drag the mouse back and forth, or click the mouse in the slider channel at the desired value for the parameter.

### Buttons

Click the mouse on the **[Clear]** button to remove all output from the graphing plane.
Click the mouse on the **[Pause]** button to stop a trajectory without canceling it.
Click the mouse on the **[Continue]** button to resume the motion of the paused trajectory.

## Lotka-Volterra with Harvest Tool

### Setting Initial Conditions

The initial conditions for the active trajectory are displayed to the right of the graphing plane. They are set using either the mouse or the keyboard.

1. Click the mouse on the *HP* graphing plane to set $H(0)$ and $P(0)$.
2. Click the mouse on the value for one of the initial conditions displayed beside the *HP* graph to activate a keyboard editor. Set a new number using the number keys, the right and left arrow keys, and the **[Delete]** key. Press the **[Return]** key or click the mouse away from the number to leave the editor, set the initial value, and start the trajectory.
3. Clicking while a trajectory is being drawn will stop the trajectory.

### Parameter Sliders

Use the sliders to set the growth rate *a*, the predation rate *b*, the predator mortality rate *c*, and the food conversion rate *d*, the harvest rate for herbivores $h_H$, and the harvest rate for predators $h_P$.

Press the mouse down on the slider knob for the parameter you want to change and drag the mouse back and forth, or click the mouse in the slider channel at the desired value for the parameter.

### Buttons

Click the mouse on the **[Clear]** button to remove all output from the graphs.
Click the mouse on the **[Pause]** button to stop a trajectory without canceling it.
Click the mouse on the **[Continue]** button to resume the motion of the paused trajectory.