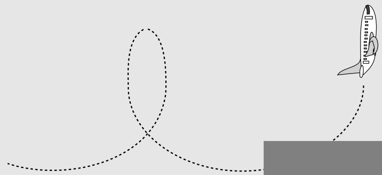


# 19 The Glider

**Tools Used in Lab 19**
The Glider

*If you've ever played with a balsa-wood glider, you know that it flies in a wavy path if you throw it gently, and it does loop-the-loops if you throw it hard. How is this all explained by phase plane analysis?*

## 1. Basic Glider Flight

The motion of a glider is approximately governed by the dimensionless equations

$$\frac{dv}{dt} = -\sin\theta - Dv^2$$
$$\frac{d\theta}{dt} = \frac{v^2 - \cos\theta}{v}$$

where $v > 0$ is the speed of the glider, and $\theta$ is the angle that its nose makes with the horizontal. The angle $\theta$ is zero when the plane is flying level with the ground.

These equations capture the important forces: $\sin\theta$ is the component of the gravitational force that tends to change the speed of the plane, and $Dv^2$ is the drag force, due to air resistance, that tends to slow the plane down; $\cos\theta$ is the orthogonal component of the gravitational force, tending to rotate the plane, and $v^2$ is the lift, tending to increase the angle, that allows the plane to fly. For further discussion of the physics behind this model, see Section 3.

Since these equations are nonlinear, there is no hope of solving them analytically, but we can *see* what will happen by plotting the path of the glider.

If you move the cursor onto the phase plane, the screen also shows the initial value of a quantity called $E$, defined as $E = v^3 - 3v\cos\theta$; you'll see why $E$ is significant when you play with **The Glider** tool.

**1.1** Find a spot in the $\theta v$ plane with a positive value of $E$ and click to start a trajectory. Sketch the resulting glider path.

Repeat for a negative value of $E$:

In Exercises **1.2–1.5**, you should assume that $D = 0$. (This is the simplest case.)

**1.2** When the glider is in steady, level flight, what are the values of $\theta$ and $v$?

**1.3** What is the most negative value of $E$ that the glider can have?

**1.4** Show that as the glider flies, the value of $E$ stays constant, even though $\theta$ and $v$ keep changing. (*Hint:* Calculate $dE/dt$ and remember to use the chain rule correctly.)

**1.5** For some initial conditions, the glider does loop-the-loops, while for others, it flies along a wavy path that is almost level. How can you tell ahead of time which type of behavior will occur?

## 2. Adding Drag

The parameter $D$ measures the strength of the drag on the plane due to air resistance. It is initially set to 0, but you can change it with a slider. For the following exercises use a drag coefficient $D > 0$.

**2.1** What is different about the flight path when $D > 0$, as compared to $D = 0$?

**2.2** If you have already learned about classification of fixed points, find and classify all the fixed points of the system.

**2.3** What is special about the value $D_c = 2\sqrt{2} \approx 2.82$ marked on the slider?

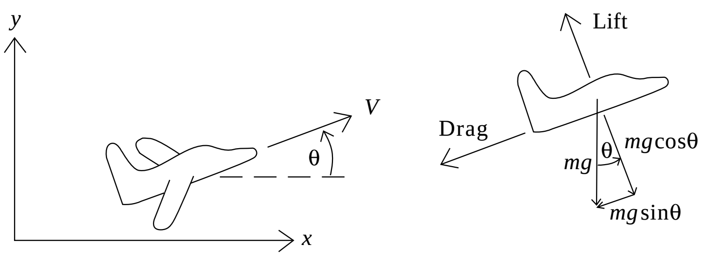
## 3. A Deeper Look at the Physics

For mathematical convenience, and to reduce the number of parameters, the equations discussed in Sections 1 and 2 were a dimensionless version of the true equations of motion. Now we consider the full equations, with all the units included, so that we can understand the physics in more detail. If we write Newton's law $F = ma$ in a moving coordinate system, with one axis tangent to the flight path, and another axis normal to it, the governing equations become

$$m\frac{dV}{dT} = -\text{drag} - mg\sin\theta$$

$$mV\frac{d\theta}{dT} = \text{lift} - mg\cos\theta$$

where $m$ is the mass of the glider, $g$ is the acceleration due to gravity, $V > 0$ is the speed of the glider, and $\theta$ is the angle that its nose makes with the horizontal.




Coordinates Forces

Let's try to understand the various forces. The terms $mg\sin\theta$ and $mg\cos\theta$ are the tangential and normal components of the gravitational force. The term $mV\frac{d\theta}{dT}$ in the normal-force equation looks strange at first, but it is simply the centripetal force. This is clear if we realize that $\frac{d\theta}{dT} = \frac{V}{R}$, where $R$ is the instantaneous radius of curvature; then $mV\frac{d\theta}{dT} = mV^2/R$, a familiar expression for the centripetal force. The drag is a tangential force due to air resistance that tends to slow the plane down, whereas the lift is the normal-force that allows the plane to fly. Experiments indicate that both the drag and lift are proportional to $V^2$, to a good approximation, so the equations become

$$m\frac{dV}{dT} = -bV^2 - mg\sin\theta$$

$$mV\frac{d\theta}{dT} = cV^2 - mg\cos\theta$$

where $b$ and $c$ are constants.

**3.1** Show that after appropriate rescaling of time and velocity, these equations can be reduced to the equations considered earlier.

**3.2** Give a physical interpretation of the characteristic velocity $V_c$.

**3.3** Given $V(T)$ and $\theta(T)$, how could you figure out the glider's flight path in space? In other words, if $x$ and $y$ are Cartesian coordinates for the horizontal and vertical location of the glider's center of mass, what are the equations that you would need to solve to find $x$ and $y$ as functions of $T$?

**3.4** For the case of no drag ($D = 0$), show that the equations for the system imply that the total energy of the glider is conserved. (Please derive this directly from the equations of motion, not just from some physical reasoning.)

**3.5** We found earlier that in the absence of drag, the quantity $E = v^3 - 3v\cos\theta$ remains constant on any given flight path. Is this equivalent to saying that energy is conserved when there is no drag?

## 4. For Further Exploration

The model discussed here is known as Lanchester's "phugoid theory," proposed in 1908—it is one of the the earliest theories for the flight path of an aircraft. For more about this model, as well as other aspects of flight dynamics, see von Mises (1959), pp. 539–545 or Miele (1962), pp. 271–273.

## References

Lanchester, F. W.. *Aerodonetics.* London, 1908.

Miele, Angelo. *Flight Mechanics.* Vol. I, *Theory of Flight Paths.* Reading, MA: Addison-Wesley, 1962.

von Mises, Richard. *Theory of Flight.* New York: Dover, 1959.

# Lab 19: Tool Instructions

## The Glider Tool

### Setting Initial Conditions

Click the mouse on the $\theta V$ plane to set the initial conditions for a trajectory.
Clicking while a trajectory is being drawn will start a new trajectory.

### Parameter Sliders

Use the slider to set the drag constant $D$.
Press the mouse down on the slider knob and drag the mouse back and forth, or click the mouse in the slider channel at the desired value for the parameter.

### Buttons

Click the **[Pause]** button to stop a trajectory without canceling it.
Click the **[Continue]** button to resume the motion of a paused trajectory.
Click the mouse on the **[Clear]** button to remove all trajectories and vectors from the graph.