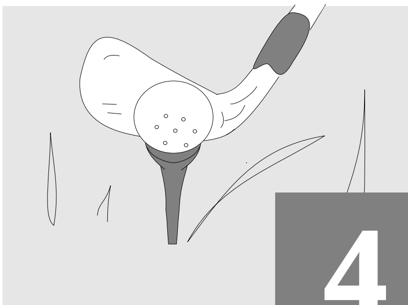


4 Mechanics: Falling Bodies and Golf

Tools Used in Lab 4

- Falling Bodies
- Golf

Suppose you're trying to hit a golf ball as far as possible. Should you launch it at a 45-degree angle? Remember, in the real world, you can't always neglect air resistance.

1. Bead Dropping Through Shampoo

In a memorable TV commercial that aired several years ago, a small bead was placed inside a transparent bottle of Prell® shampoo, and allowed to drop ever so slowly through the thick green liquid. The commercial was visually striking because the descent of the bead was so smooth, so gradual, almost hypnotic.

The bead dropping through the shampoo is an example of a body moving through a resistive medium. To take another example, we are all aware of air resistance—you can feel it by sticking your hand out the window of a moving car. Later in this lab we are going to examine the effects of air resistance on the flight path of a golf ball. But first, let's think a bit more carefully about the physics of the shampoo problem.

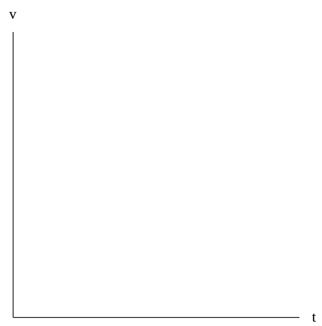
The motion of the bead is governed by Newton's law $F = ma$. If $v(t)$ denotes the bead's velocity (measured positive downward, for convenience), then Newton's law becomes

$$m\frac{dv}{dt} = mg - bv$$

where m is the mass of the bead, g is the acceleration due to gravity, and b is a measure of the viscosity, or frictional resistance, provided by the shampoo. Here, the drag force $-bv$ is taken to be proportional to velocity, as found experimentally for small objects moving slowly through a highly viscous medium.

Open the **Falling Bodies** tool. By adjusting the sliders for m and b, you can change the graph of the velocity $v(t)$. The corresponding motion of the bead is shown in the animation. The bead is assumed to be released at rest: $v(0) = 0$.

1.1 Sketch the graph of $v(t)$ for a few different choices of m and b.



1.2 The velocity appears to approach a limiting value as $t \to \infty$ (known as the **terminal velocity**). Find a formula for the terminal velocity.

1.3 Describe how the graph of $v(t)$ changes as you increase the mass m.

1.4 Assuming that the bead is released from rest, solve for $v(t)$.

1.5 Find the time required for the bead to reach 50% of the terminal velocity. Does this time increase or decrease as you increase the viscosity b? Give a physical explanation.

1.6 Notice that the graph of the height *y(t)* starts out curved, and then eventually becomes almost straight. Explain this, and find the slope of the straight part of the graph.

2. How to Hit a Golf Ball as Far as Possible

Herman Erlichson (1983) considered the question, "What angle do you need for maximum projectile range if you're not in a vacuum?" He became interested in this question because, as an avid golfer, he knew that the optimal angle for a golf tee shot is about 11 degrees from the horizontal. This is much less than the optimal launch angle of 45 degrees predicted from calculus, which is based on the simplifying (here, over-simplifying) assumption that air resistance can be neglected.

The first issue is how to model the drag force on a golf ball. There is some controversy here. At the time that Erlichson wrote his article, the available experiments indicated that the drag force on a golf ball is proportional to the velocity, over the range of typical velocities encountered in practice. More recent work (MacDonald and Hanzely, 1991) suggests that the drag force increases with the square of the velocity. As a further refinement, one should also include the aerodynamic lift on the ball, due to its backspin, but we will omit that effect here—students interested in these more realistic cases should look up the papers by Erlichson (1983) and MacDonald and Hanzely (1991).

Assuming that the drag is proportional to velocity, and neglecting lift, Newton's law yields

$$m\frac{dv_x}{dt} = -bv_x$$

$$m\frac{dv_y}{dt} = -bv_y - mg$$

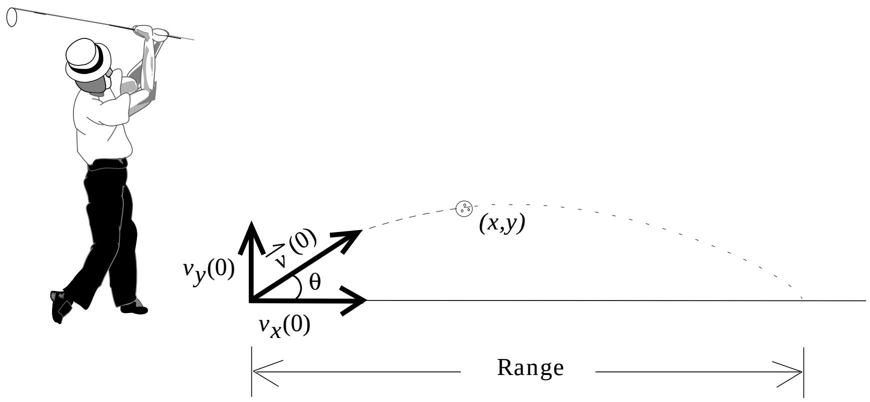
where v_x and v_y are the horizontal and vertical components of the velocity, and the convention is that y and v_y are now measured positive upward. The initial velocity is

$$v_x(0) = s\cos\theta$$

$$v_y(0) = s\sin\theta$$

where $s = 200$ ft/sec is a typical launch speed of a good drive, and θ is the launch angle. The initial position is $x = y = 0$, and the velocity and position are related by $\frac{dx}{dt} = v_x$ and $\frac{dy}{dt} = v_y$. With this information, the flight path of the golf ball is completely determined, given the values of the parameters *m, g, b, s,* θ. The values of *m, g, b, s* can be measured experimentally; thus θ is the only adjustable parameter. Furthermore, by dividing through by *m* in the equations of motion, we can see that the separate values of *b* and *m* are not important; only their ratio *b/m* matters.

2.1 Open the **Golf** tool. Using the slider for the initial angle θ, find the angle that maximizes the range of the shot, assuming that $b/m = 0.25\ \text{sec}^{-1}$ and the initial speed is 200 ft/sec. What is the approximate maximum range?



2.2 The experimentally observed optimum angle is about 11 degrees, much less than the answer you should have found for the previous question. What simplifications in the model might be responsible for this discrepancy?

2.3 The predicted flight paths are qualitatively different from those obtained in the absence of air resistance. What is the main difference? At which angles is this effect most pronounced?

2.4 Examine the effects of changing the drag parameter b. If you increase b, how does the optimal angle change? What happens to the maximum range?

2.5 To check that the tool is working correctly, examine the case where $b = 0$. What do you predict for the maximum range? Does this agree with the curves shown by the tool?

2.6 If you wanted to maximize the amount of time that the ball spends in the air, how would you set the sliders for the launch angle, the initial speed, and the air resistance?

2.7 The graph in the lower left panel shows the horizontal velocity x', the vertical velocity y', and the speed $s = \sqrt{(x')^2 + (y')^2}$, all as functions of time. The picture suggests that the graph of $s(t)$ intersects the graph of $x'(t)$ at exactly one point. Give a mathematical argument for why this must be true. In other words, explain why the two curves must intersect once, but cannot intersect more than once.

2.8 Prove that at the point where the graphs of $s(t)$ and $x'(t)$ intersect, the two curves must be tangent to each other.

2.9 Now look at the graph of the height y vs. t. Does it take longer for the ball to go up or down? In other words, which takes more time: the rise to the maximum height, or the fall back to the ground? Explain why the answer makes sense physically.

3. For Further Exploration

There are many other interesting applications of differential equations in connection with the physics of sports. How can a downhill ski racer move faster than a skydiver? What makes a curve-ball curve? Why does a well-thrown football keep its axis pointed along its trajectory? See Armenti (1992) for a fascinating collection of articles that address these and other questions. In many cases, the answers are tentative and controversial; there are opportunities for creative investigations here!

A good class project could involve thinking about the problem faced by the designers of the Prell shampoo commercial. You want the bead to drop so slowly that it takes almost the whole commercial—about 30 seconds—to reach the bottom. What should be the mass of the bead?

To solve this, you'll need to do some experiments with Prell. What measurements do you need to perform to determine the value of the viscosity parameter b? About how tall is a bottle of Prell shampoo? Are there any other measurements you need to make?

For more on the specific subject of projectiles and falling bodies, along with some ideas for class projects, see Minton (1994) and Gruzka (1994).

References

Armenti, Angelo Jr., ed. *The Physics of Sports.* New York: American Institute of Physics, 1992.

Erlichson, Herman. "Maximum Projectile Range with Drag and Lift, with Particular Application to Golf." *American Journal of Physics* 51: 357–362 (1983).

Gruzka, Thomas. "A Balloon Experiment in the Classroom." *College Mathematics Journal* 25: 442–444 (1994).

MacDonald, William M. and S. Hanzely. "The Physics of the Drive in Golf." *American Journal of Physics* 59: 213 (1991), and see the references cited therein.

Minton, Ronald. "A Progression of Projectiles: Examples from Sports." *College Mathematics Journal* 25: 436–442 (1994).

Lab 4: Tool Instructions

Falling Bodies Tool

Setting Parameters and Initial Conditions

Use the mouse to select the desired parameters and initial conditions by clicking and dragging the two sliders at the upper right (*m, b*).

Use the mouse to select the initial height of the object by moving the mouse over the bar area, which is located to the left of the first illustration. Click the mouse in the bar area to select the desired height of the object.

Other Buttons

Click the mouse on the **[Clear]** button to remove all trajectories from the graphing plane.

Golf Tool

Setting Parameters and Initial Conditions

Use the mouse to select the desired parameters and initial conditions by clicking and dragging the three sliders at the lower right (Initial Angle, Initial Speed, and Air Resistance).

Swing Button

Click the mouse on the **[Swing]** button or the *xy* plane after setting the initial conditions to see the output.

Other Buttons

Click the mouse on the **[Clear]** button to remove all trajectories from all graphing planes.