1 Newton's Law of Cooling

Tools Used in Lab 1

Newton's Law of Cooling: Curve Fitting

Newton's Law of Cooling: Cooling Rate

A cup of coffee cooling on the counter, a cake warming in the oven, and a body found in the chill autumn weather . . . are these the ingredients for a murder mystery to read by the fire or a case for that most famous detective of natural phenomena, Sir Isaac Newton? We use a first-order linear differential equation formulated by Newton to predict the temperatures of objects introduced into media with known ambient temperatures.

1. A Basic Differential Equation

A simple method to model the cooling or heating of an object placed in a constant ambient temperature is to say that the time rate of change in temperature is proportional to the difference between the temperature A of the surrounding medium (the ambient temperature) and the temperature T of the object:

$$\frac{dT}{dt} = k\big(A - T(t)\big) \quad \text{for } k > 0. \qquad (1)$$

This equation is called **Newton's Law of Cooling and Heating.** In this model Newton assumed that the heat transfer between the object and the surrounding environment is not sufficient to affect noticeably the ambient temperature. To investigate this model, open the **Newton's Law of Cooling: Curve Fitting** tool. For convenience, we will denote the time rate of change in temperature by $\dot{T} \equiv \frac{dT}{dt}$.

1.1 If $T > A$, is the object cooling or heating? Is $\frac{dT}{dt}$ positive, negative, or zero when $T > A$?

1.2 For what value of T is $\frac{dT}{dt} = 0$? This value is called the **equilibrium** value.

1.3 If $T(0) = T_0$, solve equation (1) analytically to show that $T = A + (T_0 - A)e^{-kt}$.

2. Graphical Representations

Graphical information can be displayed in several formats. To familiarize yourself with some types of graphical displays, open the **Newton's Law of Cooling: Cooling Rate** tool**.** To set an initial temperature, click the mouse on a point in the upper-left graphical window. Note that the other graphical windows also respond to this initial setting. Experiment with various values for the constant of proportionality *k* and the ambient temperature *A*.

2.1 What do you notice about the behavior of the curve on the $\dot{T}$ vs. *t* graph (in the upper-right corner)? What happens to $\dot{T}$ as *t* becomes large? Vary *k* and *A* on the sliders. Is the long-term behavior of $\dot{T}$ always the same?

2.2 Look at the graph of $A - T$ vs. $\dot{T}$. How do you interpret the straight-line graph? What does the slope of the line denote?

3. Real-World Connections

3.1 Open the **Newton's Law of Cooling: Curve Fitting** tool and select the **[Show Coffee Data]** button. Data collected from H. Hohn's cup of coffee are displayed on the graph as data points. By carefully fitting the curve to the data, determine the appropriate values for *A*, *k*, and T_0. Record them below.

$A =$
$k =$
$T_0 =$

3.2 Use the solution of Equation (1) that you found in Exercise **1.3** to determine the time when the coffee is 180 degrees. Is this time consistent with your graph?

The general solution for Equation (1) is $T = A + Ce^{-kt}$. If the ambient temperature *A* is given, then two data points are required to determine the constants *C* and *k*.

3.3 Coroners use several methods to determine time of death. If Equation (1) were used, measurements of the temperature at two different times would be required to establish k and the constant of integration. Suppose this were the only method used to determine time of death in a case where the time of death was the crucial element in the prosecution's case. How would you, as the scientific consultant, help the defense cast doubt on this estimate? Think carefully about the assumptions of the model!

Lab 1: Tool Instructions

Newton's Law of Cooling: Curve Fitting Tool

Parameter Sliders

Use the slider to change the values for the parameters T_0, A, and k.

Press the mouse down on the slider knob for the parameter you want to change and drag the mouse up and down, or click the mouse in the slider channel at the desired value for the parameter.

Buttons

Click the mouse on the **[Show Coffee Data]** button to show the coffee data on the graph.
Click the mouse on the **[Hide Coffee Data]** button to hide the coffee data.

Newton's Law of Cooling: Cooling Rate Tool

Setting Initial Conditions

Click the mouse on the tT graphing plane on the top left (labeled T) to set the initial conditions for a trajectory.
Clicking in the plane while a trajectory is being drawn will stop the trajectory and start a new one.

Parameter Sliders

Use the sliders to change the values for the parameters A and k.

Press the mouse down on the slider knob for the parameter you want to change and drag the mouse left or right, or click the mouse in the slider channel at the desired value for the parameter.

Buttons

Click the mouse on the **[Clear]** button to remove all trajectories from the graph.