

The Hog Game

Note: This exercise goes with Section 7.5 of Finite Mathematics. Part e can also be used with Section 5.2 of Calculus with Applications.

In the hog game, each player states the number of dice he or she would like to roll. The player then rolls that many dice. If a 1 comes up on any die, the player's score is 0. Otherwise, the player's score is the sum of the numbers rolled.*

- a. Find the expected value of the player's score when the player rolls one die.
- b. Find the expected value of the player's score when the player rolls two dice.
- c. Verify that the average nonzero score on a single die is 4, so that if a player rolls n dice, the average score over all rolls that do not result in a score of 0 is $4n$.
- d. Verify that if a player rolls n dice, there are 5^n possible ways to get a nonzero score, and 6^n possible ways to roll the dice. Explain why the expected value of the player's score when the player rolls n dice is then

$$E(x) = \frac{5^n \times 4n}{6^n}.$$

Confirm that your answers to parts a and b agree with this formula.

- e. Find the number of dice that maximizes the player's expected score. What is the maximum expected score? (*Hint: Make a table of $E(x)$ for various values of x . Students of calculus can also take the derivative of $E(x)$ with respect to n and use the first derivative test.*)

Answers can be found on the next page.

* James Bohan and John Shultz, "Revisiting and Extending the Hog Game," *The Mathematics Teacher*, Vol. 89, No. 9, Dec. 1996, pp. 728-733.

Answers to **The Hog Game**

a. $10/3$

b. $50/9$

e. Rolling 5 or 6 dice gives a maximum expected score of $15625/1944 \approx 8.0376$.