Blood Volume

Note: Parts a and b go with Chapter 9 of Calculus with Applications. Parts c, d, f, and j go with Chapter 2, parts e and h go with Chapter 4, and parts g and i goes with Chapter 5.

Numerous formulas have been proposed by researchers to estimate the average red blood cell volume and plasma volume of a person, which varies with a person's surface area.

a. The formula to calculate the surface area of a person used by the researchers quoted below is

$$S = .007184W^{.425}h^{.725},$$

where S is the surface area in square meters, W is the weight in kg, and h is the height in centimeters. Use this formula to calculate
i) your own surface area, or that of someone you know;
ii) the surface area of St. Louis Cardinals home run champion Mark McGwire, who is 6 ft 5 in tall and weighs 250 lbs.
(*Hint:* 1 inch corresponds to 2.54 centimeters; 1 pound corresponds to .4536 kilograms.)

b. Find and interpret $\partial S/\partial W$ and $\partial S/\partial h$ for a person who is 185 cm tall (6 ft 1 in) and weighs 79.4 kg (175 lbs).

c. A formula proposed by Hurley* for the red cell volume (RCV) in milliliters for males is

$$RCV = 1486S^2 - 4106S + 4514,$$

where S is the surface area in square meters. A formula given by Pearson et al.** is

$$RCV = 1486S - 825.$$

Verify that these two formulas never give the same answer
i) using the quadratic formula;

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* Peter J. Hurley, "Red cell and plasma volumes in normal adults," *Journal of Nuclear Medicine,* Vol. 16, 1975, pp. 46-52.

** T. C. Pearson et al., "Interpretation of measured red cell mass and plasma volume in adults," *British Journal of Haematology,* Vol. 89, 1995, pp. 748-756.

ii) using a graphing calculator.

d. Find the value of S for which the RCV values given by the two formulas in part c are the closest. Then find the value of RCV that each formula gives for this value of S.

e. For the value of S found in part d, find the rate of change of RCV with respect to S for both formulas given in part c. What does this number represent?

f. The formula for plasma volume for males given by Hurley is

$$PV = 995e^{.6085S},$$

while the formula given by Pearson et al. is

$$PV = 1578S,$$

where PV is measured in milliliters and S in square meters. Use a graphing calculator to verify that these two formulas never give the same answer.

g. Find the value of S for which the PV values given by the two formulas in part f are the closest. Then find the value of PV that each formula gives for this value of S.

h. For the value of S found in part g, find the rate of change of PV with respect to S for both formulas given in part f. What does this number represent?

i. Notice in parts e and h that both formulas give the same instantaneous rate of change at the value of S for which the function values are closest. Prove that if two functions f and g are differentiable and never cross but are closest together when $x = x_0$, then $f'(x_0) = g'(x_0)$.

j. The formula for plasma volume for females given by Hurley is

$$PV = 1278S^{.1.289},$$

while the formula given by Pearson et al. is

$$PV = 1395S,$$

where PV is measured in milliliters and S in square meters. Find the values of S for which these two formulas give the same answer. What is the predicted plasma volume at each of these values of S?

Answers can be found on the next page.

Answers to **Blood Volume**

a. i) Typical values for adults lie between about 1.3 and 2.6 square meters. ii) 2.5 square meters.

b. .011; the surface area for people of this height and weight increases approximately .011 square meters for each additional 1 kg in weight. .0080; the surface area for people of this height and weight increases approximately .0080 square meters for each additional 1 cm in height.

d. 1.88 square meters; 2049 ml (Hurley); 1971 ml (Pearson et al.)

e. 1486 ml per square meter; for people with 1.88 square meters of surface area, the red cell volume increases approximately 1486 ml for each additional square meter of surface area.

g. 1.57 square meters; 2593 ml (Hurley); 2484 (Pearson et al.)

h. 1578 ml per square meter; for people with 1.57 square meters of surface area, the red cell volume increases approximately 1578 ml for each additional square meter of surface area.

j. 0 and 1.35 square meters; 0 ml and 1889 ml